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KEY TO
ALGEBRA MADE EASY, VOL. I.
(MATRICULATION ALGEBRA.)

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KEY

10

ALGEBRA MADE EASY, VOL. I.

Exercise 1

1 1 maund and 25 seers = 65 seers Hence it contains the unit of weight (1 seer) 65 times, and the required measure is 65

2 65 seers contains the unit of weight (5 seers) 13 times, and the required measure is 13

3 300 miles contains the unit of distance 25 times, the required unit is $\frac{300 \text{ miles}}{25}$, or 12 miles

4 300 miles contains the unit of distance 40 times, the required unit is $\frac{300 \text{ miles}}{40}$, or 7½ miles

5 Rs 400 contains the unit of money 16 times, the unit is $\frac{400 \text{ Rupees}}{16}$, or Rs 25 Rs 225 contains the unit of money (Rs 25) 9 times, and the required measure is 9

6 8 ft 4 in = 100 inches, and as it contains the unit of length 25 times, the unit is $\frac{100 \text{ inches}}{25}$, or 4 inches 4 ft being equal to 48 inches, it contains the unit of length (4 in) 12 times, the required measure is 12

7 2 hrs 15 mts, i.e. 135 mts, contains the unit of time 3 times, the required unit of time is $\frac{135 \text{ mts}}{3}$, or 45 mts

8 The required time contains the unit of time (15 seconds) 60 times, it is (15 seconds) $\times 60$, or 15 minutes

9 2½ cwt = 1½ cwt = (1½ $\times 4 \times 28$) lbs = 240 lbs, the number of times that 7½ pounds is contained in 240 lbs is $\frac{240}{7½}$, or 32

10 (a) $9 \text{ sq in} = \frac{9}{144} \text{ sq ft} = \frac{1}{16} \text{ sq ft}$, and $\frac{1}{16} \text{ sq ft} = (4 \text{ sq ft}) \times \frac{1}{64}$, the required number is $\frac{1}{64}$

(b) $9 \text{ sq yds} = 81 \text{ sq ft}$, and $81 \text{ sq ft} = (4 \text{ sq ft}) \times 20\frac{1}{4}$, the required number is $20\frac{1}{4}$

11 125 sq ft is $8\frac{1}{2}$ times the unit of area, the unit of area is $(125 \text{ sq ft}) \div 8\frac{1}{2}$, or 15 sq ft . Hence 3 times the unit of area = 45 sq ft

12 The unit of money = $(\text{£}3 \text{ } 7\text{s } 6\text{d}) - 9 = 7\text{s } 6\text{d}$

13 The required measure = $(\text{£}7 \text{ } 13\text{s } 4\text{d}) - (7\text{s } 8\text{d}) - (\text{£}18 \text{ } 10\text{d}) - (92\text{d}) = 20$

14 The required measure = $(\text{Rs } 25 \text{ } 10\text{a } 3\text{p}) - (\text{Rs } 2 \text{ } 13\text{a } 7\text{p}) = (\text{Rs } 23\text{p}) - (\text{Rs } 547\text{p}) = 9$

15 The required measure = $(16 \text{ mds } 12\frac{1}{2} \text{ seers}) - (23 \text{ seers } 5 \text{ chattracks}) = (2611 \times 16 \text{ ch}) - (4 \times 373 \text{ ch}) = 28$

16 The old unit = $(\text{Rs } 20 \text{ } 10\text{a}) - 5\frac{1}{2} = (\text{Rs } 11 \text{ } 5\text{a}) - 11 = \text{Rs } 3 \text{ } 12\text{a}$ and the new unit = $(\text{Rs } 3 \text{ } 12\text{a}) \times 3 = \text{Rs } 11 \text{ } 4\text{a}$. Hence the required measure = $(\text{Rs } 45) - (\text{Rs } 11 \text{ } 4\text{a}) = 4$

17 The old unit = $(19 \text{ c } 1 \text{ } 2 \text{ qr}) - 273 = (18 \text{ qrs } 1) - 273 = 2 \text{ qr}$ and the new unit = $(2 \text{ qr}) \times \frac{1}{4} = \frac{1}{2} \text{ qr}$. Hence the required measure = $(1 \text{ ton}) - (\frac{1}{2} \text{ qr}) = (80 \text{ qrs}) - (\frac{1}{2} \text{ qr}) = 4480$

18 The old unit = $(39 \text{ yds } 2 \text{ ft}) - 84 = (119 \text{ ft}) - 84 = \frac{1}{2} \text{ ft}$ and the new unit = $(\frac{1}{2} \text{ ft}) \times \frac{1}{2} = \frac{1}{4} \text{ ft}$. Hence the required number = $(75 \text{ yds}) - \frac{1}{4} \text{ ft} = (225 \text{ ft} - \frac{1}{4} \text{ ft}) = 900$

19 The old unit = $(26 \text{ days } 10 \text{ hrs } 26 \text{ mts}) - 120 = (634 \text{ hrs } 26 \text{ mts}) - 120 = 5 \text{ hrs } 17 \text{ mts } 13 \text{ secs}$, and the new unit = $(5 \text{ hrs } 17 \text{ mts } 13 \text{ secs}) - 47 \text{ mts } 13 \text{ secs} = 4 \text{ hrs } 30 \text{ mts}$. Hence the required number = $(366 \text{ days}) - (4\frac{1}{2} \text{ hrs}) = (366 \times 24 \text{ hrs}) - (4\frac{1}{2} \text{ hrs}) = (366 \times 24 \times 2) - 9 = 122 \times 8 \times 2 = 1952$

20 The new unit = $(5 \text{ hrs } 17 \text{ mts } 13 \text{ secs}) - (6 \text{ hrs } 4 \text{ mts } 47 \text{ secs}) = 12 \text{ hrs } 12 \text{ mts} = 12\frac{1}{2} \text{ hrs}$. Hence the required number = $(366 \times 24 \text{ hrs}) - (12\frac{1}{2} \text{ hrs}) = (366 \times 24 \times 5) - 61 = 6 \times 24 \times 5 = 720$

Exercise 2

1 $a + b \times c = 2 + 1 \times 8 = 2 + 8 = 10$

2 $c - a \times b = 8 - 2 \times 4 = 8 - 8 = 0$

3 $c - b \times a = 8 - 4 \times 2 = 8 - 8 = 0$

4 $c - b \times a = 8 - (4 \times 2) = 8 - 8 = 0$

- 5 $c - 3 \times a = 8 - 3 \times 2 = 8 \times \frac{1}{2} \times 2 = 1^1 = 5\frac{1}{2}$
- 6 $c - 3a = 8 - (3 \times 2) = 8 \times \frac{1}{1 \times 2} = \frac{1}{2} = 1\frac{1}{2}$
- 7 $c - b - a = 8 - 4 - 2 = 8 - 2 = 6$
- 8 $a + c - b = 2 + 8 - 4 = 2 + 2 = 4$
- 9 $3c - 4b + 2a = (3 \times 8) - (4 \times 4) + (2 \times 2) = 24 - 16 + 4 = 12$
- 10 $c - b - a + c - b = 8 - 4 - 2 + 8 - 4 = 8 - 2 + 2 = 8$
- 11 $c - b - 2 \times a = 8 - 4 - 2 \times 2 = 8 \times \frac{1}{4} \times \frac{1}{2} \times 2 = 2$
- 12 $c - b - 2a = 8 - 4 - (2 \times 2) = 8 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{2}$
- 13 $5c - 2b = (5 \times 8) - (2 \times 4) = 5 \times 8 \times \frac{1}{2 \times 4} = 5$
- 14 $5 \times c - 2 \times b = 5 \times 8 - 2 \times 4 = 5 \times 8 \times \frac{1}{2} \times 4 = 80$
- 15 $4ab - c - 4 \times a + b - 2a = (4 \times 2 \times 4) - 8 - 4 \times 2 + 4 - (2 \times 2)$
 $= 32 - 8 \times \frac{1}{4} \times 2 + 4 \times \frac{1}{2} = 32 - 4 + 1 = 29$
- 16 $80 - b \times ca + 80 - bc \times a = 80 - 4 \times (8 \times 2) + 80 - (4 \times 8) \times 2$
 $= 80 \times \frac{1}{4} \times 8 \times 2 + 80 \times \frac{1}{4 \times 8} \times 2 = 320 + 5 = 325$
- 17 $64 - c \times b \times a - 64 - cba = 64 - 8 \times 4 \times 2 - 64 - (8 \times 4 \times 2)$
 $= 64 \times \frac{1}{8} \times 4 \times 2 - 64 \times \frac{1}{8 \times 4 \times 2} = 64 - 1 = 63$
- 18 $3bc - 16a + 5c - 16 \times a - c - 2b \times b - a \times 4$
 $= (3 \times 4 \times 8) - (16 \times 2) + (5 \times 8) - 16 \times 2 - 8 - (2 \times 4) \times 4 - 2 \times 4$
 $= 3 \times 4 \times 8 \times \frac{1}{16 \times 2} + 5 \times 8 \times \frac{1}{16} \times 2 - 8 \times \frac{1}{2 \times 4} \times 4 \times \frac{1}{2} \times 4$
 $= 3 + 5 - 8 = 8 - 8 = 0$
- 19 $10c - ab \times 2 + 32b - 2ac + 15c - 3b \times c - 5 = (10 \times 8) - (2 \times 4) \times 2$
 $+ (32 \times 4) - (2 \times 2 \times 8) + (15 \times 8) - (3 \times 4) \times 8 - 5 = 10 \times 8 \times \frac{1}{2 \times 4} \times 2 + 32 \times$
 $4 \times \frac{1}{2 \times 8} + 15 \times 8 \times \frac{1}{3 \times 4} \times 8 \times \frac{1}{2} = 20 + 4 + 16 = 40$
- 20 $48c - b - a \times 6 - 4b - 3c - 2b - 4 \times 3 - a \times 8 + 6a - c - 2 \times b - 3 \times 5$
 $= (48 \times 8) - 4 - 2 \times 6 - (4 \times 4) - (3 \times 8) - (2 \times 4) - 4 \times 3 - 2 \times 8 + (6 \times 2) -$
 $8 - 2 \times 4 - 3 \times 5 = 48 \times 8 \times \frac{1}{4} \times \frac{1}{2} \times 6 \times \frac{1}{4 \times 4} - 3 \times 8 \times \frac{1}{2 \times 4} \times \frac{1}{4} \times 3 \times \frac{1}{2} \times 8 +$
 $6 \times 2 \times \frac{1}{2} \times \frac{1}{2} \times 4 \times \frac{1}{2} \times 5 = 18 - 9 + 5 = 14$
- 21 $8m - 3p - mn + q \times 3 + 5s - 2 \times p = (8 \times 2) - (3 \times 4) - (2 \times 3) + 0$
 $+ (5 \times 10) - 2 \times 4 = 16 - 12 \times \frac{1}{2} + 50 \times \frac{1}{2} \times 4 = 16 - 2 + 100 = 114$
- 22 $s \times 6 - 5m \times 8p - 16n = 10 \times 6 - (5 \times 2) \times (8 \times 4) - (16 \times 3) = 10 \times 6$
 $\times \frac{1}{2 \times 2} \times 8 \times 4 \times \frac{1}{16 \times 3} = 4$
- 23 $24 - 3p \times 4s - 5t \times 14m = 24 - (3 \times 4) \times (4 \times 10) - (5 \times 7)$
 $\times (14 \times 2) = 24 \times \frac{1}{3 \times 4} \times 4 \times 10 \times \frac{1}{5 \times 7} \times 14 \times 2 = 64$

11 The given expression

$$\begin{aligned}
 &= 40 + 66 \times 8 \times 8 - 21 \times 8 + 8 \times 8 \times 8 \times 8 \times 8 - 65 \times 8 \times 8 \times 8 \times 8 \\
 &= 40 + 4224 - 168 + 262144 - 266240 = 4096 + 262144 - 266240 \\
 &= 266240 - 266240 = 0
 \end{aligned}$$

12 The given expression

$$\begin{aligned}
 &= 8 \times (75)^4 + 6 \times (75)^3 + 11 \times (75)^2 + 13 \times (75) + 29 \\
 &= 8 \times 75 \times 75 \times 75 \times 75 + 6 \times 75 \times 75 \times 75 \\
 &\quad + 11 \times 75 \times 75 + 13 \times 75 + 29 = 6 \times 75 \times 75 \times 75 \\
 &\quad + 6 \times 75 \times 75 \times 75 + 8 \times 75 \times 75 + 9 \times 75 + 29 \\
 &= 4 \times 5 \times 5625 + 4 \times 5 \times 5625 + 6 \times 1875 + 38 \times 75 \\
 &= 2 \times 53125 + 2 \times 53125 + 44 \times 9375 = 50
 \end{aligned}$$

$$13 \quad m^2 = \frac{16}{49}, \quad m^3 = \frac{64}{343}, \quad m^4 = \frac{256}{2401}, \quad m^5 = \frac{1024}{16807}$$

$$\begin{aligned}
 &\text{Hence } 35m^5 - 4m^3 + 7m + 15m^2 - 34m^4 - 3 \\
 &= \frac{5 \times 1024}{2401} - \frac{4 \times 16}{49} + 4 + \frac{15 \times 64}{343} - \frac{31 \times 256}{2401} - 3 \\
 &= \frac{5 \times 1024 - 4 \times 16 \times 49 + 15 \times 64 \times 7 - 31 \times 256}{2401} + 4 - 3 \\
 &= \frac{5120 - 3136 + 6720 - 7904}{2401} + 1 = 0 + 1 = 1
 \end{aligned}$$

14 The given expression

$$\begin{aligned}
 &= 25 \times (26)^8 - 27 + 20 \times 26 + 78 \times (26)^5 - 199 \times (26)^0 \\
 &= 25 \times 208827064576 - 27 + 52 + 78 \times 11881376 \\
 &\quad - 199 \times 308915776 \\
 &= 52206766144 - 27 + 52 + 926747328 - 61474239424 \\
 &= 52231766144 + 926747328 - 61474239424 \\
 &= 61499239424 - 61474239424 = 25
 \end{aligned}$$

15 The given expression

$$\begin{aligned}
 &= 50 \times (34)^7 - 51 \times (34)^4 + 35 \times (34) - 563 \times (34)^5 - 19 \\
 &= 50 \times 52523350144 - 51 \times 1356336 \\
 &\quad + 35 \times 34 - 563 \times 45435424 - 19 \\
 &= 26261675072 - 68153136 + 119 - 25580143712 - 19 \\
 &= 25580143712 + 119 - 25580143712 - 19 = 100
 \end{aligned}$$

16 The given expression

$$\begin{aligned}
 &= 64 \times \left(\frac{11}{8}\right)^{10} - 55 \times \left(\frac{11}{8}\right)^8 + 32 \times \left(\frac{11}{8}\right)^6 - 121 \times \left(\frac{11}{8}\right)^4 \\
 &\quad + 64 \times \left(\frac{11}{8}\right)^2 - 4 \times \left(\frac{11}{8}\right)^0 + 79 \\
 &= \frac{11^{10}}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} - \frac{55 \times (11)^4}{8 \times 8 \times 8 \times 8} + \frac{(11)^6}{8 \times 8 \times 8 \times 8} \\
 &\quad - \frac{121 \times (11)^2}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} + (11)^2 - \frac{(11)^0}{2 \times 8 \times 8 \times 8 \times 8} + 79 \\
 &= \frac{11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} - \frac{55 \times 14641}{8 \times 8 \times 8 \times 8} \\
 &\quad + \frac{1771561}{2 \times 8 \times 8 \times 8 \times 8} - \frac{11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} \\
 &\quad + 121 - \frac{161051}{2 \times 8 \times 8 \times 8 \times 8} + 79 \\
 &= 200 - \frac{805255}{8 \times 8 \times 8 \times 8} + \frac{1771561 - 161051}{2 \times 8 \times 8 \times 8 \times 8} \\
 &= 200 - \frac{805255}{8 \times 8 \times 8 \times 8} + \frac{805255}{8 \times 8 \times 8 \times 8} = 200
 \end{aligned}$$

$$\begin{aligned}
 17 \quad a^3 + b^3 + c^3 - 3abc &= (24)^3 + (27)^3 + (29)^3 - 3 \times 24 \times 27 \times 29 \\
 &= 13824 + 19683 + 24389 - 56376 \\
 &= 57896 - 56376 = 1520
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= (3625)^3 + (4625)^3 + (5625)^3 - 3 \times 3625 \times 4625 \times 5625 \\
 &= 47634765625 + 98931640625 + 177978515625 - \\
 &\quad 282919921875 = 324544921875 - 282919921875 \\
 &= 41625
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= \left(\frac{311}{7}\right)^3 + \left(\frac{360}{7}\right)^3 + \left(\frac{409}{7}\right)^3 - 3 \frac{311}{7} \frac{360}{7} \frac{409}{7} \\
 &= \frac{30080231}{343} + \frac{46656000}{343} + \frac{68417929}{343} - \frac{137374920}{343} \\
 &= \frac{145154160 - 137374920}{343} = \frac{7779240}{343} = 22680
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= (1659)^3 + (1667)^3 + (1674)^3 - 3 \times 1659 \times 1667 \times 1674 \\
 &= 4566034179 + 4632407963 + 4691010024 - 13888607166 \\
 &= 13889452166 - 13888607166 = 845000
 \end{aligned}$$

Exercise 4.

- 1 $\sqrt[4]{b\bar{a}d} = \sqrt[4]{2 \cdot 4 \cdot 8} = \sqrt[4]{64} = 4$
- 2 $\sqrt[3]{p\bar{a}d} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{32} = 2$
- 3 $\sqrt[4]{pn} = \sqrt[4]{8 \times 3} = 2 \times 3 = 6$
- 4 $6 \sqrt[4]{(c\bar{a})} = 6 \sqrt[4]{9 \cdot 3} = 6 \sqrt[4]{3 \times 3 \times 3 \times 3 \times 3}$
 $= 6 \times 3 = 18$
- 5 $4 \sqrt[4]{4\bar{a}} = 4 \sqrt[4]{4 \cdot 4} = 4 \sqrt[4]{16} = 4 \times 2 = 8$
- 6 $4 \sqrt[4]{4^4} = 4 \times 4 = 16$
- 7 $2 \sqrt[4]{4p^2} = 2 \times 2p = 2 \times 2 \cdot 8 = 32$
- 8 $2 \sqrt[4]{4p^8} = 2 \times 2p^2 = 2 \times 2 \cdot 8^2 = 4 \times 64 = 256$
- 9 $a + b\sqrt{c} = 5 + 2\sqrt{9} = 5 + 2 \cdot 3 = 5 + 6 = 11$
- 10 $\bar{a} + \bar{b}\sqrt{c} = 5 + 2\sqrt{9} = 7 \times 3 = 21$
- 11 $3\sqrt{c} + p = 3\sqrt{1} + 8 = 3 \cdot 1 + 8 = 3 + 8 = 11$
- 12 $3\sqrt{c+p} = 3\sqrt{1+8} = 3\sqrt{9} = 3 \cdot 3 = 9$
- 13 $\sqrt[3]{3(c+p)} = \sqrt[3]{3(1+8)} = \sqrt[3]{27} = 3$
- 14 $3 \sqrt[3]{8(n+3p)} = 3 \sqrt[3]{8(3+24)} = 3 \sqrt[3]{27} = 3 \cdot 3 = 9$
- 15 $3 \sqrt[3]{8(n+3p)} = 3 \sqrt[3]{8(3+24)} = 3 \sqrt[3]{27} = 3 \cdot 3 = 9$
- 16 $f\sqrt[3]{m+c} = 0 + 9 = 9$
- 17 $f\sqrt[3]{m+c} = 0 \times \sqrt[3]{7+9} = 0$
- 18 $3c - (2d - b) = 27 - (8 - 2) = 27 - 6 = 21$
- 19 $3c - 2(d - b) = 27 - 2(4 - 2) = 27 - 4 = 23$
- 20 $3(c - 2d) - b = 3(9 - 8) - 2 = 3 - 2 = 1$
- 21 $(3c - 2)d - b = (27 - 2)4 - 2 = 25 \cdot 4 - 2 = 100 - 2 = 98$
- 22 $(3c - 2)\bar{d} - \bar{b} = (27 - 2)4 - 2 = 25 \cdot 2 = 50$
- 23 $3\{c - (2d - b)\} = 3\{9 - (8 - 2)\} = 3 \cdot 3 = 9$
- 24 $3(c - 2)(d - b) = 3(9 - 2)(4 - 2) = 3 \cdot 7 \cdot 2 = 42$
- 25 $7p - (n^2 - b^2) = 7 \cdot 8 - (3^2 - 2^2) = 56 - (9 - 4) = 56 - 5 = 51$
- 26 $(7p - n)^2 - b^2 = (56 - 3)^2 - 2^2 = (53)^2 - 4$
 $= 2809 - 4 = 2805$
- 27 $7p - (n^2 - b)^2 = 56 - (9 - 2)^2 = 56 - 49 = 7$
- 28 $7(p - n)^2 - b^2 = 7(8 - 3)^2 - 2^2 = 7 \cdot 25 - 4 = 175 - 4 = 171$
- 29 $\{7p - (n^2 - b)\}^2 = \{56 - (9 - 2)\}^2 = (56 - 7)^2 = (49)^2 = 2401$

- 30 $\sqrt[3]{p+3c+4d}(c+n)^3 = \sqrt[3]{8+3+16}(1+3)^3$
 $= \sqrt[3]{27}(4)^3 = 3 \ 64 = 192$
- 31 $\sqrt[3]{p+3c+4d}(c+n)^3 = \sqrt[3]{8+3+16}(1+3)^3 = 2+3+16 \ 64$
 $= 5+1024 = 1029$
- 32 $\sqrt[3]{p+3c+4d}(c+n)^3 = \sqrt[3]{8+(3+16)(1+3)^3} = 2+19 \ 64$
 $= 2+1216 = 1218.$
- 33 $\sqrt[3]{p+(3c+4d)c+n^3} = \sqrt[3]{8+(3+16)+3^3} = 2+19+27 = 48$
- 34 $\sqrt[3]{p+3\{(c+4)d+c+n^3\}}$
 $= \sqrt[3]{8+3\{(1+4)4+3^3\}} = 2+3(5 \ 4+27)$
 $= 2+3(20+27) = 2+3 \ 47 = 2+141 = 143$
- 35 $a(1+j)^2(a-\overline{c-s})^3 = 6(2+3)^2(6-\overline{5-4})^3$
 $= 6 \ 5^2 \ 5^3 = 6 \ 25 \ 125 = 150 \ 125 = 18750.$
- 36 $4\{n-a(d-\overline{a+p})\} \sim 4\{\overline{n-a(d-a)+p}\}$
 $= 4\{9-6(8-\overline{6+1})\} \sim 4\{\overline{9-6(8-6)+1}\}$
 $= 4\{9-6(8-7)\} \sim 4\{3 \ 2+1\}$
 $= 4(9-6) \sim 4(6+1) = 12 \sim 28 = 16$
- 37 $5\{c+\overline{r^2+j(n-d-s)}\} \sim 5\{\overline{(c+r^2+j)n-d-s}\}$
 $= 5\{5+4+3(9-\overline{8-4})\} \sim 5\{\overline{(5+4+3)9-8-4}\}$
 $= 5(5+7(9-4)) \sim 5\{(9+3) \ 1-4\}$
 $= 5(5+7 \ 5) \sim 5(12 \ 1-4)$
 $= 5(5+35) \sim 5(12-4)$
 $= 5 \ 40 \sim 40 = 200 - 40 = 160$
- 38 $[r+j^2\{ap-s(c-\overline{a-r})\}] \sim [\overline{1+j^2\{(ap-s)c-a\}}-1]$
 $= [2+3^2\{6-4(5-\overline{6-2})\}] \sim [\overline{2+3^2\{(6-4)5-6\}}-2]$
 $= [2+9\{6-4(5-4)\}] \sim [5^2\{2 \ 5-6\}-2]$
 $= [2+9\{6-4\}] \sim [25\{10-6\}-2]$
 $= [2+9 \ 2] \sim [25 \ 4-2]$
 $= [2+18] \sim [100-2]$
 $= 20 \sim 98 = 78$

Exercise 5

- 1 A's loss = + 50 or £100
- 2 Loss of £1 is evidently represented by -1 , and a gain of £70 by -70
- 3 Income of £1 is represented by $+1$, a debt of £1 must be represented by -1 , and a debt of £100 by -1×100 or -25
- 4 A debt of £1 is represented by $+1$, in income of £1 must be represented by -1 , and in income of £800 by -1×800 or -100
- 5 A distance of 1 mile to the north is represented by $+1$, a distance of 1 mile to the South must be represented by -1 , and a distance of 150 miles to the South by -1×150 or -30
- 6 The rise (8 in) on the 1st day = $\frac{8 \text{ in}}{2 \text{ in}} = 4$
 The fall (6 in) on the 2nd day = $\frac{6 \text{ in}}{2 \text{ in}} = 3$;
 the rise on the 2nd day must be represented by -3
 The rise (10 in) on the 3rd day = $\frac{10 \text{ in}}{2 \text{ in}} = 5$
- 7 The gain in the 1st year = $\frac{\text{Rs } 30}{\text{Rs } 2} = 15$
 The loss in the 2nd year = $\frac{\text{Rs } 20}{\text{Rs } 2} = 10$, the gain in the 2nd year must be represented by -10
 The loss in the 3rd year = $\frac{\text{Rs } 40}{\text{Rs } 2} = 20$, the gain in the 3rd year must be represented by -20
 The gain in the 4th year = $\frac{\text{Rs } 60}{\text{Rs } 2} = 30$
- 8 The gain in the 1st year = 15,
 the loss in the 1st year = -15
 The loss in the 2nd year = 10,
 and the loss in the 3rd year = 20
 The gain in the 4th year = 30,
 the loss in the 4th year = -30

Exercise 6

$$1 \quad (-3) - (-7) + (-12) = -(3-7+12) = -22$$

$$2 \quad a - (-b) + (-5c) = (-5) - (-3) + \{-5(2)\} \\ = -(5+3+10) = -18$$

$$3 \quad -5-a-b = (-5) + (-6) - (-20) = -(5+6+20) \\ = -31, \text{ and } (-10) - (-31) = -(10+31) = -41$$

$$4 \quad -a - (b-c) = -5 - \{(-8) + (-6)\} = -(5+8+6) = -19$$

$$5 \quad (-a^2b^2) - (-b^2c^2) - \{-(a^2-b^2)\} \\ = (-2564) + (-169) + \{-(16-4)\} \\ = (-1024) + (-144) + (-12) \\ = -(1024+144+12) = -1180$$

$$6 \quad (-3a^2b^2) - d + e - (-20c^2) + (d+e) \\ = (-318) - (-1) - (-5) + (-209) + \{(-4) + (-5)\} \\ = (-24) + (-1) - (-5) - (-180) - (-9) \\ = -(24+1+5-180-9) = -222$$

$$7 \quad \{-a^2(b-c)\} - \{-b^2(c-a)\} - \{-c^2(a-b)\} \\ = \{-16(5-4)\} - \{-625(4-2)\} + \{-256(5-2)\} \\ = (-16) - (-1250) + (-768) \\ = -(16-1250-768) = -2034$$

$$8 \quad \{-(x^2-y^2)\} - \{-(x^2-y^2)\} - \{-(x^2-y^2)\} \\ = \{-(25-9)\} - \{-(125-27)\} + \{-(625-81)\} \\ = (-16) - (-98) + (-544) = -(16+98-544) = -658$$

$$9 \quad \{-x^2(y^2-z^2)\} - \{-y^2(z^2-x^2)\} + \{-z^2(x^2-y^2)\} \\ = \{-27(36-25)\} + \{-216(25-9)\} - \{-125(36-9)\} \\ = \{-2711\} - \{-21616\} - \{-12527\} \\ = (-297) - (-3456) + (-3375) \\ = -(297-3456-3375) \\ = -7128$$

$$10 \quad a^4 - b^4 - c^4 = 60^4 - 4^4 - 2^4 \\ = 60 \times 60 \times 60 \times 60 \times \frac{1}{2 \times 4 \times 4 \times 4} - 16 \\ = 50625 - 16 = 50609; \\ a^4 - (b^4 - c^4) = 60^4 - (256 - 16) \\ = 60 \times 60 \times 60 \times 60 \times \frac{1}{2 \times 4} = 54000$$

$$\begin{aligned}
 a^4 - b^4 \times c^4 &= 12960000 - 256 \times 16 \\
 &= 12960000 - 4096 = 12955904, \\
 \text{and } (a^4 - b^4) \times c^4 &= (12960000 - 256) \times 16 \\
 &= 12959744 \times 16 = 207355904
 \end{aligned}$$

Hence the given expression

$$\begin{aligned}
 &= (-50609) + (-54000) + (-12955904) + (-207355904) \\
 &= -(50609 + 54000 + 12955904 + 207355904) \\
 &= -220416417
 \end{aligned}$$

Exercise 7

- 1 $7 + (-4) = 7 - 4 = 3$
- 2 $8 + (-13) = 8 - 13 = -5$
- 3 $b - c + d = 13 - 25 + 8 = -12 + 8 = -4$
- 4 $3a + (-5b) + c + d + 4e$
 $= 6 - 15 + 4 - 7 + 12 = -9 - 3 + 12 = -12 + 12 = 0$
- 5 $3m - 5n + 6p + r = 12 - 30 + 12 - 8 = -18 + 4 = -14$
- 6 $2x^2y - 3y^2x - 5x^2y^2 + x^2y^4$
 $= 2 \cdot 2^2 \cdot 2 - 3 \cdot 2^2 \cdot 2 - 5 \cdot 2^2 \cdot 2^2 + 2^3 \cdot 2^4$
 $= 2 \cdot 4 \cdot 2 - 3 \cdot 8 \cdot 2 - 5 \cdot 4 \cdot 4 + 8 \cdot 16$
 $= 16 - 48 - 80 + 128 = -32 + 48 = 16$
- 7 $a^2c = 4 \cdot 3 = 12$, $ba^2 = 5 \cdot 36 = 180$,
 $cb^2 = 3 \cdot 25 = 75$, and $a^2a^2 = 4 \cdot 36 = 144$

Hence the given expression

$$\begin{aligned}
 &= (-12) + (180) + (-75) + (-144) \\
 &= -12 + 180 - 75 - 144 = 168 - 219 = -51
 \end{aligned}$$

- 8 The given expression
 $= 3^3 + 3 \cdot 9 \cdot 5 + 3 \cdot 3 \cdot 25 - 5^2$
 $= 27 - 135 + 225 - 125 = -108 + 100 = -8$
- 9 The given expression
 $= 16 - 4 \cdot 8 \cdot 4 + 6 \cdot 4 \cdot 16 - 4 \cdot 2 \cdot 64 + 256$
 $= 16 - 128 + 384 - 512 + 256$
 $= -112 - 128 + 256 = -240 + 256 = 16$

10 The given expression

$$\begin{aligned} &= 5^6 - 5\ 625\ 7 + 10\ 125\ 49 - 10\ 25\ 343 + 5\ 5\ 2401 - 7^6 \\ &= 3125 - 21875 + 61250 - 85750 + 60025 - 16807 \\ &= -18750 - 24500 + 43218 = -43250 + 43218 = -32 \end{aligned}$$

Exercise 8

- 1 $a + b + c - 2a - 3b + 4c$
 $= (a - 2a) + (b - 3b) + (4c - c) = \underline{-a - 2b + 3c}$
- 2 $5a^2 - 7b^3 + 8c^2 + 5b^2 - 7c^2 - 6a^3$
 $= (5a^2 - 6a^3) + (5b^2 - 7b^3) + (8c^2 - 7c^2) = \underline{-a^3 - 2b^3 + c^2}$
- 3 $8a - ab - 7a + 5c - 3a + 5ab$
 $= (8a - 3a - 7a) + (5ab - ab) + 5c = \underline{-2a + 4ab + 5c}$
- 4 $5mnp^3 - 6ab1 - 7c^2 - 8mnp^3 + 9c^2 + 4ab1 - 5c^2$
 $= (5mnp^3 - 8mnp^3) + (4ab1 - 6ab1) + (9c^2 - 5c^2 - 7c^2)$
 $= \underline{-3mnp^3 - 2ab1 - 3c^2}$
- 5 $-7a^2b - 5b^2c^2 + 10a^3b - 3b^2c^2 + 3df - a^3b - b^2c^2 - 5df$
 $= (-7a^2b + 10a^3b - a^3b) - (5b^2c^2 + 3b^2c^2 + b^2c^2) + (3df - 5df)$
 $= \underline{2a^3b - 9b^2c^2 - 2df}$
- 6 $8x^4y - 5xyz - 17x^4y + 20x^2y^2 - 2xyz - 35x^2y^2$
 $+ 3x^4y - 4xyz + 5x^2y^2 = (8x^4y - 17x^4y + 3x^4y)$
 $- (5xyz + 2xyz + 4xyz) + (20x^2y^2 - 35x^2y^2 + 5x^2y^2)$
 $= \underline{-6x^4y - 11xyz - 10x^2y^2}$
- 7 $-13a^3bc + 15ab^2c - 27abc^3 - 5a^2bc + 13abc^2 - 23abc^2$
 $+ 7a^2bc + 6abc^2 + 19ab^2c$
 $= (-13a^3bc - 5a^2bc + 7a^2bc) + (15ab^2c - 23ab^2c + 19ab^2c)$
 $+ (13abc^2 - 27abc^2 + 6abc^2)$
 $= \underline{-11a^3bc + 11ab^2c - 8abc^2}$
- 8 $20x^3mn - 23m^3nr + 14n^3rm - 37x^3mn - 47n^3rm$
 $+ 54m^3nr - 8x^3mn + 13n^3rm$
 $= (20x^3mn - 37x^3mn - 8x^3mn) - 23m^3nr + 54m^3nr$
 $+ (14n^3rm - 47n^3rm + 13n^3rm) + 20n^3rm$
 $= \underline{-25x^3mn + 16m^3nr}$

- 9 The required sum $= (-2 + 9 + 20) + (25 - 24 - 42)$
 $= -2 + 9 + 20 + 25 - 24 - 42 = (9 + 20 + 25) - (2 + 24 + 42)$
 $= 54 - 68 = -14$
- 10 The required sum $= (15 - 14) + (-70 - 96) + (15 + 6 - 90)$
 $= 15 - 14 - 70 - 96 + 15 + 6 - 90$
 $= (15 + 15 + 6) - (14 + 70 + 96 + 90)$
 $= 36 - 270 = -234$
- 11 The required sum
 $= (34) + (-59 + 716) + (-26 + 510 - 12) + (105 - 79)$
 $= 12 - 45 + 112 - 12 + 50 - 12 + 50 - 63 = (12 + 112 + 50 + 50)$
 $- (45 + 12 + 12 + 63) = 224 - 132 = 92$
- 12 The required sum
 $= (-4 + 9 - 16) + (-5 - 30 + 42) + (24 - 45 - 30 + 60)$
 $= -4 + 9 - 16 - 5 - 30 + 42 + 24 - 45 - 30 + 60$
 $= (9 + 42 + 24 + 60) - (4 + 16 + 5 + 30 + 45 + 30)$
 $= 135 - 130 = 5$
- 13 The required sum
 $= (-6 + 72) + (120 - 60 + 7) + (-16 - 25 + 90)$
 $+ (48 - 35 - 42 + 24) = -6 + 72 + 120 - 60 + 7 - 16 - 25$
 $+ 90 + 48 - 35 - 42 + 24 = (72 + 120 + 7 + 90 + 48 + 24)$
 $- (6 + 60 + 16 + 25 + 35 + 42) = 361 - 184 = 177$
- 14 The required sum
 $= (24 - 120) + (490 - 72 - 180) + (-80 + 243 - 810)$
 $+ (80 - 2800 + 72 - 1500) = 24 - 120 + 490 - 72 - 180$
 $- 80 + 243 - 810 + 80 - 2800 + 72 - 1500$
 $= -(24 + 490 + 243 + 80 + 72) - (120 + 72 + 180 + 80$
 $+ 810 + 2800 + 1500) = 909 - 5562 = -4653$
- 15 The required sum
 $= (3810 - 5816 - 41008) + (-13410 + 4645 - 7254) +$
 $(-51444 + 87100 + 9125) + (5649 - 710012 - 421610$
 $+ 8459) = (2430 - 2430 - 3200) + (-520 + 1280 - 700)$
 $+ (-2880 + 5600 + 1125) + (2880 - 8400 - 8640 + 1440)$
 $= -3200 + 60 + 3845 + (-12720) = (3845 + 60) -$
 $(12720 + 3260) = 3905 - 15920 = -12015$

Exercise 9

$$\begin{array}{r} 1 \quad a-2b+5c \\ -7a+3b-8c \\ \hline -6a+b-3c \end{array}$$

$$\begin{array}{r} 2 \quad -3x+5y-9z \\ +5x-3y+7z \\ \hline -2y+z \\ 2x \quad -z \end{array}$$

$$\begin{array}{r} 3 \quad x^3+3x^2-5x+4 \\ 2x^3-6x^2+7x-8 \\ -x^3+7x^2-2x+9 \\ +5x^2 \quad +2 \\ \hline 2x^3+9x^2 \quad +7 \end{array}$$

$$\begin{array}{r} 4 \quad 3a-2b+7c-8d \\ -5a \quad +2c+6d \\ +3b-10c+d \\ a-4b+c \\ +5b \quad -7d \\ \hline -a+2b \quad -8d \end{array}$$

$$\begin{array}{r} 5 \quad x^3+2xy+3y^3-x+y+2 \\ -5x^3 \quad +y^2+2x \quad -5 \\ -3xy-7y^2 \quad +3y+1 \\ 6x^2+xy \quad -x-4y+2 \\ \hline 2x^2 \quad -3y^2 \end{array}$$

$$\begin{array}{r} 6 \quad 2x^2-5xy+y^2 \\ -7x^3 \quad +4y^2-5x+2y \\ 3xy-6y^3 \quad +y-5 \\ \hline 3x-4y+3 \\ -5x^3-2xy-y^2-2x-y-2 \end{array}$$

$$\begin{array}{r} 7 \quad a^3b+abc-b^2c^2 \\ 5a^3b-3abc-12b^2c^2 \\ -4a^3b+2abc+8b^2c^2 \\ 2a^3b \quad +5b^2c^2 \\ \hline 4a^3b \end{array}$$

$$\begin{array}{r} 8 \quad m^3n^2-3mnp+2m^2n^2+6m^2n^2 \\ 5m^3n^2+7mnp-m^2n^3-10m^2n^2 \\ -5mnp+3m^2n^2+2m^2n^2 \\ -7m^3n^2 \quad -4m^2n^3+m^2n^3 \\ \hline -m^3n^2-mnp \quad -m^2n^2 \end{array}$$

$$\begin{array}{r} 9 \quad 12a^3b^2x-29b^3x^2a+37a^3a^2b+45a^2b^2x^2 \\ -18a^3b^2x+25b^3x^2a-5x^3a^2b-16a^2b^2x^2 \\ 20a^3b^2x-28b^3x^2a-23x^3a^2b+32a^2b^2x^2 \\ -14a^3b^2x+32b^3x^2a-9x^3a^2b-60a^2b^2x^2 \\ \hline a^2b^2x^2 \end{array}$$

$$\begin{array}{r} 10 \quad -15a^4b^4c^4+7c^4a^3b^5-24b^4c^4a^5+27a^4b^3c^5 \\ 23a^4b^4c^4+19c^4a^3b^5-8b^4c^4a^5-15a^4b^3c^5 \\ 11a^4b^4c^4-16c^4a^3b^5+29b^4c^4a^5-9a^4b^3c^5 \\ -18a^4b^4c^4-10c^4a^3b^5+3b^4c^4a^5-3a^4b^3c^5 \\ \hline a^4b^4c^4 \end{array}$$

$$\begin{array}{r}
 11 \quad 25a^3b^3 - 8b^2c^2 - 23c^2a^2 + 19a^2b^2c^2 \\
 - 19a^2b^2 - 12b^2c^2 + 16c^2a^2 - 14a^2b^2c^2 \\
 13a^2b^2 - 20b^2c^2 + 17c^2a^2 + 27a^2b^2c^2 \\
 - 21a^2b^2 + 29b^2c^2 - 13c^2a^2 - 6a^2b^2c^2 \\
 \hline
 3a^2b^2 + 10b^2c^2 + 4c^2a^2 - 27a^2b^2c^2 \\
 a^2b^2 - b^2c^2 + c^2a^2 - a^2b^2c^2
 \end{array}$$

$$\begin{array}{r}
 12 \quad 5a^2 - 18b^2 - 53c^2 - 25abc \\
 - 37a^2 + 29b^2 + 38c^2 - 7abc \\
 43a^2 + 11b^2 - 17c^2 + 26abc \\
 4a^2 + 13b^2 + 21c^2 - 18abc \\
 - 14a^2 - 34b^2 + 12c^2 + 21abc \\
 \hline
 a^2 + b^2 + c^2 - 3abc
 \end{array}$$

$$\begin{array}{r}
 13 \quad 31^2 + 5^2 - 20a^2 + 49b^2 \\
 - 23x^2 - y^2 + 17a^2 - 27b^2 \\
 + 20x^2 - 4y^2 - 3a^2 + 3b^2 \\
 \hline
 + 7a^2 - 23b^2 \\
 a^2 + 2b^2
 \end{array}$$

Hence the required value
 $= 25 + 128 = 153$

$$\begin{array}{r}
 14 \quad 10a^2 - 26x^2y^4 - 30x^2b^5 - 17a^2y^7 \\
 - 304a^2 + 35x^2y^4 - 28x^2b^5 + 16a^2y^7 \\
 - 9x^2y^7 - 7x^2b^5 - 8a^2y^7 \\
 + 289a^2 + 51x^2b^5 - 25a^2y^7 \\
 \hline
 - 5a^2
 \end{array}$$

Hence the required value $= -125$

$$\begin{array}{r}
 15 \quad 2a^2 - 7b^2 + 9x^2 - 13y^2 + 15ab - 21xy \\
 - 6a^2 + 8b^2 - 20x^2 + 5y^2 - 8ab + 17xy \\
 + 5a^2 - 2b^2 + 13x^2 - 10y^2 - 20ab - 16xy \\
 - a^2 + 3b^2 - 2x^2 + 18y^2 + 13ab + 23xy \\
 \hline
 2b^2 + 3xy
 \end{array}$$

Hence the required value $= 32 + 168 = 200$

$$\begin{array}{r}
 16 \quad 29abx - 39bxy + 491ya - 59jab \\
 - 19abx + 29bxy - 39xya + 49jab \\
 2abx + 6bxy - 1211a + 21yab \\
 - 13abx + 4bx + 3xya - 14yab \\
 \hline
 - abx + xya
 \end{array}$$

Hence the required value $= -160 + 280 + 120$

$$\begin{aligned}
 17 \quad & 18a^2b^2 - 43b^2x^2 + 62x^2y^2 - 23abxy \\
 & - 25a^2b^2 + 28b^2x^2 - 42x^2y^2 + 39abxy \\
 & 37a^2b^2 + 19b^2x^2 + 35x^2y^2 - 25abxy \\
 & - 29a^2b^2 - 4b^2x^2 - 55x^2y^2 + 9abxy \\
 & \hline
 & a^2b^2
 \end{aligned}$$

Hence the required value = $25 \times 16 = 400$

$$\begin{aligned}
 18 \quad & 46a^4 + 38b^4 - 87abx^2 - 105y^4 \\
 & - 56a^4 - 58b^4 + 47abx^2 + 85y^4 \\
 & + 23a^4 + 75b^4 + 63abx^2 + 57y^4 \\
 & - 39a^4 - 33b^4 - 27abx^2 + 8y^4 \\
 & + 26a^4 - 22b^4 + 5abx^2 - 45y^4 \\
 & \hline
 & abx^2
 \end{aligned}$$

Hence the required value = $4 \times 5 \times 8 \times 8 = 1280$

$$\begin{aligned}
 19 \quad & 35xy^4 + 207ab^4 - 98bx^4 - 62y^4a^4 - 83abx^2y \\
 & - 65xy^4 - 87ab^4 + 68bx^4 + 102y^4a^4 + 53abx^2y \\
 & + 53xy^4 - 75ab^4 + 43bx^4 - 25y^4a^4 + 26abx^2y \\
 & - 29xy^4 + 45ab^4 + 26bx^4 + 28y^4a^4 - 65abx^2y \\
 & + 6xy^4 - 89ab^4 - 39bx^4 - 43y^4a^4 + 69abx^2y \\
 & \hline
 & ab^4
 \end{aligned}$$

Hence the required value = $5 \times 4 \times 4 \times 4 \times 4 = 1280$

$$\begin{aligned}
 20 \quad & 57a^4bx + 25b^4xy - 143x^4y^2a + 37y^4ab - 253a^4b^2x - \\
 & - 63a^4bx - 85b^4xy + 63x^4y^2a - 92y^4ab + 73a^4b^2x^2 \\
 & + 96a^4bx + 132b^4xy + 36x^4y^2a + 35y^4ab + 82a^4b^2x^2 \\
 & - 78a^4bx - 52b^4xy - 17x^4y^2a + 27y^4ab - 50a^4b^2x^2 \\
 & - 12a^4bx - 20b^4xy + 61x^4y^2a - 7y^4ab + 148a^4b^2x^2 \\
 & \hline
 & 0
 \end{aligned}$$

Hence the required value = 0

Exercise 10

- 1 $-3 + (-5) - (-6) + (-8) = -3 - 5 + 6 - 8$
 $= -(3 + 5 + 8) + 6 = -10$
- 2 $3 - (-5) + (-6) - (-8) = 3 + 5 - 6 + 8 = (3 + 5 + 8) - 6 = 10$
- 3 $-6 - (-8) - \{-(-5)\} - 3 = -6 + 8 - (5) - 3 = -6 + 8 - 5 - 3$
 $= -(6 + 5 + 3) + 8 = -6$

- 4 $-6 - \{-(-8)\} + (-5) - 3 = -6 - (8) - 5 - 3 = -6 - 8 - 5 - 3$
 $= -(6 + 8 + 5 + 3) = -22$
- 5 $-(-3) + (-5) - \{-(-6)\} - (-8) = 3 - 5 - (6) + 8 = 3 - 5 - 6 + 8$
 $= (3 + 8) - (5 + 6) = 11 - 11 = 0$
- 6 $50 - (-47) - \{-(-154)\} + (-234) = 50 + 47 - (154) - 234$
 $= 50 + 47 - 154 - 234 = (50 + 47) - (154 + 234)$
 $= 97 - 388 = -291$
- 7 $-\{-(-47)\} + (-234) - (-50) - (-154)$
 $= -(47) - 234 + 50 + 154$
 $= -47 - 234 + 50 + 154$
 $= -(47 + 234) + (50 + 154)$
 $= -281 + 204 = -77$
- 8 $-\{-(-154)\} + (-47) - (-234) - (-50)$
 $= -154 - 47 + 234 + 50 = -(154 + 47) + (234 + 50)$
 $= -201 + 284 = 83$
- 9 $-\{-(-234)\} - (-47) - (-154) - (-50)$
 $= -234 + 47 + 154 + 50 = -234 + (47 + 154 + 50)$
 $= -234 + 251 = 17$
- 10 $-(-50) - (-234) - \{-(-154)\} - (-47) = 50 + 234 - 154 + 47$
 $= (50 + 234 + 47) - 154 = 331 - 154 = 177$

Exercise 11

- | | |
|---|---|
| 1 $\frac{(2a + 3b - c) + (-a + b - c)}{a + 4b - 2c}$ | 2 $\frac{(-a - 2b + 8c) + (-2a + 5b - 4c)}{-3a + 3b + 4c}$ |
| 3 $\frac{(2x + 3y - 4z) + (x - y + z)}{3x + 2y - 3z}$ | 4 $\frac{(-5m^2 - mn + 4) + (3m^2 - 2mn - 5)}{-2m^2 - 3mn - 1}$ |
| 5 $\frac{(3x^2 - y^2 + 2z^2) + (-x^2 + 2y^2 - 3z^2)}{2x^2 + y^2 - z^2}$ | 6 $\frac{(2ax + 3y - 6y^2) + (-3ax + 4xy - 5y^2)}{-ax + 5xy - 11y^2}$ |

$$\begin{array}{r} 7 \quad (a^2 - 5ab - 8b^2) \\ + (3a^2 - 2ab + 7b^2) \\ \hline 4a^2 - 7ab - 8b^2 \end{array} \qquad \begin{array}{r} 8 \quad (5bc - c^2 + 2xj) \\ + (2bc - 6c^2 + 8ij) \\ \hline 7bc - 7c^2 + 10ij \end{array}$$

$$\begin{array}{r} 9 \quad (x^3 - 3x^2 + 6x + 7) \\ + (-2x^3 + 4x^2 - 7x - 5) \\ \hline -x^3 + x^2 - x + 2 \end{array}$$

$$\begin{array}{r} 10 \quad (2x^4 - 2x^3y - 3x^2y^2 + 4xy^3 - y^4) \\ + (-x^4 - 2x^3y + 3x^2y^2 - 6xy^3 + y^4) \\ \hline x^4 - 4x^3y - 2xy^3 \end{array}$$

$$\begin{array}{r} 11 \quad (2m^4 - 13m^3n + 15m^2n^2 - 37n^3) \\ + (-3m^4 + 7m^3n - 8m^2n^2 + 13n^3) \\ \hline -m^4 - 6m^3n + 7m^2n^2 - 24n^3 \end{array}$$

$$\begin{array}{r} 12 \quad (5p^4 - 12p^3q + 7p^2q^2 - 23pq^3 + 3q^4) \\ + (-8p^4 + 7p^3q - 10p^2q^2 + 13pq^3 - 5q^4) \\ \hline -3p^4 - 5p^3q - 3p^2q^2 - 10pq^3 - 2q^4 \end{array}$$

$$\begin{array}{r} 13 \quad (3x^5 - 5x^4y + 2x^3y^2 - 7x^2y^3 + 6y^4) \\ + (7x^5 - 6x^4y + 8x^3y^2 + 13x^2y^3 - 9y^4) \\ \hline 10x^5 - 11x^4y + 10x^3y^2 + 6x^2y^3 - 3y^4 \end{array}$$

$$\begin{array}{r} 14 \quad (5m^3n^2 - 17m^2nm + 26m^3mn - 13m^2n^2x - 19m^2n^2m) \\ + (-3m^3nx + 10m^3nm - 14x^3mn + 20m^3n^2x + 27m^3n^2m) \\ \hline 2m^3nx - 7m^3nm + 12x^3mn + 7m^3n^2x + 8m^3n^2m \end{array}$$

$$\begin{array}{r} 15 \quad (48x^6 - 31x^5y - 7x^4y^2 - 39x^3y^3 - 41x^2y^4 + 65xy^5 - 53y^6) \\ + (-37x^6 + 28x^5y - 43x^4y^2 + 54x^3y^3 + 67x^2y^4 - 84xy^5 + 93y^6) \\ \hline 11x^6 - 3x^5y - 50x^4y^2 + 15x^3y^3 + 26x^2y^4 - 19xy^5 + 40y^6 \end{array}$$

Exercise 12

$$\begin{array}{r} 1 \quad (2a + 3b - 5c) \\ + (-3a + 4b - 5c) \\ \hline -a + 7b - 10c \end{array} \qquad \begin{array}{r} 2 \quad (7m - 2n + 5r) \\ + (3m - 5n + 7r) \\ \hline 10m - 7n + 12r \end{array}$$

$$\begin{array}{r} 3 \quad (2x^2 - 3xy + 4y^2) \\ + (x^2 - xy + y^2) \\ \hline 3x^2 - 4xy + 5y^2 \end{array} \qquad \begin{array}{r} 4 \quad (2x^3 + 3x^2 - 3x + 5) \\ + (-x^3 + 2x^2 - 5x - 6) \\ \hline x^3 + 5x^2 - 8x - 1 \end{array}$$

$$5 \quad \frac{(2a^2 - 3ab + 3bc) + (-a^2 + 5ab - 7bc - 2b^2)}{a^2 + 2ab - 4bc - 2b^2}$$

$$6 \quad \frac{(3x^3 - 2x^2y + 8xy^2 - 5y^3) + (5x^3 - 6x^2y + 4xy^2 - 7y^3)}{8x^3 - 8x^2y + 12xy^2 - 12y^3}$$

$$7 \quad \frac{(4 - 6x + 7x^2 - 9x^3) + (2 - 3x + 5x^2 - 7x^3)}{6 - 9x + 12x^2 - 16x^3}$$

$$8 \quad \frac{(3x^4 - 2x^3 - 8x^2 + 7) + (-x^4 + 7x^2 - 9)}{2x^4 - 2x^3 - x^2 - 2}$$

$$9 \quad \frac{(x^5 - 6x^4 - 4x^2 + 5) + (5x^5 - 2x^4 + x^3 - 6x^2 - 9x - 8)}{6x^5 - 8x^4 + x^3 - 10x^2 - 9x - 3}$$

$$10 \quad \frac{(x^3 - 4x^2y - 8xy^2 - 5ab) + (-2x^3 + 3x^2y - 7xy^2 + 3ab + 8y^3 - a^3)}{-x^3 - x^2y - 15xy^2 - 2ab + 8y^3 - a^3}$$

$$11 \quad \frac{(12x^4y - 5x^3y^2 + 8xy^2z - 9x^2yz) + (2x^4y - 2x^3y^2 - 3xy^2z + 7x^2yz)}{14x^4y - 7x^3y^2 + 5xy^2z - 2x^2yz}$$

$$12 \quad \frac{(2a^3 - 3b^3 + 5c^3 - 6ab + 7bc - 8ac) + (-5a^3 + 4b^3 + 3c^3 + 7ab - 2bc - ac)}{-3a^2 + b^3 + 8c^3 + ab + 5bc - 9ac}$$

$$13 \quad \frac{(2x^3 - 3xy + 5y^3 - 6x - 8y + 9) + (3x^3 - 2xy - 4y^2 + 5x + 7y - 12)}{5x^3 - 5xy + y^2 - x - y - 3}$$

$$14 \quad \frac{(a^3 - 3a^2b + 5ab^2 - 9b^3 + 3a^2 - 2ab + b^3) + (-2a^3 + 2a^2b + 7ab^2 + 7b^3 + 5a^2 - 5ab - 4b^2)}{-a^3 - a^2b + 12ab^2 - 2b^3 + 8a^2 - 7ab - 3b^2}$$

$$15 \quad \frac{(3ax^4 - 5a^2x^3 + 6y^2bc^2 - 7y^2zbc + 8yz^2bc) + (2ax^4 - 3a^2x^3 + 2y^2bc^2 + 9y^2zbc - 4yz^2bc)}{5ax^4 - 8a^2x^3 + 8y^2bc^2 + 2y^2zbc + 4yz^2bc}$$

$$16 \quad \frac{(25 - 16x^2y^5z - 17xy^3z^5 + 21x^2z^5y - 6x^2y^2z^2 + 8xy^2z^4) + (-27 + 15x^2y^5z + 19xy^3z^5 - 19x^2z^5y + 12x^2y^2z^2 - 11xy^2z^4)}{-2 - x^2y^5z + 2xy^3z^5 + 2x^2z^5y + 6x^2y^2z^2 - 3xy^2z^4}$$

$$17 \quad \frac{(29x^4y^3z^2 - 37x^3y^4z^2 + 54x^2y^3z^4 - 45x^2y^2z^4 - 67x^4y^2z^3 + 89x^2y^4z^3) + (-25x^4y^3z^2 - 43x^3y^4z^2 - 26x^2y^3z^4 + 23x^2y^2z^4 - 35x^4y^2z^3 + 66x^2y^4z^3)}{4x^4y^3z^2 - 80x^3y^4z^2 + 28x^2y^3z^4 - 22x^2y^2z^4 - 102x^4y^2z^3 + 155x^2y^4z^3}$$

$$18 \quad (41x^3y^4z^5 - 87x^3y^5z^4 - 28x^4y^5z^3 + 63x^4y^3z^5 - 55x^5y^3z^4 + 37x^6y^4z^3) \\ + \frac{(-53x^3y^4z^5 - 13x^3y^5z^4 + 86x^4y^5z^3 + 29x^4y^3z^5 + 94x^5y^3z^4 - 75x^5y^4z^3) - 12x^3y^4z^5 - 100x^3y^5z^4 + 58x^4y^5z^3 + 92x^4y^3z^5 + 39x^5y^3z^4 - 38x^5y^4z^3}{-12x^3y^4z^5 - 100x^3y^5z^4 + 58x^4y^5z^3 + 92x^4y^3z^5 + 39x^5y^3z^4 - 38x^5y^4z^3}$$

$$19 \quad (-x^2 - y^2 - yz) \\ + \frac{(-3x^2 + 5xy - 6y^2 - 7yz)}{-4x^2 + 5xy - 7y^2 - 8yz}$$

$$20 \quad (x^3 + x^2y^2 + a^2bx - 2bxy^2 - 2xyab) \\ + \frac{(5x^3 - 13x^2y^2 + a^2bx - 5bxy^2 - 7xyab)}{6x^3 - 12x^2y^2 + 2a^2bx - 7bxy^2 - 9xyab}$$

$$21 \quad (3x^4 + 5x^2y^2 - 12y^4) \\ + \frac{(-5x^4 + 6x^3y - 7x^2y^2 + 8xy^3 + 19y^4)}{-2x^4 + 6x^3y - 2x^2y^2 + 8xy^3 + 7y^4}$$

$$22 \quad (-7x^5 - 4x^3y^2 + 13x^2y^3 + 29y^5) \\ + \frac{(5x^5 + 3x^4y - 6x^3y^2 - 17x^2y^3 - 13xy^4 + 21y^5)}{-2x^5 + 3x^4y - 10x^3y^2 - 4x^2y^3 - 13xy^4 + 50y^5}$$

$$23 \quad (2a^2 + 5ab - 6b^2) \\ + \frac{(-a^2 - 2b^2)}{a^2 + 5ab - 8b^2}$$

$$24 \quad (5x^2 - 6xy + 4y^2 - 8x - 10y + 15) \\ + \frac{(-x^2 - 2xy - 3y^2 - 4x - 5y - 6)}{4x^2 - 8xy + y^2 - 12x - 15y + 9}$$

$$25 \quad (3a^3 - 4a^2b + 5ab^2 - 8b^3) \\ + \frac{(-a^3 + 2ab^2 - 7b^3)}{2a^3 - 4a^2b + 7ab^2 - 15b^3}$$

$$26 \quad (-8x^3y + 4x^2y^2 - 11xy^3 + 12x^3 - 13y + 27) \\ + \frac{(-4x^3y + 3x^2y^2 + 11xy^3 - 20x^2 + 30y - 56)}{-12x^3y + 7x^2y^2 - 8x^2 + 17y - 29}$$

$$27 \quad (3a^3 - 7ab - 8bc + 9b^2) \\ + \frac{(2a^2 + 3ab + 3bc + 2b^2)}{5a^2 - 4ab - 5bc + 11b^2}$$

$$28 \quad (-3x^3 + 5y^2 - 7xy + 8x - 9) \\ + \frac{(x^3 - 8y^2 + 2xy - 11x + 7)}{-2x^3 - 3y^2 - 5xy - 3x - 2}$$

$$\begin{aligned}
 29 \quad & (-7a^2 - 8b^2 - 13ac^2 + 3b^3) \\
 & + (4a^2 - 3b^2c + 7ac^2 - 8b^3) \\
 & - 3a^2 - 11b^2c - 6ac^2 - 5b^3
 \end{aligned}$$

$$\begin{aligned}
 30 \quad & (21x^3 - 37xy^2 + 42y^3 - 18x^2 + 19xy - 39) \\
 & + (-25x^3 + 15xy^2 - 87y^3 + 7x^2 - 43xy + 24) \\
 & - 4x^3 - 22xy^2 - 45y^3 - 11x^2 - 24xy - 15
 \end{aligned}$$

Exercise 13

$$\begin{aligned}
 1 \quad & \text{The given expression} \\
 & = 2a - 3b - 4a + 6b - 2a + 5b = -\underline{4a + 8b}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & \text{The given expression} \\
 & = 1 - y + 4x + 2x - 3y = \underline{7x - 4y}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & \text{The given expression} \\
 & = -5x + y - 3x + y - 2y + 6x = \underline{-2x}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & \text{The given expression} = 3a - 6a + (2b - a) \\
 & = 3a - 6a + 2b - a = -\underline{4a + 2b}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \text{The given expression} = -a - 2b + (6a + 4b) \\
 & = -a - 2b + 6a + 4b = \underline{5a + 2b}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \text{The given expression} = 2a - \{5b - 7b + 2a\} \\
 & = 2a - 5b + 7b - 2a = \underline{2b}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad & \text{The given expression} = 3 - \{5 - (6 - 7 + 9)\} \\
 & = 3 - \{5 - 6 + 7 - 9\} \\
 & = 3 - 5 + 6 - 7 + 9 \\
 & = 18 - 12 = 6
 \end{aligned}$$

$$\begin{aligned}
 8 \quad & \text{The given expression} \\
 & = -2 + 3 + \{-4 - (-5 - 6)\} \\
 & = -2 + 3 - 4 - (-5 - 6) \\
 & = -2 + 3 - 4 + 5 + 6 \\
 & = -6 + 14 = 8
 \end{aligned}$$

9 The given expression

$$\begin{aligned} &= -a + 3b + \{-3a - (-a - 4b)\} \\ &= -a + 3b - 2a + a + 4b \\ &= -\underline{2a + 7b} \end{aligned}$$

10 The given expression

$$\begin{aligned} &= a - [2b - \{3c - (a - 2b + 3c)\}] \\ &= a - [2b - \{3c - a + 2b - 3c\}] \\ &= a - [2b - 3c + a - 2b + 3c] \\ &= a - a = 0 \end{aligned}$$

11 The given expression

$$\begin{aligned} &= 3x - [5y - \{10z - (5x - 10y + 3z)\}] \\ &= 3x - [5y - \{10z - 5x + 10y - 3z\}] \\ &= 3x - [5y - 10z + 5x - 10y + 3z] \\ &= 3x - 5y + 10z - 5x + 10y - 3z \\ &= -\underline{2x + 5y + 7z} \end{aligned}$$

12 The given expression

$$\begin{aligned} &= -a - [-b - \{-c - (-a - b - c)\}] \\ &= -a - [-b - \{-c - (-a + b + c)\}] \\ &= -a - [-b - \{-c + a - b - c\}] \\ &= -a - [-b + c - a + b + c] \\ &= -a + b - c + a - b - c \\ &= -\underline{2c} \end{aligned}$$

13 The given expression

$$\begin{aligned} &= 2x - 5y + \{9x - (10y - 4x)\} \\ &= 2x - 5y + 9x - (10y - 4x) \\ &= 2x - 5y + 9x - 10y + 4x \\ &= \underline{15x - 15y} \end{aligned}$$

14. The given expression

$$\begin{aligned} &= -5a - 3b + \{6a - (5b - 7a)\} \\ &= -5a - 3b + 6a - (5b - 7a) \\ &= -5a - 3b + 6a - 5b + 7a \\ &= \underline{8a - 8b} \end{aligned}$$

15 The given expression

$$\begin{aligned}
 &= -7m - 3n + \{8m - (4n - 10m)\} \\
 &= -7m - 3n + 8m - (4n - 10m) \\
 &= m - 3n - 4n + 10m \\
 &= \underline{11m - 7n}
 \end{aligned}$$

16 The given expression

$$\begin{aligned}
 &= -2a + 4b + \{-6c - (-8a - \overline{-10b - 12c})\} \\
 &= -2a + 4b - 6c - (-8a - \overline{-10b - 12c}) \\
 &= -2a + 4b - 6c + 8a + \overline{-10b - 12c} \\
 &= -2a + 4b - 6c + 8a - 10b - 12c \\
 &= \underline{6a - 6b - 18c}
 \end{aligned}$$

17 The given expression

$$\begin{aligned}
 &= -3x + 5y + \{-7z - (-9x - \overline{-11y - 13z})\} \\
 &= -3x + 5y - 7z - (-9x - \overline{-11y - 13z}) \\
 &= -3x + 5y - 7z + 9x + \overline{-11y - 13z} \\
 &= -3x + 5y - 7z + 9x - 11y - 13z \\
 &= \underline{6x - 6y - 20z}
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= -2x + 4y + \{-6z - (-3x - \overline{-5y - 7z})\} \\
 &= -2x + 4y - 6z - (-3x - \overline{-5y - 7z}) \\
 &= -2x + 4y - 6z + 3x + \overline{-5y - 7z} \\
 &= -2x + 4y - 6z + 3x - 5y - 7z \\
 &= \underline{x - y - 13z}
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= -x + 3y - \{5z - (-2x + \overline{-4y - 6z})\} \\
 &= -x + 3y + 5z + (-2x + \overline{-4y - 6z}) \\
 &= -x + 3y + 5z - 2x + \overline{-4y - 6z} \\
 &= -x + 3y + 5z - 2x - 4y - 6z \\
 &= \underline{-3x - y - z}
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= -2a + \{-5b - \{-8c + (-3a - \overline{-6b + 9c})\}\} \\
 &= -2a - 5b - \{-8c + (-3a - \overline{-6b + 9c})\} \\
 &= -2a - 5b + 8c - (-3a - \overline{-6b + 9c}) \\
 &= -2a - 5b + 8c + 3a + \overline{-6b + 9c} \\
 &= -2a - 5b + 8c + 3a - 6b + 9c \\
 &= \underline{a - 11b - 17c}
 \end{aligned}$$

21 The given expression

$$\begin{aligned}
 &= -r - 5j - \{-az + (-3z - \overline{-7j + 11z})\} \\
 &= -r - 5j + 9z - (-3z - \overline{-7j + 11z}) \\
 &= -r - 5j + 9z + 3z + \overline{-7j + 11z} \\
 &= -r - 5j + 9z + 3z - 7j + 11z \\
 &= \underline{2z - 12j + 20z}
 \end{aligned}$$

22 The given expression

$$\begin{aligned}
 &= \{2a - 3b - 5c\} - a + 2b - (c - 4a) + 7c \\
 &= 2a - 3b - 5c - a + 2b - c + 4a + 7c \\
 &= \underline{5a - b + 11c}
 \end{aligned}$$

23 The given expression

$$\begin{aligned}
 &= r - j - (z - r) - y - z - z + r - (j - z) \\
 &= 2r - 2j + z - z - j - z \\
 &= \underline{-j - 3z}
 \end{aligned}$$

24 The given expression

$$\begin{aligned}
 &= 2a - (b - c) - 3b + (2z - c) - 2a - (c - 4b) + 3b + \\
 &\quad (2a - 4c) - \{6c - (2b - 3a)\} - 5c - (6a - 7b) \\
 &= 4a - b + c - 3b + 2c - c - c - 4b + 3b + 2a - 4c - 6c + 2b \\
 &\quad - 3z - 5c + 6z - 7b \\
 &= \underline{11a - 2b - 16c}
 \end{aligned}$$

25 The given expression

$$\begin{aligned}
 &= a - (-b - c - d) + (-m + n - x) - j - z \\
 &= a - (b - c - d) + (-m - n - x) - j - z
 \end{aligned}$$

26 The given expression

$$\begin{aligned} &= a - (b + c - d + m - n + 1 - y + z) \\ &= a - \{b + c - d + m + (-n + 1 - y + z)\} \end{aligned}$$

27 The given expression

$$\begin{aligned} &= (a - b - c + d - m) - (-n + 1 - y + z) \\ &= \{a - b - (c - d + m)\} - \{-n - (-1 + y - z)\} \end{aligned}$$

28 The given expression

$$\begin{aligned} &= -(-a + b + c) - (-d + m - n) - (x - y + z) \\ &= -\{-a - (-b - c)\} - \{-d - (-m + n)\} - \{1 - (y - z)\} \end{aligned}$$

Miscellaneous Exercises (I).

I

1 (i) 5 hours $= 10 \times \frac{1}{2}$ hr (unit of time), and
10 is the required number

(ii) 5 hours $= \frac{1}{2} \times 10$ hrs (unit of time), and
 $\frac{1}{2}$ is the required number

2 $x - y = 17 - 25 = 25 - 17 = 8$

3 See Art 10, Page 9

The respective coefficients are 15, $2a$, $7ab^2$ and $16m^2pq$

4 \sqrt{ab} stands for "the square root of ab ," while $\sqrt[1]{ab}$ stands for " b multiplied by the square root of a "

$$\sqrt{ab} - \sqrt[1]{ab} = \sqrt[1]{9} \sqrt[1]{4} - \sqrt[1]{9} \sqrt[1]{4} = 3 \sqrt[1]{2} - 3 \sqrt[1]{2} = 6 - 12 = 12 - 6 = 6$$

5 A distance of 880 yds to the north is represented by 40,
a distance of 1 yd to the north must be represented by $\frac{10}{880}$,
or $\frac{1}{88}$, 1 yard to the south must be represented by $-\frac{1}{88}$, and
a distance of 11 yds to the south must be represented by
 $-\frac{1}{88} \times 11$, or $-\frac{1}{8}$

6 See Art 4, Page 22

7 See Art I, Page 30

8 9, 7, 5, 2, -1, -3, -4, 8, -12

II

1. (i) $ab - a \times b = ab - ab = 0$

(ii) $45 - ab = 45 - 4 \times 5 = 45 - 20 = 25$

(iii) $74 - 7a = 74 - 7 \times 4 = 74 - 28 = 46$

(iv) $85 - 8b = 85 - 8 \times 5 = 85 - 40 = 45$.

2 a^n means $a \times a \times a \times a \times \dots$ to n factors,
whereas n^a means $n \times n \times n \times n \times \dots$ a factors

The given expression

$$\begin{aligned} &= 2401 - 4 \times 343 \times 5 + 6 \times 49 \times 25 - 4 \times 7 \times 125 + 625 \\ &= 2401 - 6860 + 7350 - 3500 + 625 \\ &= (2401 + 7350 + 625) - (6860 + 3500) \\ &= 10376 - 10360 = 16 \end{aligned}$$

3 $\sqrt[n]{a}$ = a number whose n^{th} power is " a "
 $\sqrt[5]{a}$ = 5th power is " a "
 $\sqrt[8]{a}$ = 8th power is " a "
 $\sqrt[n]{a}$ = n th power is " a "

The given expression

$$\begin{aligned} &= \sqrt{64 - 15} \times \sqrt[3]{343 - 216 - 2} \\ &= \sqrt{49} \times \sqrt[3]{125} = 7 \times 5 = 35 \end{aligned}$$

4 See Note 2 to Art. 3, Page 19

5 The required sum
 $= (3x^2y - 8x^2y - 19x^2y + 17x^2y)$
 $= (3x^2y + 17x^2y) - (8x^2y + 19x^2y)$
 $= 20x^2y - 27x^2y = \underline{-7x^2y}$

And the numerical value $= -7 \times 4^2 \times 5 = -560$

6 The required sum
 $= 16x^4 - 81y^3 + 24x^2y^2 + y^4 - 32x^2y$, and its numerical value
 $= 16 \times 256 - 8 \times 4 \times 125 + 24 \times 16 \times 25 + 625 - 32 \times 64 \times 5$
 $= 4096 - 4000 + 9600 + 625 - 10240$
 $= (4096 + 9600 + 625) - (4000 + 10240)$
 $= 14321 - 14240 = 81$

$$\begin{array}{r}
 7 \quad (17b - 12c - 19a) \\
 + (13b + 25c - 4a) \\
 \hline
 30b + 13c - 23a
 \end{array}$$

8 The given expression

$$\begin{aligned}
 &= 3x - [4y + \{2z - (1 - 5y + 3z)\}] - (3x - 7y) \\
 &= 3x - 4y - \{2z - (1 - 5y + 3z)\} - 3x + 7y \\
 &= 3x - 4y - 3x + 7y - 2z + (1 - 5y + 3z) \\
 &= 3x - 4y - 3x + 7y - 2z + 1 - 5y + 3z \\
 &= \underline{1 - 2y + z}
 \end{aligned}$$

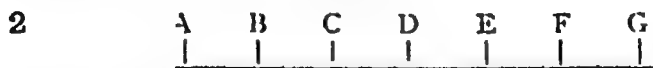
III

1 (i) $(a+b)c = 1 - yz$

(ii) $(x+y)^2 = x^2 + y^2 + 2xy$

(iii) $\sqrt{(m-n)} - m^2n^2 < \sqrt{1} + \sqrt{1}y$

(iv) $a > b \quad 3a > 3b$



The length of CD is 6 inches, the length of DC must be -6 inches. Since DC is represented by 3, the unit of length must be $(-6 \text{ in} \div 3)$, or -2 in

$$DB = DC + CB = (-CD) + (-BC) = (-4) + (-6) = -10,$$

DB will be represented by $(-10 \text{ in} \div -2 \text{ in})$ i.e. by 5,

$$DF = (DE + EF) = 5 + 8 = 13 \text{ in}, \quad \text{it will be represented by } (13 \text{ in} \div -2 \text{ in}) \text{ i.e. by } -\frac{13}{2},$$

$$DA = (-CD) + (-BC) + (-AB) = -13 \text{ in}, \quad \text{it will be represented by } \frac{-13 \text{ in}}{-2 \text{ in}} \text{ i.e. by } \frac{13}{2}$$

$$DG = (DE + EF + FG) = 20 \text{ in}, \quad \text{it will be represented by } \frac{20 \text{ in}}{-2 \text{ in}} \text{ i.e. by } -10$$

3 See Art 3, page 20

$$\begin{aligned}
 \text{The required sum} &= (-a^3) + (-3a^2b) + (-3ab^2) + (-b^3) \\
 &\quad - -(a^3 + 3a^2b + 3ab^2 + b^3) \\
 &= -(216 + 3 \cdot 364 + 3 \cdot 616 + 64) \\
 &= -(216 + 1332 + 288 + 64) = -1000
 \end{aligned}$$

4 See Art 7, Page 6

5 The given expression

$$= \sqrt[3]{5(2+3)^2} + \sqrt[3]{(2+6)(5-4)} + \sqrt[3]{2(5-3)^2}$$

$$= \sqrt[3]{5 \cdot 5^2} + \sqrt[3]{6 \cdot 1} + \sqrt[3]{2 \cdot 2^2} = 5 + 2 + 2 = 9$$

6 A receives x pounds, B receives $(x+a)$ pounds and C $(x+a+b)$ pounds

Hence the whole sum $= x + (x+a) + (x+a+b) = \underline{3x+2a+b}$

$$7 \quad \begin{array}{r} a^2 - 3ab - \frac{1}{2}b^2 \\ + \frac{1}{2}b^2 - \frac{1}{2}b^2 + c^2 \\ + ab - \frac{1}{2}b^2 + \frac{1}{2}b^2 \\ + \frac{2ab}{a^2} - \frac{b^2}{b^2} + c^2 \end{array}$$

$$8 \quad \{2x^2 - (y^2 - 1)\} - \{y^2 - (4x^2 - y^2)\} + \{2y^2 - (3xy - x^2)\}$$

$$= 2x^2 - y^2 + 1 - y^2 + 4x^2 - y^2 + 2y^2 - 3xy + x^2$$

$$= 2x^2 - y^2 + 1 - y^2 + 4x^2 - y^2 + 2y^2 - 3xy + x^2$$

$$= \underline{7x^2 - 2xy - y^2 + 1}$$

V

1 See Arts 16 & 17, Pages 15, 16

2 Follow the directions given in Art 6, (Page 6), in finding separately the values of (i) $a \times b$, (ii) $c - d \times e$ and (iii) $f + gh$

Now subtract (ii) from (i), and to the remainder add (iii)

3 See Art 9, Page 9

The required factors are 2, a , b and $(a+b)$

4 See Art 6, Page 23

5 The given expression

$$= \frac{2 \cdot 4 \cdot 4}{(4-2)^2} - \frac{6 \sqrt[3]{4 \cdot 2}}{4^2 \sqrt{2 \cdot 4 + 4 \cdot 2}} - \frac{29 \cdot 4}{6 \cdot 4}$$

$$= \frac{2 \cdot 4 \cdot 4}{4} - \frac{6 \cdot 2}{4 \cdot 4 \cdot 4 \sqrt{16}} - \frac{29}{6 \cdot 4}$$

$$= 8 - \frac{6 \cdot 2}{4 \cdot 4 \cdot 4 \cdot 4} - \frac{29}{6 \cdot 4}$$

$$= 8 - \frac{3}{8} - \frac{29}{24}$$

$$= 8 - \frac{1}{2} = 7\frac{1}{2}$$

$$\begin{array}{r}
 6 \quad \quad \quad x^3 - 7x^2 + 6x + 5 \\
 \quad \quad \quad 2x^3 + 5x^2 - 3x + 4 \\
 -4x^3 - 7x^2 + 2x - 11 \\
 \hline
 \quad \quad \quad 5x^3 + 9x^2 - 4x + 2 \\
 \quad \quad \quad 4x^3 \quad \quad + x \\
 \quad \quad \quad \text{the required value} \\
 \quad \quad \quad = 4 \cdot 5^3 + 5 = 4 \cdot 125 + 5 \\
 \quad \quad \quad = 500 + 5 = 505
 \end{array}$$

7 See Art. 4, Page 32

8 The given expression

$$\begin{aligned}
 &= 2x - [3x - 9y - \{2x - 3y - x - 5y\}] \\
 &= 2x - [3x - 9y - 2x + 3y + x + 5y] \\
 &= 2x - 3x + 9y + 2x - 3y - x - 5y = y
 \end{aligned}$$

VI

1 See Art. 22, Page 9

$$\begin{aligned}
 2 \quad (16-1) \sqrt{24 \cdot 10 \cdot 5 + 25} + \sqrt{(16-5)(10+1)} \\
 = 15 \sqrt{1225} + \sqrt{11 \cdot 11} \\
 = 15 \cdot 35 + 11 = 525 + 11 = 536
 \end{aligned}$$

3 (i) The left-hand expression

$$\begin{aligned}
 &= 3^3 + 4^3 + 5^3 - 3 \cdot 3 \cdot 4 \cdot 5 \\
 &= 27 + 64 + 125 - 180 = 216 - 180 = 36
 \end{aligned}$$

and the right-hand expression

$$= (3+4+5)(9+16+25-12-15-20) = 12 \cdot 3 = 36$$

Therefore the two expressions are equal

(ii) The left hand expression

$$\begin{aligned}
 &= \left(\frac{3}{2}\right)^3 + \left(\frac{5}{2}\right)^3 + \left(\frac{7}{2}\right)^3 - 3 \cdot \frac{3}{2} \cdot \frac{5}{2} \cdot \frac{7}{2} \\
 &= \frac{27}{8} + \frac{125}{8} + \frac{343}{8} - \frac{315}{8} = \frac{495 - 315}{8} = \frac{180}{8} = 22\frac{1}{2}
 \end{aligned}$$

and the right-hand expression

$$= \left(\frac{3}{2} + \frac{5}{2} - \frac{7}{2} \right) \left(\frac{9}{4} + \frac{25}{4} + \frac{49}{4} - \frac{15}{4} - \frac{21}{4} - \frac{35}{4} \right) \\ = \frac{15}{2} \cdot \frac{12}{4} = \frac{45}{2} = 22\frac{1}{2}$$

Therefore the two expressions are equal

4 See Cor 2, Art 6, Page 25

5 See Art 3, Page 31

$$\begin{array}{r} 71^2 - 25\sqrt{3z + z^2} \\ - 121^2 + 19\sqrt{1z - 3z^2} \\ \hline 51^2 + 7\sqrt{3z + 2z^2} \\ \hline \sqrt{3z} \end{array}$$

Hence the required value = $\sqrt{16} \cdot 15 = 4 \cdot 15 = 60$

7 Subtract $7c$ from $2d$, then subtract the remainder from $3e$, then subtract the remainder from $4b$ and lastly subtract the remainder thus obtained from $5a$

8 The given expression

$$\begin{aligned} &= [(64 + 27 + 8 + 1)\{4 + 3 - (2 - 4)\} + 16 \cdot 3 + 4 \cdot 1] \\ &\quad \times \{16 - (9 + 4) + 1\} \\ &= [100\{4 + 3 + 2\} + 48 + 4] \times \{16 - 13 + 1\} \\ &= [900 + 48 + 4] \times 4 = 952 \times 4 = 3808 \end{aligned}$$

VII

1 (i) $a \div bc$ means that a is to be divided by the product of b and c , whereas $a \div b \times c$ means that c is to be multiplied by the quotient of a by b

(ii) a^4 stands for $a \times a \times a \times a$, while $4a$ stands for $4 \times a$

(iii) $3\sqrt{a}$ means that the square root of a is to be multiplied by 3, while $\sqrt[3]{a}$ means the quantity whose cube is a

(iv) $\sqrt{a+b}$ means the square root of the sum of a and b , while $\sqrt{a} + b$ means that b is to be added to the square root of a

(v) \sqrt{ab} stands for "the square root of ab ," while $\sqrt{a}b$ stands for " b multiplied by the square root of a "

2 (i) The given expression

$$= \frac{12+43+90+01}{(1+2)(3+0)-\{(1-0)+(3-2)\}}$$

$$= \frac{2+12}{33-12} = \frac{14}{9-2} = \frac{14}{7} = 2$$

(ii) The given expression

$$= \sqrt[3]{2-1} + \sqrt[3]{4(3-1)} - \sqrt[4]{3(81+52+33-20)}$$

$$= \sqrt[3]{1} + \sqrt[3]{4 \cdot 2} - \sqrt[4]{3(8+10+9)} = 1 + \sqrt[3]{8} - \sqrt[4]{3 \cdot 27}$$

$$= 1+2-3=0$$

3 (i) The first expression $= (2+3+4)^3 + 2^3 + 3^3 + 4^3$

$$= 9^3 + 8 + 27 + 64 = 729 + 8 + 27 + 64 = 828,$$

the second expression

$$= (2+3)^3 + (3+4)^3 + (4+2)^3 + 6 \cdot 2 \cdot 3 \cdot 4$$

$$= 125 + 343 + 216 + 144 = 828,$$

and the third expression

$$= 2 \cdot 2^3 + 3 \cdot 3^2(2+4) + 2 \cdot 3^3 + 3 \cdot 4^2(2+3)$$

$$+ 2 \cdot 4^3 + 3 \cdot 2^2(3+4) + 6 \cdot 2 \cdot 3 \cdot 4$$

$$= 2 \cdot 8 + 3 \cdot 9 \cdot 6 + 2 \cdot 27 + 3 \cdot 16 \cdot 5 + 2 \cdot 64 + 3 \cdot 4 \cdot 7 + 6 \cdot 2 \cdot 3 \cdot 4$$

$$= 16 + 162 + 54 + 240 + 128 + 84 + 144 = 828$$

the three expressions are equal to one another

(ii) The first expression $= (7+4+1)^3 + 7^3 + 4^3 + 1^3$

$$= (12)^3 + 343 + 64 + 1 = 1728 + 343 + 64 + 1 = 2136,$$

the second expression $= (7+4)^3 + (4+1)^3 + (1+7)^3 + 6 \cdot 7 \cdot 4 \cdot 1$

$$= 11^3 + 5^3 + 8^3 + 168 = 1331 + 125 + 512 + 168 = 2136,$$

and the third expression

$$= 2 \cdot 7^3 + 3 \cdot 4^2(7+1) + 2 \cdot 4^3 + 3 \cdot 1^2(7+4) + 2 \cdot 1^3$$

$$+ 3 \cdot 7^2(4+1) + 6 \cdot 7 \cdot 4 \cdot 1 = 2 \cdot 343 + 3 \cdot 16 \cdot 8 + 2 \cdot 64$$

$$+ 3 \cdot 11 + 2 \cdot 3 \cdot 49 \cdot 5 + 6 \cdot 7 \cdot 4 = 686 + 384 + 128$$

$$+ 33 + 2 + 735 + 168 = 2136$$

the three expressions are equal to one another

4 (i) $1 - [1 - \{1 - (1 - 1 + x)\}]$

$$= 1 - [1 - \{1 - (1 - 1 + x)\}]$$

$$= 1 - [1 - \{1 - 1 + x\}]$$

$$= 1 - [1 - 1 + 1 - x]$$

$$= 1 - 1 + 1 - 1 + x = \underline{1+x}$$

(ii) The given expression

$$\begin{aligned} &= 3a - b + 2c - \{a + c - 3a + b + 2c\} - 2a + 3b - 4c \\ &= 3a - b + 2c - a - c + 3a - b - 2c - 2a + 3b - 4c \\ &= \underline{3a + b - 5c} \end{aligned}$$

5 (i) $(x+y)(x-y) = x^2 - y^2$ (ii) $(x+y)^2 = x^2 + y^2 + 2xy$

6 The given expression

$$\begin{aligned} &= 17a - 5b - [7a - 3b - \{4a - 4b - 2a - 3b\}] \\ &= 17a - 5b - [7a - 3b - 4a + 4b + 2a + 3b] \\ &= 17a - 5b - 7a + 3b + 4a - 4b - 2a - 3b \\ &= 12a - 9b \end{aligned}$$

Hence the required value $= 12 \times 39 - 9 \times 52 = 468 - 468 = 0$

7 $V - (V + V) + V$

$$\begin{aligned} &= (5a + 4b - 6c) - (-3a - 9b + 7c + 20a + 7b - 5c) \\ &\quad + 13a - 5b + 9c \\ &= 5a + 4b - 6c + 3a + 9b - 7c - 20a - 7b + 5c + 13a - 5b + 9c \\ &= \underline{a + b + c} \end{aligned}$$

8

$a - \frac{1}{2}b$	$+\frac{1}{4}c - \frac{1}{8}d$	$(\frac{1}{2}a - \frac{1}{4}b + \frac{1}{8}c - \frac{1}{16}d)$
$\frac{1}{2}a - \frac{1}{4}b$	$-\frac{1}{8}c + \frac{1}{16}d$	$+(-\frac{1}{4}a + \frac{1}{8}b - \frac{1}{16}c + \frac{1}{32}d)$
$-a - \frac{1}{2}b$	$+c + \frac{1}{4}d$	$\frac{1}{2}a - \frac{1}{4}b + \frac{1}{8}c - \frac{1}{16}d$
$\frac{1}{2}a + b$	$-\frac{1}{8}c - \frac{1}{16}d$	
$\frac{8a - 6b}{2} + \frac{3c - 4d}{8}$		which is the required difference

Exercise 14

- $6 \times 5 = 5 + 5 + 5 + 5 + 5 = 30$
- $7 \times 4 = 4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$
- $3 \times 12 = 12 + 12 + 12 = 36$
- $-8 \times 4 = (-8) + (-8) + (-8) + (-8) = 32$
- $5 \times (-9) = (-9) + (-9) + (-9) + (-9) + (-9) = -45$
- $6 \times (-13) = (-13) + (-13) + (-13) + (-13) + (-13) + (-13) = -78$

- 7 $(-3) \times 8 = -8 - 8 - 8 = -24$
- 8 $(-5) \times 14 = -14 - 14 - 14 - 14 - 14 = -70$
- 9 $(-3) \times 15 = -15 - 15 - 15 = -45$
- 10 $(-4) \times (-9) = -(-9) - (-9) - (-9) - (-9)$
 $= 9 + 9 + 9 + 9 = 36$
- 11 $(-5) \times (-12) = -(-12) - (-12) - (-12) - (-12) - (-12)$
 $= 12 + 12 + 12 + 12 + 12 = 60$
- 12 $(-4) \times (-16) = -(-16) - (-16) - (-16) - (-16)$
 $= 16 + 16 + 16 + 16 = 64$

Exercise 15

- 1 $ab - cd = (-2)(-3) - (-8)6 = 6 + 48 = 54$
- 2 $(a^2 - b^2)c = (25 - 36)(-9) = (-11)(-9) = 99$
- 3 $abc + b^2c - c^2d = 3(-5)(-7) + 25(-7) - 49(-4)$
 $= 105 - 175 + 196 = 126$
- 4 $(-a)b^2 - cd^2 + b(-c)^2 = (-5)49 - 49 - 716$
 $= -245 - 36 - 112 = -393$
- 5 $\{a^2 + (-b)^2 - (-c)^2\}(bc - ad) = \{9 + 16 - 36\}(24 - 27)$
 $= (-11)(-3) = 33$
- 6 $a^2(b - c) + b^2(c - a) + c^2(a - b)$
 $= 4\{(-5) - (-7)\} + 25\{(-7) - (-2)\} + 49\{(-2) - (-5)\}$
 $= 4(-5 + 7) + 25(-7 + 2) + 49(-2 + 5)$
 $= 4 \cdot 2 + 25(-5) + 49 \cdot 3 = 8 - 125 + 147 = 30$
- 7 $x^2(y - z) + y^2(z - x) + z^2(x - y)$
 $= -27\{8 - (-5)\} + 512\{(-5) - (-3)\} + (-125)\{(3) - (8)\}$
 $= -27 \cdot 13 + 512(-2) + (-125)(-5)$
 $= -351 - 1024 + 1375 = 0$
- 8 $p^2(q^2 - r^2) + q^2(r^2 - p^2) + r^2(p^2 - q^2)$
 $= (-27)(25 - 49) + (-125)(49 - 9) + (-343)(9 - 2)$
 $= (-27)(-24) + (-125)40 + (-343)(-16)$
 $= 648 - 5000 + 5488 = 1136$

$$\begin{aligned}
 9 \quad a^3 + b^3 + c^3 - 3abc &= (-1728) + (-2197) \\
 &\quad + (-3375) - 3(-12)(-13)(-15) \\
 &= -(1728 + 2197 + 3375) + 39180 \\
 &= -7300 + 7020 = -280
 \end{aligned}$$

$$10 \quad (a+b)^5 = (-5+3)^5 = -32$$

Again the right-hand expression

$$\begin{aligned}
 &= (-5)^5 + 5(-5)^4 \times 3 + 10(-5)^3 \times 9 + 10(-5)^2 \times 27 + 5(-5)81 + 243 \\
 &= -3125 + 9375 - 11250 + 6750 - 2025 + 243 = -32,
 \end{aligned}$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Exercise 16

$$1 \quad 4 \times 5 = 5 + 5 + 5 + 5 = 20,$$

$$\text{also } 5 \times 4 = 4 + 4 + 4 + 4 + 4 = 20, \quad 4 \times 5 = 5 \times 4$$

$$2 \quad 6 \times 3 = 3 + 3 + 3 + 3 + 3 + 3 = 18,$$

$$\text{also } 3 \times 6 = 6 + 6 + 6 = 18, \quad 6 \times 3 = 3 \times 6$$

$$3 \quad 7 \times 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 = 35,$$

$$\text{also } 5 \times 7 = 7 + 7 + 7 + 7 + 7 = 35, \quad 7 \times 5 = 5 \times 7$$

$$4 \quad 4 \times 8 = 8 + 8 + 8 + 8 = 32,$$

$$\text{also } 8 \times 4 = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 32, \quad 4 \times 8 = 8 \times 4$$

$$5 \quad 9 \times 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45,$$

$$\text{also } 5 \times 9 = 9 + 9 + 9 + 9 + 9 = 45, \quad 9 \times 5 = 5 \times 9$$

Exercise 17.

$$1 \quad (-a) \times 3b = (-1) \times a \times 3 \times b = (-1) \times 3 \times a \times b = -3ab$$

$$\begin{aligned}
 2 \quad (2a) \times (-4b) &= 2 \times a \times (-1) \times 4 \times b \\
 &= (-1) \times 2 \times 4 \times a \times b = -8ab
 \end{aligned}$$

$$\begin{aligned}
 3 \quad (-3a) \times (-6a^2) &= (3a) \times (6a^2) = 3 \times a \times 6 \times a \times a \\
 &= 3 \times 6 \times a \times a \times a = 18a^3
 \end{aligned}$$

$$4 \quad (-4b) \times (-5a) = (4b) \times (5a) = 4 \times b \times 5 \times a = 4 \times 5 \times a \times b = 20ab$$

$$\begin{aligned}
 5 \quad (-7c) \times (-3ab) &= (7c) \times (3ab) = 7 \times c \times 3 \times a \times b \\
 &= 7 \times 3 \times a \times b \times c = 21abc
 \end{aligned}$$

$$6 \quad 10 \times 35 = 5 \times 2 \times 5 \times 7 = 5 \times 5 \times 2 \times 7 = 25 \times 14$$

$$7 \quad 15 \times 75 = 5 \times 3 \times 5 \times 5 \times 3 = 5 \times 5 \times 5 \times 3 \times 3 = 5^3 \times 3^2$$

$$8 \quad (-a)^3 = (-1) a \times (-1) a \times (-1) a = (-1)^3 \times a \times a \times a = -a^3$$

$$9 \quad (ab)^3 = a \times b \times a \times b \times a \times b = a \times a \times a \times b \times b \times b = a^3 b^3$$

$$\begin{aligned} 10 \quad (-ab)^3 &= (-ab) \times (-ab) \times (-ab) \\ &= (-1) \times a \times b \times (-1) \times a \times b \times (-1) \times a \times b \\ &= (-1)^3 \times a \times a \times a \times b \times b \times b = (-1)^3 \times a^3 \times b^3 = -a^3 b^3 \end{aligned}$$

$$\begin{aligned} 11 \quad (-a^3 b^6)^3 &= (-a^3 b^6) \times (-a^3 b^6) \times (-a^3 b^6) = a^3 b^6 \times a^3 b^6 \\ &= a \times a \times a \times b \times b \times b \times b \times b \times a \times a \times a \times b \times b \times b \times b \times b \\ &= a \times a \times a \times a \times a \times a \times a \times b \times b \times b \times b \times b \times b \times b \times b \times b \\ &= a^9 b^{18} \end{aligned}$$

$$\begin{aligned} 12 \quad (-r)^5 &= (-1) \times (-r) \times (-1) \times (-r) \times (-1) \\ &= (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times r \times r \times r \times r \times r \\ &= -r^5 \end{aligned}$$

$$\begin{aligned} 13 \quad (-xy)^5 &= (-xy) \times (-xy) \times (-xy) \times (-xy) \times (-xy) \\ &= (-1) \times x \times y \times (-1) \times x \times y \times (-1) \times x \times y \times (-1) \times x \times y \\ &\quad \times (-1) \times x \times y \\ &= (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times x \times x \times x \times x \times x \\ &\quad \times y \times y \times y \times y \times y = (-1)^5 \times x^5 \times y^5 = -x^5 y^5 \end{aligned}$$

$$14 \quad 6x^2 y \times (-7x^5 y^4) = -\{6 \times 7 \times x^2 \times x^5 \times y \times y^4\} = -42x^7 y^5$$

$$15 \quad (-3x^4 y^3) \times (8x^7 y^8) = -(3 \times 8 \times x^4 \times x^7 \times y^3 \times y^8) = -24x^{11} y^{11}$$

$$16 \quad (-5x^{12} y^3) \times (-8x^6 y^{13}) = 5 \times 8 \times x^{12} \times x^6 \times y^3 \times y^{13} = 40x^{18} y^{16}$$

$$\begin{aligned} 17 \quad (-12x^3 y^3 z^2) \times (13x^7 y^6 z^4) \\ &= -(12 \times 13 \times x^3 \times x^7 \times y^3 \times y^6 \times z^2 \times z^4) = -156x^{10} y^9 z^6 \end{aligned}$$

$$\begin{aligned} 18 \quad (-14x^6 y^3 z^8) \times (-10x^5 y^3 z^{12}) &= (14x^6 y^3 z^8) \times (10x^5 y^3 z^{12}) \\ &= 14 \times 10 \times x^6 \times x^5 \times y^3 \times y^3 \times z^8 \times z^{12} = 140x^{11} y^6 z^{20} \end{aligned}$$

$$\begin{aligned} 19 \quad (-3ab) \times (4a^2 b^2) \times (-2a^3 b^3) \\ &= (-3) \times 4 \times (-2) \times a \times a^2 \times a^3 \times b \times b^2 \times b^3 = 24a^6 b^5 \end{aligned}$$

- 20 $(-2a^2) + (-7a^4b^7) \times (-5a^3b^9)$
 $= (-2) \times (-7) \times (-5) \times a^2 \times a^4 \times a^3 \times b^7 \times b^9$
 $= \underline{-70a^{16}b^{16}}$
- 21 $(-6x^5y^2z) \times 2x^2y^3z \times (-4y^3z^2x^3)$
 $= -6 \times 2 \times -4 \times x^5 \times x^2 \times x^3 \times y^2 \times y^3 \times y^3 \times z \times z \times z^2 \times x^3$
 $= \underline{48x^{10}y^8z^5}$
- 22 $(-3x^2y) \times 4xy^2z \times (-1x^5y^4) \times 2xyz$
 $= (-3) \times 4 \times (-1) \times 2 \times x^2 \times x \times x^5 \times y \times y^2 \times y^4 \times y \times z \times z \times x$
 $= \underline{24x^{13}y^7z^2}$

Exercise 18

The examples should be worked out mentally by the method illustrated in Art. 6, CHAP. V

Exercise 19

- | | | | |
|---|--|---|--|
| 1 | $\begin{array}{r} 2a - 3b \\ -ab \\ \hline -2a^2b + 3ab^2 \end{array}$ | 2 | $\begin{array}{r} a - 2b + 3c \\ -5a \\ \hline -5a^2 + 10ab - 15ac \end{array}$ |
| 3 | $\begin{array}{r} 2a^2 - 3b^2 - c^2 \\ abc \\ \hline 2a^2bc - 3ab^2c - abc^2 \end{array}$ | 4 | $\begin{array}{r} x^2y - 2xy^2 - y^3 \\ -3xy \\ \hline -3x^2y^2 + 6x^2y^3 + 3xy^4 \end{array}$ |
| 5 | $\begin{array}{r} (-a^2b + ab^2 - 3a^2b^2 + 5a^3) \\ -7b^3 \\ \hline 7a^2b^3 - 7ab^4 + 21a^2b^3 - 35a^3b^3 \end{array}$ | | |
| 6 | $\begin{array}{r} (2ab - 3bc + 4ac - 5abc) \\ -8a^2b \\ \hline -16a^2b^2c + 24a^2b^3c^2 - 32a^3bc^3 + 40a^4b^3c^2 \end{array}$ | | |
| 7 | $\begin{array}{r} 3a^2x - 4a^2x^2 + 5a^3 \\ -2a^2 \\ \hline -6a^4x + 8a^5x^2 - 10a^6x \end{array}$ | 8 | $\begin{array}{r} -2m^3 + 3m^2n - 5mn^2 \\ 4mn \\ \hline -8m^4n + 12m^3n^2 - 20m^2n^3 \end{array}$ |

- 9
$$\begin{array}{r} 5x^4 - 6x^3 + 7x^2 - 8x \\ - 4x^8 \\ \hline - 20x^7 + 24x^6 - 28x^5 + 32x^4 \end{array}$$
- 10
$$\begin{array}{r} - 2c^2d + 3d^2c - 5cd^2 - 4c^2d^3 \\ - 6c^2d^4 \\ \hline 12c^4d^5 - 18c^3d^7 + 30c^2d^9 + 24c^4d^{10} \end{array}$$
- 11
$$\begin{array}{r} 8a^4 - 6a^3b + 5a^2b^2 - 4ab^3 \\ - 2a^3b^3 \\ \hline - 16a^7b^3 + 12a^6b^4 - 10a^5b^5 + 8a^4b^6 \end{array}$$
- 12
$$\begin{array}{r} - a^2bc + ab^2c - abc^2 + a^3b^3 - b^3c^2 \\ - a^3b^3c^3 \\ \hline a^6b^4c^4 - a^4b^5c^4 + a^4b^4c^5 - a^5b^5c^3 + a^7b^5c^5 \end{array}$$
- 13 The given expression

$$\begin{aligned} &= 7x^4 - 14x^3 - 2x^2 + 6x^2 - 8x^2 + 16x^2 \\ &= 7x^4 + (-14x^3 - 2x^2 + 16x^2) + (6x^2 - 8x^2) \\ &= \underline{7x^4 - 2x^2} \end{aligned}$$
- 14 The given expression

$$\begin{aligned} &= -5x^4 - 35x^3 - 21x^2 + 35x^3 + 21x^2 - 12x + 17x \\ &= -5x^4 + (-35x^3 + 35x^3) + (-21x^2 + 21x^2) \\ &\quad + (-12x + 17x) \\ &= \underline{-5x^4 + 5x} \end{aligned}$$
- 15 The given expression

$$\begin{aligned} &= 9x^6 - 18x^3y^2 + 15x^3y^2 + 5y^4 + 3x^3y^2 - 30y^4 \\ &= 9x^6 + (-18x^3y^2 + 15x^3y^2 + 3x^3y^2) + (5y^4 - 30y^4) \\ &= \underline{9x^6 - 25y^4} \end{aligned}$$
- 16 The given expression

$$\begin{aligned} &= x^6 + 2x^5 + 2x^4 - 2x^5 - 4x^4 - 4x^3 + 2x^4 + 4x^3 + 4x^2 \\ &= \underline{x^6 + 4x^2} \end{aligned}$$
- 17 The given expression

$$\begin{aligned} &= a^{12}b^6 - 2a^{10}b^6 + 2a^8b^4 + 2a^{10}b^4 - 4a^8b^4 \\ &\quad + 4a^6b^3 + 2a^8b^4 - 4a^6b^3 + 4a^4b^2 \\ &= \underline{a^{12}b^6 + 4a^4b^2} \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= 4a^{15}b^{11} + 12a^{14}b^{10} + 18a^{13}b^9 - 12a^{16}b^{10} \\
 &\quad - 36a^{15}b^9 - 54a^{14}b^8 + 18a^{13}b^8 + 54a^{12}b^7 + 81a^{11}b^6 \\
 &= 4a^{15}b^{12} + 81a^{11}b^4
 \end{aligned}$$

19

$$\begin{aligned}
 &a^2(2x - 3y) + a^2(3x + 4y) - a^2(5x - 2y) \\
 &= a^2\{(2x - 3y) + (3x + 4y) - (5x - 2y)\} \\
 &= a^2\{2x - 3y + 3x + 4y - 5x + 2y\} \\
 &= a^2(3y) = 3a^2y
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= 2ab\{(3a^2 - 4b^2) - (2a^2 - 3b^2) - (2a^2 - b^2)\} \\
 &= 2ab\{3a^2 - 4b^2 - 2a^2 + 3b^2 - 2a^2 + b^2\} \\
 &= 2ab(-a^2) = \underline{\underline{-2a^2b}}
 \end{aligned}$$

21 The given expression

$$\begin{aligned}
 &= x^2\{(2a + 3b + 4c) - (a + 2b + 3c) - (b + c)\} \\
 &= x^2\{2a + 3b + 4c - a - 2b - 3c - b - c\} \\
 &= x^2(a) = \underline{\underline{ax^3}}
 \end{aligned}$$

Exercise 20

$$1 \quad (2a + 3b)(a + b) = 2a \cdot a + 3b \cdot a + 2a \cdot b + 3b \cdot b$$

$$= 2a^2 + 3ab + 2ab + 3b^2 = 2a^2 + 5ab + 3b^2$$

$$2 \quad (2m - 3n)(m - n) = 2m \cdot m - 3n \cdot m - 2m \cdot n + (-3n)(-n)$$

$$= 2m^2 - 3mn - 2mn + 3n^2 = 2m^2 - 5mn + 3n^2$$

$$3 \quad (a + b + c)(a + b + c)$$

$$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$4 \quad (a - b + c)(a - b + c)$$

$$= a^2 - ab + ac - ab + b^2 - bc + ac - bc + c^2$$

$$= a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$$

$$5 \quad (a - b - c)(a - b - c)$$

$$= a^2 - ab - ac - ab + b^2 + bc - ac + bc + c^2$$

$$= a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$$

- 6 $(a-2b-3c)(2a-b-c)$
 $= 2a^2 - 4ab - 6ac - ab + 2b^2 + 3bc - ac + 2bc + 3c^2$
 $= 2a^2 + 2b^2 + 3c^2 - 5ab - 7ac + 5bc$
- 7 $(2x-3y-4z)(x-y-z)$
 $= 2x^2 - 3xy - 4xz - 2xy + 3y^2 + 4yz - 2xz + 3yz + 4z^2$
 $= 2x^2 + 3y^2 + 4z^2 - 5xy - 6xz + 7yz$
- 8 $(-5x+2a-3b)(-x-a+b)$
 $= 5x^2 - 2ax + 3bx + 5ax - 2a^2 + 3ab - 5bx + 2ab - 3b^2$
 $= 5x^2 - 2a^2 - 3b^2 + 3ax - 2bx + 5ab$
- 9 $(x^2+y^2+z^2)(x-y-z)$
 $= x^3 + xy^2 + xz^2 - x^2y - y^3 - yz^2 - x^2z - y^2z - z^3$
 $= x^3 - y^3 - z^3 - x^2y - x^2z + y^2z - y^2x + xz^2 - yz^2$
- 10 $(xy+yz+zx)(xy-yz-zx)$
 $= x^2y^2 + xy^2z + x^2yz - xy^2z - y^2z^2 - xyz^2 - x^2yz - yz^2 - x^2z^2$
 $= x^2y^2 - y^2z^2 - x^2z^2 - 2xyz^2$

Exercise 21

- 1
$$\begin{array}{r} x^3 - x + 1 \\ x^3 + x + 1 \\ \hline x^4 - x^2 + x^2 \\ + x^3 - x^2 + x \\ \hline + x^2 - x + 1 \\ \hline x^4 + x^2 + 1 \end{array}$$
- 2
$$\begin{array}{r} a^3 - ab + b^2 \\ a + b \\ \hline a^3 - a^2b + ab^2 \\ + a^2b - ab^2 + b^3 \\ \hline a^3 + b^3 \end{array}$$
- 3
$$\begin{array}{r} a^3 + ab + b^3 \\ a - b \\ \hline a^3 + a^2b + ab^2 \\ - a^2b - ab^2 - b^3 \\ \hline a^3 - b^3 \end{array}$$
- 4
$$\begin{array}{r} a^3 - 2ab + b^2 \\ a^2 + 2ab + b^2 \\ \hline a^4 - 2a^2b + a^2b^2 \\ + 2a^2b - 4a^2b^2 + 2ab^3 \\ \hline + a^2b^2 - 2ab^3 + b^4 \\ \hline a^4 - 2a^2b^2 + b^4 \end{array}$$
- 5
$$\begin{array}{r} x^4 + x^2 + 1 \\ x^4 - x^2 + 1 \\ \hline x^8 + x^6 + x^4 \\ - x^6 - x^4 - x^2 \\ \hline + x^4 + x^2 + 1 \\ \hline x^8 + x^2 + 1 \end{array}$$
- 6
$$\begin{array}{r} x^3 - x^2y^2 + y^3 \\ x^3 + x^2y^2 + y^3 \\ \hline x^6 - x^2y^2 + x^2y^2 + y^6 \\ - x^5y^2 - x^4y^4 + x^2y^5 \\ \hline + x^3y^3 - x^2y^5 + y^6 \\ \hline x^6 - x^4y^2 + 2x^2y^3 + y^6 \end{array}$$

$$\begin{array}{rcl}
 7 & m^4 - m^2 n^2 + n^4 & 8 \quad p^4 + p^2 q^2 + q^4 \\
 & \frac{m^2 + n^2}{m^6 - m^4 n^2 + m^2 n^4} & \frac{p^2 - q^2}{p^6 + p^4 q^2 + p^2 q^4} \\
 & + \frac{m^4 n^2 - m^2 n^4 + n^6}{m^6} + n^6 & \frac{-p^4 q^2 - p^2 q^4 - q^6}{p^6 - q^6}
 \end{array}$$

$$\begin{array}{rcl}
 9 & 9a^2 + 30ab + 25b^2 & \\
 & \frac{3a - 5b}{27a^2 + 90ab + 75b^2} & \\
 & - 45a^2b - 150ab^2 - 125b^3 & \\
 & \frac{27a^2 + 45ab + 75ab^2 - 125b^3}{27a^2 + 45ab + 75ab^2 - 125b^3} &
 \end{array}$$

$$\begin{array}{rcl}
 10 & a^3 - 6a^2b + 5ab^2 & \\
 & \frac{a^2 + 6ab + 5b^2}{a^6 - 6a^4b + 5a^2b^2} & \\
 & + 6a^4b - 36a^2b^2 + 30ab^3 & \\
 & + \frac{5a^2b^2 - 30a^2b^3 + 25ab^4}{-26a^2b^2 + 25ab^4} &
 \end{array}$$

$$\begin{array}{rcl}
 11 & 2a - 3b + 4c & \\
 & \frac{2a + 3b - 4c}{4a^2 - 6ab + 8ac} & \\
 & + 6ab - 9b^2 + 12bc & \\
 & - 8ac & + 12bc - 16c^2 \\
 & \frac{4a^2}{-9b^2 + 24bc - 16c^2} &
 \end{array}$$

$$\begin{array}{rcl}
 12 & x^3 - 3x^2 + 3x - 1 & \\
 & \frac{x^2 + 3x + 1}{x^6 - 3x^4 + 3x^2 - x^2} & \\
 & + 3x^4 - 9x^2 + 9x^2 - 3x & \\
 & + \frac{x^2 - 3x^2 + 3x - 1}{-5x^2 + 5x^2 - 1} &
 \end{array}$$

$$\begin{array}{rcl}
 13 & x^4 + 2ax^3 + 3a^2x^2 + 2a^3x + a^4 & \\
 & \frac{x^4 - 2ax^3 + a^4}{x^4 + 2ax^3 + 3a^2x^2 + 2a^3x^2 + a^4} & \\
 & - 2ax^3 - 4a^2x^2 - 6a^3x^2 - 4a^2a^4 - 2a^5x & \\
 & + a^2x^4 + 2a^3x^3 + 3x^2a^4 + 2a^5x + a^6 & \\
 & \frac{x^4}{-2a^2x^2} + a^6 &
 \end{array}$$

$$\begin{array}{r}
 14 \quad a^3 + 3a^2b + 3ab^2 + b^3 \\
 \underline{a^3 - 3a^2b + 3ab^2 - b^3} \\
 a^6 + 3a^5b + 3a^4b^2 + a^3b^3 \\
 - 3a^5b - 9a^4b^2 - 9a^3b^3 - 3a^2b^4 \\
 + 3a^4b^2 + 9a^3b^3 + 9a^2b^4 + 3ab^5 \\
 - a^7b^3 - 3a^2b^4 - 3ab^5 - b^6 \\
 \hline
 a^6 - 3a^4b^2 + 3a^2b^4 - b^6
 \end{array}$$

$$\begin{array}{r}
 15 \quad x^4 + 2x^3 + x^2 - 4x - 11 \\
 \underline{x^2 - 2x + 3} \\
 x^6 + 2x^5 + x^4 - 4x^3 - 11x^2 \\
 - 2x^5 - 4x^4 - 2x^3 + 8x^2 + 22x \\
 + 3x^4 + 6x^3 + 3x^2 - 12x - 33 \\
 \hline
 x^6 + 10x^3 - 33
 \end{array}$$

$$\begin{array}{r}
 16 \quad x^4 + 2x^3 + 3x^2 + 2x + 1 \\
 \underline{x^2 - 2x + 1} \\
 x^6 + 2x^5 + 3x^4 + 2x^3 + x^2 \\
 - 2x^5 - 4x^4 - 6x^3 - 4x^2 - 2x \\
 + x^4 + 2x^3 + 3x^2 + 2x + 1 \\
 \hline
 x^6 - 2x^2 + 1
 \end{array}$$

$$\begin{array}{r}
 17 \quad a^4 + a^3b + a^2b^2 + ab^3 + b^4 \\
 \underline{a^4 - a^3b + a^2b^2 - ab^3 + b^4} \\
 a^8 + a^7b + a^6b^2 + a^5b^3 + a^4b^4 \\
 - a^7b - a^6b^2 - a^5b^3 - a^4b^4 - a^3b^5 \\
 + a^6b^3 + a^5b^4 + a^4b^5 + a^3b^6 + a^2b^7 \\
 - a^6b^7 - a^4b^4 - a^3b^5 - a^2b^6 - ab^7 \\
 + a^4b^4 + a^3b^5 + a^2b^6 + ab^7 + b^8 \\
 \hline
 a^8 + a^6b^2 + a^5b^3 + a^4b^4 + a^3b^5 + a^2b^6 + ab^7 + b^8
 \end{array}$$

$$\begin{array}{r}
 18 \quad x^3 - 1y - 1z + y^2 - yz + z^2 \\
 \underline{x + y + z} \\
 x^3 - x^2y - x^2z + xy^2 - xyz + xz^2 \\
 - 1x^2y - 1y^3 - 1yz^2 + y^3 - y^2z + yz^2 \\
 + 1x^2z - xyz - xz^2 + y^2z - yz^2 + z^3 \\
 \hline
 x^3 - 3xyz + y^3 + z^3
 \end{array}$$

$$\begin{array}{r}
 19 \quad a^2 - ab - ac + b^2 - bc + c^2 \\
 \hline
 a + b + c \\
 a^2 - a^2b - a^2c + ab^2 - abc + a^2c \\
 + a^2b \quad - ab^2 - abc \quad + b^3 - b^2c + bc^2 \\
 \hline
 \quad + a^2c \quad - abc - ac^2 \quad + b^2c - bc^2 + c^3 \\
 \hline
 a^2 \quad \quad - 5abc \quad + b^3 \quad \quad + c^3
 \end{array}$$

$$\begin{array}{r}
 20 \quad 2a^7 + 5a^2b - 3ab^2 + 4b^7 \\
 \hline
 a^7 - 3a^3b + 2ab^2 - 5b^3 \\
 2a^6 + 5a^5b - 3a^4b^2 + 4a^3b^3 \\
 - 6a^5b - 15a^4b^2 + 9a^3b^3 - 12a^2b^4 \\
 + 4a^4b^3 + 10a^3b^4 - 6a^2b^4 + 8ab^5 \\
 - 10a^3b^4 - 25a^2b^4 + 15ab^5 - 20b^6 \\
 \hline
 2a^6 - a^6b - 14a^4b^2 + 13a^3b^3 - 43a^2b^4 + 23ab^5 - 20b^6
 \end{array}$$

$$\begin{array}{r}
 21 \quad a + b \qquad \qquad a^2 - b^2 \\
 a - b \qquad \qquad \frac{a^2 + b^2}{a^4 - a^2b^2} \\
 a^2 + ab \qquad \qquad \frac{\quad + a^2b^2 - b^4}{a^4 - b^4} \\
 \hline
 -ab - b^2 \\
 a^2 \quad - b^2
 \end{array}$$

$$\begin{array}{r}
 22 \quad a + b \qquad \qquad a^4 + a^2b^2 + b^4 \\
 a - b \qquad \qquad \frac{a^2 - b^2}{a^6 + a^4b^2 + a^2b^4} \\
 \hline
 a^2 + ab \qquad \qquad \frac{\quad - a^4b^2 - a^2b^4 - b^6}{a^6 - b^6} \\
 \hline
 -ab - b^2 \\
 a^2 \quad - b^2
 \end{array}$$

$$\begin{array}{r}
 23 \quad x - y \qquad \qquad x^8 + x^4y^4 + y^8 \\
 x + y \qquad \qquad \frac{x^4 - y^4}{x^{12} + x^8y^4 + x^4y^8} \\
 \hline
 x^2 - xy \qquad \qquad \frac{\quad - x^8y^4 - x^4y^8 - y^{12}}{x^{12} - y^{12}} \\
 \hline
 \quad + xy - y^2 \qquad \qquad \frac{\quad + x^4y^4 - y^8}{x^4 - y^4} \\
 \hline
 x^2 \quad + y^2 \\
 x^4 \quad - x^2y^2 \\
 \hline
 \quad + x^2y^2 - y^4 \\
 x^4 \quad - y^4
 \end{array}$$

$$\begin{array}{r}
 24 \quad \begin{array}{r} x^3 + 3xy + 5y^2 \\ \underline{x^2 - 3xy + 5y^2} \\ x^4 + 3x^2y + 5x^2y^2 \\ \quad - 3x^2y - 9x^2y^2 - 15xy^2 \\ \quad \quad + 5x^2y^2 + 15xy^2 + 25y^4 \\ \hline x^4 \quad \quad + x^2y^2 \quad \quad + 25y^4 \end{array} \\
 \end{array}
 \quad
 \begin{array}{r}
 x^4 + x^2y^2 + 25y^4 \\ \underline{x^4 - x^2y^2 + y^4} \\ 1^8 + 1^6y^2 + 25x^4y^4 \\ \quad - 1^6y^2 - x^4y^4 \quad - 251^2y^6 \\ \quad \quad + 1^4y^4 \quad \quad + x^2y^6 \quad + 25y^8 \\ \hline x^8 \quad \quad + 25x^4y^4 \quad - 24x^2y^6 + 25y^8
 \end{array}$$

$$\begin{array}{r}
 25 \quad \begin{array}{r} a - b \\ \underline{a + b} \\ a^2 - ab \\ \quad + ab - b^2 \\ \hline a^2 \quad \quad - b^2 \end{array} \\
 \end{array}
 \quad
 \begin{array}{r}
 a^4 + a^2b^2 + b^4 \\ \underline{a^2 - b^2} \\ a^6 + a^4b^2 + a^2b^4 \\ \quad - a^4b^2 - a^2b^4 - b^6 \\ \hline a^6 \quad \quad \quad - b^6
 \end{array}
 \quad
 \begin{array}{r}
 a^{12} + a^6b^6 + b^{12} \\ \underline{a^6 - b^6} \\ a^{18} + a^{12}b^6 + a^6b^{12} \\ \quad - a^{12}b^6 - a^6b^{12} - b^{18} \\ \hline a^{18} \quad \quad \quad - b^{18}
 \end{array}$$

$$\begin{array}{r}
 26 \quad \begin{array}{r} ax^2 + bx - c \\ \underline{px - q} \\ apx^3 + bpx^2 - cpx \\ \quad - aqx^2 - bqx + cq \\ \hline apx^3 + (bp - aq)x^2 - (cp + bq)x + cq \end{array}
 \end{array}$$

$$\begin{array}{r}
 27 \quad \begin{array}{r} mx^3 - nx - r \\ \underline{mx - r} \\ mnrx^3 - n^2x^3 - mx \\ \quad - mx^2 + mx + r^2 \\ \hline mnrx^3 - (n^2 + m)x^3 + r^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 28 \quad \begin{array}{r} ax^2 - bx + c \\ \underline{x^3 - bx - c} \\ ax^4 - bx^3 + cx^3 \\ \quad - abx^3 + b^2x^3 - bcx \\ \quad \quad - acx^3 + bcr - c^2 \\ \hline ax^4 - (b + ab)x^3 + (c + b^2 - ac)x^2 - c^2 \\ \quad \quad \quad = ax^4 - (1 + a)bx^3 + (c + b^2 - ac)x^2 - c^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 29 \quad \begin{array}{r} ax^3 - bx^2 + cx - d \\ \underline{bx^2 - cx + d} \\ abx^5 - b^2x^4 + bcx^3 - bdx^2 \\ \quad - acx^4 + bcr^3 - c^2x^2 + cdx \\ \quad \quad + adx^3 - bdx^2 + cdx - d^2 \\ \hline abx^5 - (b^2 + ac)x^4 + (2bc + ad)x^3 - (2bd + c^2)x^2 + 2cdx - d^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 30 \quad \frac{px^2 - (q-1)x + 1}{mx^2 - nx - s} \\
 \frac{m \cancel{px^2} - m(q-1)x^2 + m \cancel{nx} - 1 \cancel{px^2} + n(q-1)x^2 - nsx}{m \cancel{px^2} - (mq - m + np)x^2 + (ms + nq - m - ps)x^2 + (q-1-n)sx - s^2}
 \end{array}$$

$$\begin{array}{r}
 31 \quad \frac{1+a}{1+b} \quad \frac{1^2 + (a+b)1 + ab}{1^2 + (a+b)1^2 + ab1} \\
 \frac{+b1 + ab}{1^2 + (a+b)1 + ab} \quad \frac{+c1^2 + (a+b)c1 + abc}{1^2 + (a+b+c)1^2 + (ab+ac+bc)1 + abc}
 \end{array}$$

$$\begin{array}{r}
 32 \quad \frac{1-a}{1^2-a1} \quad \frac{1^2 - (a+b)1 + ab}{1^2 - (a+b)1^2 + ab1} \\
 \frac{-b1 + ab}{1^2 - (a+b)1 + ab} \quad \frac{-cx^2 + (a+b)c1 - abc}{1^2 - (a+b+c)1^2 + (ab+ac+bc)1 - abc}
 \end{array}$$

33 Worked out in the book

$$34 \quad x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2} + \frac{1}{2}} = \underline{x^1}$$

35 and 36 worked out in the book

$$37 \quad (x^{\frac{1}{2}})' = x^{\frac{1}{2}} \times x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = x^{\frac{3}{2}}, \quad x^{\frac{3}{2}} = \underline{\sqrt[3]{x^3}}$$

$$38 \quad (y^{\frac{1}{4}})^{\frac{1}{2}} = y^{\frac{1}{4}} \times y^{\frac{1}{4}} \times y^{\frac{1}{4}} \times y^{\frac{1}{4}} \times y^{\frac{1}{4}} = y^{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = y^{\frac{5}{4}}, \quad y^{\frac{5}{4}} = \underline{\sqrt[4]{y^5}}$$

$$39 \quad (z^{\frac{1}{4}})^{\frac{1}{2}} = z^{\frac{1}{4}} \times z^{\frac{1}{4}} \times z^{\frac{1}{4}} \times z^{\frac{1}{4}} = z^{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = z^1, \quad z^1 = \underline{\sqrt[4]{z^8}}$$

$$40 \quad b^{\frac{1}{2}} \times b^{\frac{5}{2}} = b^{\frac{1}{2} + \frac{5}{2}} = \underline{b^3}$$

$$41 \quad c^{\frac{1}{2}} \times c^{\frac{1}{2}} \times c^{\frac{8}{2}} = c^{\frac{1}{2} + \frac{1}{2} + \frac{8}{2}} = c^{\frac{10}{2}} = \underline{c^5}$$

$$42 \quad y^{\frac{1}{2}} \times y^{\frac{1}{2}} \times y^{\frac{1}{2}} = y^{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = y^{\frac{3}{2}} = \underline{y^{\frac{3}{2}}}$$

43 Worked out in the book

$$44 \quad y^4 \times y^{-7} = y^{4-7} = \underline{y^{-3}}$$

$$45 \quad z^{\frac{3}{2}} \times z^{-\frac{1}{2}} = z^{\frac{3}{2}-\frac{1}{2}} = z^{\frac{2}{2}} = z^1 = \underline{z}$$

46 Worked out in the book

$$47 \quad (b^{-\frac{5}{3}})^3 = b^{-5} \times b^{-\frac{5}{3}} \times b^{-\frac{5}{3}} = b^{-\frac{5}{3}-\frac{5}{3}-\frac{5}{3}} = b^{-\frac{15}{3}} b^{-5} ,$$

$$b^{-\frac{5}{3}} = \underline{\sqrt[3]{b^{-5}}}$$

$$48 \quad (c^{-\frac{2}{5}})^5 = c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} = c^{-2}$$

$$= c^{-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}} = c^{-\frac{10}{5}} = c^{-2} , \quad c^{-\frac{2}{5}} = \underline{\sqrt[5]{c^{-2}}}$$

$$49 \quad a^{-\frac{3}{4}} \times a^{\frac{1}{4}} = a^{-\frac{3}{4}+\frac{1}{4}} = a^{-\frac{2}{4}} = \underline{a^{-1}}$$

$$50 \quad x^{-\frac{5}{3}} \times x^{-\frac{4}{3}} = x^{-\frac{5}{3}-\frac{4}{3}} = x^{-\frac{9}{3}} = \underline{x^{-3}}$$

$$51 \quad -3x^{\frac{1}{2}} \times 2x^{\frac{1}{2}} = -6x^{\frac{1}{2}+\frac{1}{2}} = -6x^1 = \underline{-6x}$$

$$52 \quad 5y^{\frac{1}{2}} \times (-2y^{\frac{5}{2}}) = -2y^{\frac{1}{2}+\frac{5}{2}} = -2y^3 = \underline{-2y^3}$$

$$53 \quad 2x^{\frac{1}{2}} y^{\frac{1}{2}} \times 3x^{\frac{1}{2}} y^{\frac{1}{2}} = 6x^{\frac{1}{2}+\frac{1}{2}} y^{\frac{1}{2}+\frac{1}{2}} = 6x^1 y^1 = \underline{6x^2 y}$$

$$54 \quad (-5xy^4) \times (-3x^{\frac{2}{3}}y^{\frac{1}{3}}) = 15x^{1+\frac{2}{3}}y^{4+\frac{1}{3}} = \underline{15x^{\frac{5}{3}}y^{\frac{13}{3}}}$$

$$55 \quad 4a^{-2}b^3 \times (-\frac{1}{4}a^3b^{-5}) = -3a^{-2+3}b^{3-5} = \underline{-3ab^{-2}}$$

$$56 \quad \frac{1}{5}a^{\frac{5}{2}}y^3 \times (-\frac{1}{5}a^{\frac{5}{2}}y^{-4}) = -a^{\frac{5}{2}+\frac{5}{2}}y^{3-4} = -a^5y^{-1} = \underline{-ay^{-1}}$$

$$57 \quad (-4a^{\frac{1}{2}}b^{\frac{2}{3}}c^{\frac{1}{4}}) \times (-3a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{5}{4}}) = 12a^{-\frac{1}{2}+\frac{1}{2}}b^{\frac{2}{3}+\frac{1}{3}}c^{\frac{1}{4}+\frac{5}{4}} = 12a^1b^1c^{\frac{6}{4}}$$

$$= \underline{12a^2b^2c^2}$$

$$58 \quad (-5x^{\frac{2}{3}}y^{\frac{1}{5}}z^{\frac{4}{5}}) \times (-3x^{\frac{1}{3}}y^{\frac{2}{5}}z^{-\frac{1}{5}})$$

$$= 15x^{\frac{2}{3}+\frac{1}{3}}y^{\frac{1}{5}+\frac{2}{5}}z^{\frac{4}{5}-\frac{1}{5}} = 15x^1y^{\frac{3}{5}}z^{\frac{3}{5}}$$

$$= \underline{15xyz}$$

$$\begin{aligned}
 59 \quad & (-6a^7 b^{-4} c^8) \times (5a^3 b^{\frac{1}{2}} c^{-\frac{1}{2}}) \\
 &= -30a^{\frac{7}{2}+\frac{1}{2}} b^{-4+\frac{1}{2}} c^{8-\frac{1}{2}} \\
 &= -30a^4 b^{\frac{7}{2}} c^{\frac{15}{2}} = -30a^2 b^{3\frac{1}{2}} c^7
 \end{aligned}$$

$$\begin{aligned}
 60 \quad & (-4a^{\frac{5}{2}} x^{\frac{3}{2}} y^{-4}) \times (-19a^{-1} x^{-\frac{1}{2}} y^{-1}) \\
 &= 76a^{\frac{5}{2}+\frac{1}{2}} x^{\frac{3}{2}-\frac{1}{2}} y^{-4-1} \\
 &= 76a^3 x^1 y^{-5} \\
 &= \underline{76a^3 x y^{-5}}
 \end{aligned}$$

$$\begin{aligned}
 61 \quad & \frac{a^{\frac{1}{2}} + b^{\frac{1}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}} \\
 & \frac{a^{\frac{1}{2}} + b^{\frac{1}{2}}}{a + a^{\frac{1}{2}} b^{\frac{1}{2}}} \\
 & \frac{+ a^{\frac{1}{2}} b^{\frac{1}{2}} + b}{a + 2a^{\frac{1}{2}} b^{\frac{1}{2}} + b}
 \end{aligned}$$

$$\begin{aligned}
 62 \quad & \frac{a^{\frac{1}{2}} - b^{\frac{1}{2}}}{a^{\frac{1}{2}} - b^{\frac{1}{2}}} \\
 & \frac{a^{\frac{1}{2}} - b^{\frac{1}{2}}}{a - a^{\frac{1}{2}} b^{\frac{1}{2}}} \\
 & \frac{- a^{\frac{1}{2}} b^{\frac{1}{2}} + b}{a - 2a^{\frac{1}{2}} b^{\frac{1}{2}} + b}
 \end{aligned}$$

$$\begin{aligned}
 63 \quad & \frac{3x^3 - 4y^3}{3x^2 + 4y^3} \\
 & \frac{9x^{\frac{3}{2}} - 12x^{\frac{1}{2}} y^3}{9x^{\frac{3}{2}} - 12x^{\frac{1}{2}} y^3 + 16y^{\frac{3}{2}}} \\
 & \frac{+ 12x^{\frac{1}{2}} y^3 - 16y^{\frac{3}{2}}}{9x^{\frac{3}{2}} - 16y^{\frac{3}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 64 \quad & \frac{a^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}} \\
 & \frac{a^{\frac{1}{2}} + b^{\frac{1}{2}}}{a - a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}}} \\
 & \frac{+ a^{\frac{1}{2}} b^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + b}{a + b}
 \end{aligned}$$

$$\begin{aligned}
 65 \quad & \frac{x^{\frac{1}{2}} + x^{\frac{1}{2}} y^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}} \\
 & \frac{x + x^{\frac{1}{2}} y^{\frac{1}{2}} + x^{\frac{1}{2}} y^{\frac{1}{2}}}{x + x^{\frac{1}{2}} y^{\frac{1}{2}} + x^{\frac{1}{2}} y^{\frac{1}{2}} - x^{\frac{1}{2}} y^{\frac{1}{2}} - x^{\frac{1}{2}} y^{\frac{1}{2}} - y} \\
 & \frac{- x^{\frac{1}{2}} y^{\frac{1}{2}} - x^{\frac{1}{2}} y^{\frac{1}{2}} - y}{-y}
 \end{aligned}$$

$$\begin{aligned}
 66 \quad & \frac{a^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}}{a^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}} \\
 & \frac{a^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}}}{a^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}}} \\
 & \frac{+ a^{\frac{1}{2}} b^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}}}{+ a^{\frac{1}{2}} b^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}} \\
 & \frac{+ a^{\frac{1}{2}} b^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}}{a^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\frac{1}{2}} + b^{\frac{1}{2}}}
 \end{aligned}$$

$$\begin{array}{r}
 67 \quad 2x^{\frac{1}{3}} - 5x^{\frac{1}{3}}y^{\frac{2}{3}} - 3y^{\frac{4}{3}} \\
 \underline{2x^{\frac{1}{3}} + 5x^{\frac{1}{3}}y^{\frac{2}{3}} - 3y^{\frac{4}{3}}} \\
 4x^{\frac{2}{3}} - 10x^{\frac{2}{3}}y^{\frac{2}{3}} - 6x^{\frac{4}{3}}y^{\frac{1}{3}} \\
 \quad + 10x^{\frac{2}{3}}y^{\frac{1}{3}} - 25x^{\frac{1}{3}}y^{\frac{4}{3}} - 15x^{\frac{2}{3}}y^{\frac{2}{3}} \\
 \quad \quad - 6x^{\frac{1}{3}}y^{\frac{4}{3}} + 15x^{\frac{2}{3}}y^{\frac{2}{3}} + 9y^{\frac{8}{3}} \\
 \hline
 4x^{\frac{2}{3}} \quad \quad - 37x^{\frac{1}{3}}y^{\frac{4}{3}} \quad \quad + 9y^{\frac{8}{3}}
 \end{array}$$

$$\begin{array}{r}
 68 \quad a^{\frac{7}{3}} + a^{\frac{2}{3}}b^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} + ab + a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{5}{3}} \\
 \underline{a^{\frac{1}{3}} - b^{\frac{1}{3}}} \\
 a^{\frac{2}{3}} + a^{\frac{5}{3}}b^{\frac{1}{3}} + a^{\frac{4}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b + ab^{\frac{4}{3}} + a^{\frac{1}{3}}b^{\frac{5}{3}} \\
 \quad - a^{\frac{5}{3}}b^{\frac{1}{3}} - a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b - ab^{\frac{4}{3}} - a^{\frac{1}{3}}b^{\frac{5}{3}} - b^{\frac{2}{3}} \\
 \hline
 a^{\frac{2}{3}} \quad - b^{\frac{2}{3}}
 \end{array}$$

$$\begin{array}{r}
 69 \quad x^{\frac{1}{2}} - xy^{\frac{1}{2}} + x^{\frac{1}{2}}y - y^{\frac{3}{2}} \\
 \underline{x^{\frac{1}{2}} + y^{\frac{1}{2}}} \\
 x^2 - x^{\frac{1}{2}}y^{\frac{1}{2}} + xy - x^{\frac{1}{2}}y^{\frac{1}{2}} \\
 \quad - x^{\frac{1}{2}}y^{\frac{1}{2}} - xy + x^{\frac{1}{2}}y^{\frac{3}{2}} - y^2 \\
 \hline
 x^2 \quad - y^2
 \end{array}$$

$$\begin{array}{r}
 70 \quad a^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b + b^{\frac{1}{2}} \\
 \underline{a^{\frac{1}{2}} - b^{\frac{1}{2}}} \\
 a + a^{\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b + a^{\frac{1}{2}}b^{\frac{1}{2}} \\
 \quad - a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b - a^{\frac{1}{2}}b^{\frac{3}{2}} - b^{\frac{1}{2}} \\
 \hline
 a \quad - b^{\frac{3}{2}}
 \end{array}$$

$$\begin{array}{r}
 71 \quad x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{1}{3}} + y^{\frac{2}{3}} - y^{\frac{1}{3}}z^{\frac{1}{3}} + z^{\frac{2}{3}} \\
 \underline{x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}}} \\
 x - x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{2}{3}}z^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + x^{\frac{1}{3}}z^{\frac{2}{3}} \\
 \quad - x^{\frac{1}{3}}y^{\frac{1}{3}} \quad \quad - x^{\frac{1}{3}}y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} \quad \quad + y - y^{\frac{2}{3}}z^{\frac{1}{3}} + y^{\frac{1}{3}}z^{\frac{2}{3}} \\
 \quad \quad \quad + x^{\frac{2}{3}}z^{\frac{1}{3}} \quad \quad - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{2}{3}} \quad \quad + y^{\frac{2}{3}}z^{\frac{1}{3}} - y^{\frac{1}{3}}z^{\frac{2}{3}} + z \\
 \hline
 x \quad - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} \quad \quad + y \quad \quad \quad + z
 \end{array}$$

$$72 \quad a^{2n} - a^{2n-1} + a^{2n-2}$$

$$\frac{a^{2n} + a^{2n-1}}{a^{2n} - a^{2n-1} + a^{2n-2}} \\ + \frac{a^{2n-1} - a^{2n-2}}{a^{2n-1} - a^{2n-2} + a^{2n-3}}$$

$$73 \quad a^5 - 4a^4b + 4a^3b^2 - b^5$$

$$\frac{a^5 - 4a^4b + 4a^3b^2 - b^5}{a^5 - 4a^4b + 4a^3b^2 - a^2b^3} \\ - \frac{2a^4b + 8a^3b^2 - 8a^2b^3 + 2ab^4}{a^5 - 6a^4b + 13a^3b^2 - 13a^2b^3 + 6ab^4 - b^5}$$

$$74 \quad x^3 + 3x^2y^2 + 2y^3$$

$$\frac{x^3 + 3x^2y^2 + 2y^3}{x^3 + 3x^2y^2 + 2x^2y^3} \\ - \frac{3x^2y^2 - 6x^2y^3 + 6x^2y^4 + 4y^5}{-5x^2y^3 + 4y^5}$$

$$75 \quad 2a^5 + 3a^{\frac{5}{2}}b^{\frac{5}{2}} - 5b^5$$

$$\frac{2a^5 + 3a^{\frac{5}{2}}b^{\frac{5}{2}} + 5b^5}{4a^{10} + 6a^{\frac{15}{2}}b^{\frac{15}{2}} - 10a^5b^5} \\ + \frac{6a^{\frac{15}{2}}b^{\frac{15}{2}} + 9a^5b^5 - 15a^{\frac{5}{2}}b^{\frac{5}{2}}}{+ 10a^5b^5 - 15a^{\frac{5}{2}}b^{\frac{5}{2}} - 25b^5} \\ \frac{4a^{10} + 12a^{\frac{15}{2}}b^{\frac{15}{2}} + 9a^5b^5}{- 25b^5}$$

$$76 \quad 3x^4 - 5x^{\frac{7}{2}}y^{\frac{1}{2}} + 7y^4$$

$$\frac{3x^4 + 5x^{\frac{7}{2}}y^{\frac{1}{2}} - 7y^4}{9x^{\frac{7}{2}} - 15x^{\frac{7}{2}}y^{\frac{1}{2}} + 21x^{\frac{7}{2}}y^{\frac{1}{2}}} \\ + \frac{15x^{\frac{7}{2}}y^{\frac{1}{2}} - 25x^{\frac{7}{2}}y^{\frac{1}{2}} + 35x^{\frac{7}{2}}y^{\frac{1}{2}}}{- 21x^{\frac{7}{2}}y^{\frac{1}{2}} + 35x^{\frac{7}{2}}y^{\frac{1}{2}} - 49y^{\frac{1}{2}}} \\ \frac{9x^{\frac{7}{2}} - 25x^{\frac{7}{2}}y^{\frac{1}{2}} + 70x^{\frac{7}{2}}y^{\frac{1}{2}} - 49y^{\frac{1}{2}}}{- 25x^{\frac{7}{2}}y^{\frac{1}{2}} + 70x^{\frac{7}{2}}y^{\frac{1}{2}} - 49y^{\frac{1}{2}}}$$

77 The given expression

$$= 2x^5 - 3x^{\frac{5}{2}}a + 4x^2a^2 - 5x^{-1}a^3 + 6x^{-1}a^4 - 7x^{-\frac{7}{2}}a^5 \\ + 8x^{-4}a^6$$

78 The given expression

$$= -3a^{-\frac{7}{2}} - 2a^{-4}b^{-2} + 5a^{-2}b^{-4} - 7a^{-\frac{5}{2}}b^{-7} + 6a^{\frac{5}{2}}b^{-4} \\ - 5a^{-\frac{1}{2}}b^{-4} + 8a^{\frac{5}{2}}b^{-2}$$

Exercise 22

$$1 \quad (3x+4)^2 = (3x)^2 + 2(3x)4 + 4^2 \\ = 9x^2 + 24x + 16$$

$$2 \quad (7x+8)^2 = (7x)^2 + 2(7x)8 + 8^2 \\ = 49x^2 + 112x + 64$$

$$3 \quad (a+5b)^2 = a^2 + 2(a)(5b) + (5b)^2 \\ = a^2 + 10ab + 25b^2$$

$$4 \quad (2b+7b)^2 = (2a)^2 + 2(2a)(7b) + (7b)^2 \\ = 4a^2 + 28ab + 49b^2$$

$$5 \quad (3x+8y)^2 = (3x)^2 + 2(3x)(8y) + (8y)^2 \\ = 9x^2 + 48xy + 64y^2$$

$$6 \quad (5m+8n)^2 = (5m)^2 + 2(5m)(8n) + (8n)^2 \\ = 25m^2 + 80mn + 64n^2$$

$$7 \quad (ax+3by)^2 = (ax)^2 + 2(ax)(3by) + (3by)^2 \\ = a^2x^2 + 6abxy + 9b^2y^2$$

$$8 \quad (4ab+c^2)^2 = (4ab)^2 + 2(4ab)c^2 + (c^2)^2 \\ = 16a^2b^2 + 8abc^2 + c^4$$

$$9 \quad (2a^2+5b^2)^2 = (2a^2)^2 + 2(2a^2)(5b^2) + (5b^2)^2 \\ = 4a^4 + 20a^2b^2 + 25b^4$$

$$10 \quad (4m^3+n^2)^2 = (4m^3)^2 + 2(4m^3)(n^2) + (n^2)^2 \\ = 16m^6 + 8m^3n^2 + n^4$$

- 11 $(a+2b+3c)^2 = \{(a+2b)+3c\}^2$
 $= (a+2b)^2 + 2 \cdot 3c(a+2b) + (3c)^2$
 $= a^2 + 2 \cdot a \cdot 2b + 4b^2 + 6ac + 12bc + 9c^2$
 $= a^2 + 4b^2 + c^2 + 4ab + 6ac + 12bc$
- 12 $(ab+bc+ca)^2 = \{(ab+bc)+ca\}^2$
 $= (ab+bc)^2 + 2(ab+bc)ca + (ca)^2$
 $= a^2b^2 + 2ab^2c + b^2c^2 + 2a^2bc + 2abc^2 + c^2a^2$
 $= a^2b^2 + b^2c^2 + c^2a^2 + 2a^2bc + 2ab^2c + 2abc^2$
- 13 $(2p+3q+4r)^2 = \{(2p+3q)+4r\}^2$
 $= (2p+3q)^2 + 2(2p+3q)4r + (4r)^2$
 $= 4p^2 + 12pq + 9q^2 + 16pr + 24qr + 16r^2$
 $= 4p^2 + 9q^2 + 16r^2 + 12pq + 16pr + 24qr$
- 14 $(x^2+y^2+z^2)^2 = \{(x^2+y^2)+z^2\}^2$
 $= (x^2+y^2)^2 + 2(x^2+y^2)z^2 + (z^2)^2$
 $= x^4 + 2x^2y^2 + y^4 + 2x^2z^2 + 2y^2z^2 + z^4$
 $= x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2y^2z^2$
- 15 $(2x+3y+4z)^2 = \{(2x+3y)+4z\}^2$
 $= (2x+3y)^2 + 2(2x+3y)4z + (4z)^2$
 $= 4x^2 + 12xy + 9y^2 + 16xz + 24yz + 16z^2$
 $= 4x^2 + 9y^2 + 16z^2 + 12xy + 16xz + 24yz$
- 16 $(x^2+y^2+z^2)^2 = \{(x^2+y^2)+z^2\}^2$
 $= (x^2+y^2)^2 + 2(x^2+y^2)z^2 + (z^2)^2$
 $= x^4 + 2x^2y^2 + y^4 + 2x^2z^2 + 2y^2z^2 + z^4$
 $= x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2y^2z^2$
- 17 $(x+y+2a+3b)^2 = \{(x+y)+(2a+3b)\}^2$
 $= (x+y)^2 + 2(x+y)(2a+3b) + (2a+3b)^2$
 $= x^2 + y^2 + 2xy + 2(2ax+2ay+3bx+3by) + 4a^2 + 9b^2 + 12ab$
 $= x^2 + y^2 + 2xy + 4ax + 4ay + 6bx + 6by + 4a^2 + 9b^2 + 12ab$
 $= x^2 + y^2 + 4a^2 + 9b^2 + 2xy + 4ax + 6bx + 4ay + 6by + 12ab$
- 18 $(3a+4b+c+2d)^2 = \{(3a+4b)+(c+2d)\}^2$
 $= (3a+4b)^2 + 2(3a+4b)(c+2d) + (c+2d)^2$
 $= 9a^2 + 24ab + 16b^2 + 2(3ac+4bc+6ad+8bd) + c^2 + 4cd + 4d^2$
 $= 9a^2 + 16b^2 + c^2 + 4d^2 + 24ab + 6ac + 12ad + 8bc + 16bd + 4cd$

- 19 $(2a+x+4y+3z)^2 = \{(2a+x) + (4y+3z)\}^2$
 $= (2a+x)^2 + 2(2a+x)(4y+3z) + (4y+3z)^2$
 $= 4a^2 + 4ax + x^2 + 2(8ay + 6az + 4xy + 3xz) + 16y^2 + 24yz + 9z^2$
 $= 4a^2 + 4ax + x^2 + 16ay + 12az + 8xy + 6xz + 16y^2 + 24yz + 9z^2$
 $= 4a^2 + x^2 + 16y^2 + 9z^2 + 4ax + 16ay + 12az + 8xy + 6xz + 24yz$
- 20 $(4m+3n+3p+2q)^2$
 $= \{(4m+3n) + (3p+2q)\}^2$
 $= (4m+3n)^2 + 2(4m+3n)(3p+2q) + (3p+2q)^2$
 $= 16m^2 + 24mn + 9n^2 + 2(12pm + 9pn + 8qm + 6qn)$
 $\quad + 9p^2 + 12pq + 4q^2$
 $= 16m^2 + 24mn + 9n^2 + 24pm + 18pn + 16qm + 12qn$
 $\quad + 9p^2 + 12pq + 4q^2$
 $= 16m^2 + 9n^2 + 9p^2 + 4q^2 + 24mn + 24pm + 16qm + 18pn$
 $\quad + 12nq + 12pq$
- 21 Putting a for $(x+y)$ and b for $(x-y)$, we have the given expression $= a^2 + 2ab + b^2 = (a+b)^2$
 $= \{(x+y) + (x-y)\}^2 = (2x)^2 = \underline{4x^2}$
- 22 Putting a for $(x-y+z)$ and b for $(y+z-x)$, we have the given expression $= a^2 + b^2 + 2ab = (a+b)^2$
 $= \{(x-y+z) + (y+z-x)\}^2$
 $= (2z)^2 = \underline{4z^2}$
- 23 Putting x for $(2a-3b+4c)$ and y for $(2a+3b-4c)$, we have the given expression $= x^2 + y^2 + 2xy = (x+y)^2$
 $= \{(2a-3b+4c) + (2a+3b-4c)\}^2$
 $= (4a)^2 = \underline{16a^2}$
- 24 Putting x for $(5a-7b)$ and y for $(9b-4a)$ we have the given expression $= x^2 + 2xy + y^2 = (x+y)^2$
 $= \{(5a-7b) + (9b-4a)\}^2 = (a+2b)^2$
 $= \underline{a^2 + 4ab + 4b^2}$
- 25 Putting a for $(2x-5y-3z)$ and b for $(6y+3z-x)$ we have the given expression $= a^2 + b^2 + 2ab = (a+b)^2$
 $= \{2x-5y-3z+6y+3z-x\}^2$
 $= (x+y)^2 = \underline{x^2 + 2xy + y^2}$

$$\begin{aligned}
 23 \quad \text{The given expression} &= (2x)^2 + 2(2x)7 + 7^2 \\
 &= (2x+7)^2 = (-16+7)^2 = (-9)^2 = 81
 \end{aligned}$$

$$\begin{aligned}
 27 \quad \text{The given expression} &= (5a)^2 + 2(5a)(4b) + (4b)^2 \\
 &= (5a+4b)^2 = (-90+92)^2 = (2)^2 = 4
 \end{aligned}$$

$$\begin{aligned}
 28 \quad \text{The given expression} &= (9x)^2 + 2(9x)(5y) + (5y)^2 \\
 &= (9x+5y)^2 = (135-135)^2 = 0
 \end{aligned}$$

$$\begin{aligned}
 29 \quad \text{The given expression} &= (4m)^2 + 2(4m)(7n) + (7n)^2 \\
 &= (4m+7n)^2 = (-52+49)^2 = (3)^2 = 9
 \end{aligned}$$

$$\begin{aligned}
 30 \quad \text{The given expression} &= (8a)^2 + 2(8a)c + c^2 \\
 &= (8a+c)^2 = (48-49)^2 = (-1)^2 = 1
 \end{aligned}$$

$$\begin{aligned}
 31 \quad \text{The given expression} &= (9x)^2 + 2(9x)z + z^2 \\
 &= (9x+z)^2 = (63-67)^2 = (-4)^2 = 16
 \end{aligned}$$

$$\begin{aligned}
 32 \quad \text{The given expression} &= 6p + 2(6p)(11q) + (11q)^2 \\
 &= (6p+11q)^2 = (72-77)^2 = (-5)^2 = 25
 \end{aligned}$$

$$\begin{aligned}
 33 \quad m^2 + \left(\frac{1}{m}\right)^2 &= \left(m + \frac{1}{m}\right)^2 - 2m \cdot \frac{1}{m} = \left(m + \frac{1}{m}\right)^2 - 2 \\
 &= (4)^2 - 2 = 16 - 2 = 14
 \end{aligned}$$

Exercise 23

$$\begin{aligned}
 1 \quad (2x-7)^2 &= (2x)^2 - 2(2x)7 + (7)^2 \\
 &= 4x^2 - 28x + 49
 \end{aligned}$$

$$\begin{aligned}
 2 \quad (8x-5)^2 &= (8x)^2 - 2(8x)5 + (5)^2 \\
 &= 64x^2 - 80x + 25
 \end{aligned}$$

$$\begin{aligned}
 3 \quad (ax-by)^2 &= (ax)^2 - 2(ax)(by) + (by)^2 \\
 &= a^2x^2 - 2abxy + b^2y^2
 \end{aligned}$$

- 4 $(-5x - 3y)^2 = \{(-5x) - 3y\}^2$
 $= (-5x)^2 - 2(-5x)(3y) + (3y)^2$
 $= 25x^2 + 30xy + 9y^2$
- 5 $(3m - 8n)^2 = (3m)^2 - 2(3m)(8n) + (8n)^2$
 $= 9m^2 - 48mn + 64n^2$
- 6 $(-ab - cd)^2 = \{(-ab) - cd\}^2$
 $= (-ab)^2 - 2(-ab)cd + (cd)^2$
 $= a^2b^2 + 2abcd + c^2d^2$
- 7 $(a^2b - c^2d)^2 = (a^2b)^2 - 2(a^2b)(c^2d) + (c^2d)^2 = a^4b^2 - 2a^2bc^2d + c^4d^2$
- 8 $(x^2 - 2yz)^2 = (x^2)^2 - 2(x^2)(2yz) + (2yz)^2 = x^4 - 4x^2yz + 4y^2z^2$
- 9 $(-mnp - q)^2 = \{(-mnp) - q\}^2$
 $= (-mnp)^2 - 2(-mnp)(q) + (q)^2$
 $= m^2n^2p^2 + 2mnpq + q^2$
- 10 $(2a^3 - 5b^4)^2 = (2a^3)^2 - 2(2a^3)(5b^4) + (5b^4)^2$
 $= 4a^6 - 20a^3b^4 + 25b^8$
- 11 $(a - 2b - 3c)^2 = \{a - (2b + 3c)\}^2$
 $= a^2 - 2a(2b + 3c) + (2b + 3c)^2$
 $= a^2 - 4ab - 6ac + 4b^2 + 12bc + 9c^2$
 $= a^2 + 4b^2 + 9c^2 - 4ab - 6ac + 12bc$
- 12 $(2x - 3y - 4z)^2 = \{2x - (3y + 4z)\}^2$
 $= (2x)^2 - 2(2x)(3y + 4z) + (3y + 4z)^2$
 $= 4x^2 - 12xy - 16xz + 9y^2 + 24yz + 16z^2$
 $= 4x^2 + 9y^2 + 16z^2 - 12xy - 16xz + 24yz$
- 13 $(3m - 4n - 5q)^2 = \{3m - (4n + 5q)\}^2$
 $= (3m)^2 - 2(3m)(4n + 5q) + (4n + 5q)^2$
 $= 9m^2 - 24mn - 30mq + 16n^2 + 40nq + 25q^2$
 $= 9m^2 + 16n^2 + 25q^2 - 24mn - 30mq + 40nq$
- 14 $(a^2 - 3b^2 - 5c^2)^2 = \{a^2 - (3b^2 + 5c^2)\}^2$
 $= (a^2)^2 - 2a^2(3b^2 + 5c^2) + (3b^2 + 5c^2)^2$
 $= a^4 - 6a^2b^2 - 10a^2c^2 + 9b^4 + 30b^2c^2 + 25c^4$
 $= a^4 + 9b^4 + 25c^4 - 6a^2b^2 - 10a^2c^2 + 30b^2c^2$

- 15 $(1 - y - a - b)^2 = \{(1 - y) - (a + b)\}^2$
 $= (1 - y)^2 - 2(1 - y)(a + b) + (a + b)^2$
 $= 1^2 - 2 \cdot 1 \cdot y + y^2 - 2(ax + bx - ay - by)$
 $\quad \quad \quad + a^2 + 2ab + b^2$
 $= 1^2 + y^2 + a^2 + b^2 - 2 \cdot 1 \cdot y - 2ax - 2bx + 2ay$
 $\quad \quad \quad + 2by + 2ab$
- 16 $(a - 2x - 3b - 4y)^2 = \{(a - 2x) - (3b + 4y)\}^2$
 $= (a - 2x)^2 - 2(a - 2x)(3b + 4y) + (3b + 4y)^2$
 $= a^2 - 4ax + 4x^2 - 2(3ab + 4ay - 6bx - 8xy)$
 $\quad \quad \quad + 9b^2 + 24by + 16y^2$
 $= a^2 + 4x^2 + 9b^2 + 16y^2 - 4ax - 6ab - 8ay$
 $\quad \quad \quad + 12bx + 16xy + 24by$
- 17 $(90 - 1)^2 = (90)^2 - 2 \cdot 90 + 1 = 8100 - 180 + 1 = 7921$
- 18 $(120 - 3)^2 = (120)^2 - 2(120) \cdot 3 + (3)^2 = 14400 - 720 + 9 = 13689$
- 19 $(500 - 2)^2 = (500)^2 - 2(500)(2) + (2)^2 = 250000 - 2000 + 4$
 $= 248004$
- 20 $(1000 - 7)^2 = (1000 - 7)^2 = (1000)^2 - 2(1000) \cdot 7 + (7)^2$
 $= 1000000 - 14000 + 49 = 986049$
- 21 Putting x for $(a + 3b)$ and y for $(a - 3b)$ we have the given expression $= x^2 - 2xy + y^2 = (x - y)^2 = \{(a + 3b) - (a - 3b)\}^2$
 $= (6b)^2 = \underline{36b^2}$
- 22 Putting x for $(2a - 4b + 5c)$ and y for $(2a + 4b + 5c)$, we have the given expression $= x^2 + y^2 - 2xy = (x - y)^2$
 $= \{(2a - 4b + 5c) - (2a + 4b + 5c)\}^2 = (-8b)^2 = \underline{64b^2}$
- 23 Putting x for $(3a + 5b + 7c)$ and y for $(7c - 4a + 5b)$, we have the given expression $= x^2 + y^2 - 2xy = (x - y)^2$
 $= \{(3a + 5b + 7c) - (7c - 4a + 5b)\}$
 $= (7a)^2 = \underline{49a^2}$
- 24 Putting a for $(2x^2 - y^2 - 5z^2)$ and b for $(6x^2 + 2x^2 - y^2)$, we have the given expression $= a^2 - 2ab + b^2$
 $= (a - b)^2 = \{(2x^2 - y^2 - 5z^2) - (6x^2 + 2x^2 - y^2)\}^2$
 $= (-11x^2)^2 = \underline{121x^4}$

25 Putting x for $(ab - bc + ca)$ and y for $(ab + 4bc + 2ca)$, we have
 the given expression $= x^2 + y^2 - 2xy = (x - y)^2$
 $= \{(ab - bc + ca) - (ab + 4bc + 2ca)\}^2$
 $= (-5bc - ac)^2 = (-5bc)^2 - 2(-5bc)(ac) + (ac)^2$
 $= \underline{25b^2c^2 - 10abc^2 + a^2c^2}$

26 The given expression
 $= (ab)^2 - 2(ab)(bc) + (bc)^2$
 $= (ab - bc)^2 = (28 - 30)^2 = (-2)^2 = 4$

27 The given expression
 $= (xy)^2 - 2(xy)(12x) + (12x)^2$
 $= (xy - 12x)^2 = (63 - 72)^2 = (-9)^2 = 81$

28 The given expression
 $= \{5(x + y)\}^2 + x^2 - 2(x)\{5(x + y)\}$
 $= \{5(x + y) - x\}^2 = \{5(47 - 22) - 129\}^2$
 $= \{5 \cdot 25 - 129\}^2 = \{125 - 129\}^2 = (-4)^2 = 16.$

29 The given expression
 $= (3c)^2 - 2(3c)\{7(a + b)\} + \{7(a + b)\}^2$
 $= \{3c - 7(a + b)\}^2 = \{135 - 7(-37 + 57)\}^2$
 $= \{135 - 7 \cdot 20\}^2 = \{135 - 140\}^2 = (-5)^2 = 25$

30 The given expression
 $= \{8(7p - 5q)\}^2 - 2\{8(7p - 5q)\}(6r) + (6r)^2$
 $= \{8(7p - 5q) - 6r\}^2 = \{8(196 - 160) - 276\}^2$
 $= \{8 \cdot 36 - 276\}^2 = (288 - 276)^2 = (12)^2 = 144$

31 We have $c^2 + \frac{1}{c^2} = \left(c - \frac{1}{c}\right)^2 + 2c \cdot \frac{1}{c}$
 $= \left(c - \frac{1}{c}\right)^2 + 2 = (4)^2 + 2 = 16 + 2 = 18$

Exercise 24

1 $(ax + by)(ax - by) = (ax)^2 - (by)^2 = a^2x^2 - b^2y^2$

2 $(5x + 8)(5x - 8) = (5x)^2 - (8)^2 = 25x^2 - 64$

3 $(cx + d^2)(cx - d^2) = (cx)^2 - (d^2)^2 = c^2x^2 - d^4$

- 4 $(ab+bc)(ab-bc)=(ab)^2-(bc)^2=a^2b^2-b^2c^2$
- 5 $(7a+3b)(7a-3b)=(7a)^2-(3b)^2=49a^2-9b^2$
- 6 $(4a^2-5b^2)(4a^2+5b^2)=(4a^2)^2-(5b^2)^2=16a^4-25b^4$
- 7 $(m^4-3n)(m^2+3n)=(m^2)^2-(3n)^2=m^4-9n^2$
- 8 $(2ab-7bc)(2ab+7bc)=(2ab)^2-(7bc)^2=4a^2b^2-49b^2c^2$
- 9 $(\tau+1)(\tau-1)(\tau^2+1)=(\tau^2-1)(\tau^2+1)=\tau^4-1$
- 10 $(a^2+b^2)(a^2-b^2)(a^4+b^4)=(a^4-b^4)(a^4+b^4)=a^8-b^8$
- 11 $(a+b+c)(a+b-c)$
 $=\{(a+b)+c\}\{(a+b)-c\}$
 $=(a+b)^2-c^2=a^2+2ab+b^2-c^2$
- 12 $(a+b+c)(a-b-c)$
 $=\{a+(b+c)\}\{a-(b+c)\}$
 $=a^2-(b+c)^2=a^2-(b^2+2bc+c^2)$
 $=a^2-b^2-2bc-c^2$
- 13 $(m^2+mn+n^2)(m^2-mn+n^2)$
 $=\{(m^2+n^2)+mn\}\{(m^2+n^2)-mn\}$
 $=m^4+2m^2n^2+n^4-m^2n^2=m^4+m^2n^2+n^4$
- 14 $(ax+by-cz)(ax-by+cz)$
 $=\{ax+(by-cz)\}\{ax-(by-cz)\}$
 $=a^2x^2-(by-cz)^2=a^2x^2-(b^2y^2-2bcyz+c^2z^2)$
 $=a^2x^2-b^2y^2+2bcyz-c^2z^2$
- 15 $(a-2b+3c)(a+2b-3c)$
 $=\{a-(2b-3c)\}\{a+(2b-3c)\}$
 $=a^2-(2b-3c)^2=a^2-(4b^2-12bc+9c^2)$
 $=a^2-4b^2+12bc-9c^2$
- 16 $(x^2-x+1)(x^2+x+1)$
 $=\{(x^2+1)-x\}\{(x^2+1)+x\}$
 $=(x^2+1)^2-x^2=x^4+2x^2+1-x^2=x^4+x^2+1$
- 17 $(x^4-x^2+1)(x^4+x^2+1)$
 $=\{(x^4+1)-x^2\}\{(x^4+1)+x^2\}$
 $=(x^4+1)^2-x^4=x^8+2x^4+1-x^4=x^8+x^4+1$

$$\begin{aligned}
 18 \quad & (x^2 + 2xy + y^2)(x^2 - 2xy + y^2) \\
 &= (x^2 + y^2)^2 - (2xy)^2 \\
 &= x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2 = x^4 + 4y^4
 \end{aligned}$$

$$\begin{aligned}
 19 \quad & (a^2 + ab\sqrt{2} + b^2)(a^2 - ab\sqrt{2} + b^2) \\
 &= \{(a^2 + b^2) + ab\sqrt{2}\}\{(a^2 + b^2) - ab\sqrt{2}\} \\
 &= (a^2 + b^2)^2 - (ab\sqrt{2})^2 \\
 &= a^4 + 2a^2b^2 + b^4 - 2a^2b^2 = a^4 + b^4
 \end{aligned}$$

$$\begin{aligned}
 20 \quad & (x^2 - 2x + 1)(x^2 + 2x + 1) \\
 &= \{(x^2 + 1)^2 - 2x\}\{(x^2 + 1) + 2x\} \\
 &= (x^2 + 1)^2 - 4x^2 = (x^2 + 1) - 2x^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence the given expression} &= \{(x^4 + 1) - 2x^2\}\{(x^4 + 1) + 2x^2\} \\
 &= (x^4 + 1)^2 - 4x^4 = x^8 - 2x^4 + 1
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & (a + b - c)^2 - (a - b + c)^2 \\
 &= (a + b - c + a - b + c)\{(a + b - c) - (a - b + c)\} \\
 &= 2a\{a + b - c - a + b - c\} = 2a(2b - 2c) = 4a(b - c)
 \end{aligned}$$

$$\begin{aligned}
 22 \quad & (a - 2b + 3c)^2 - (a + 2b - 3c)^2 \\
 &= \{(a - 2b + 3c) + (a + 2b - 3c)\} \times \{(a - 2b + 3c) - (a + 2b - 3c)\} \\
 &= \{2a\}\{-4b + 6c\} = 2a \cdot 2(3c - 2b) = 4a(3c - 2b)
 \end{aligned}$$

$$\begin{aligned}
 23 \quad & (x^2 + xy + y^2)^2 - (x^2 - xy + y^2)^2 \\
 &= \{(x^2 + xy + y^2) + (x^2 - xy + y^2)\} \times \{(x^2 + xy + y^2) - (x^2 - xy + y^2)\} \\
 &= \{2x^2 + 2y^2\}\{2xy\} = 2(x^2 + y^2) \cdot 2xy = 4xy(x^2 + y^2)
 \end{aligned}$$

$$\begin{aligned}
 24 \quad & (x + y - a + b)^2 - (x - y + a - b)^2 \\
 &= \{(x + y - a + b) + (x - y + a - b)\} \times \{(x + y - a + b) - (x - y + a - b)\} \\
 &= \{2x\} \times \{2y - 2a + 2b\} = 2x \cdot 2\{y - a + b\} = 4x(y - a + b)
 \end{aligned}$$

$$\begin{aligned}
 25 \quad & (2a + 3b - 5c + 7d)^2 - (2a - 3b + 5c - 7d)^2 \\
 &= \{(2a + 3b - 5c + 7d) + (2a - 3b + 5c - 7d)\} \\
 &\quad \times \{(2a + 3b - 5c + 7d) - (2a - 3b + 5c - 7d)\} \\
 &= \{4a\}\{6b - 10c + 14d\} = 4a \cdot 2(3b - 5c + 7d) = 8a(3b - 5c + 7d)
 \end{aligned}$$

$$\begin{aligned}
 26 \quad & 2345 \times 2345 - 2343 \times 2343 = (2345)^2 - (2343)^2 = \{2345 + 2343\} \\
 &\quad \times \{2345 - 2343\} = 4688 \times 2 = 9376
 \end{aligned}$$

$$\begin{aligned} 27 \quad (53497)^2 - (53487)^2 &= (53497 + 53487)(53497 - 53487) \\ &= 106984 \times 10 = 1069840 \end{aligned}$$

$$\begin{aligned} 28 \quad 498567 \times 498567 - 498562 \times 498562 &= (498567)^2 - (498562)^2 \\ &= (498567 + 498562)(498567 - 498562) \\ &= 997129 \times 5 = 4985645 \end{aligned}$$

$$29 \quad 351^2 - 36 = (5x)^2 - (6)^2 = (5x + 6)(5x - 6)$$

$$30 \quad 9a^2 - 16c^2 = (3a)^2 - (4c)^2 = (3a + 4c)(3a - 4c)$$

$$31 \quad 16m^2 - 49n^2 = (4m)^2 - (7n)^2 = (4m + 7n)(4m - 7n)$$

$$32 \quad 4p^2 - 81q^2 = (2p)^2 - (9q)^2 = (2p + 9q)(2p - 9q)$$

$$33 \quad a^2x^2 - 64b^2 = (ax)^2 - (8b)^2 = (ax + 8b)(ax - 8b)$$

$$34 \quad 36x^2 - 121y^2 = (6x)^2 - (11y)^2 = (6x + 11y)(6x - 11y)$$

$$35 \quad 49 - 64d^2 = (7)^2 - (8d)^2 = (7 + 8d)(7 - 8d)$$

$$36 \quad 144c^2 - 25d^2 = (12c)^2 - (5d)^2 = (12c + 5d)(12c - 5d)$$

$$37 \quad (a+b)^2 - c^2 = (a+b+c)(a+b-c)$$

$$38 \quad (a+2b)^2 - 25c^2 = (a+2b)^2 - (5c)^2 = (a+2b+5c)(a+2b-5c)$$

$$39 \quad 4x^2 - (3a-4b)^2 = (2x)^2 - (3a-4b)^2 = (2x+3a-4b)(2x-3a+4b)$$

$$\begin{aligned} 40 \quad a^2 - (2b-3c)^2 &= \{a+(2b-3c)\}\{a-(2b-3c)\} \\ &= (a+2b-3c)(a-2b+3c) \end{aligned}$$

$$\begin{aligned} 41 \quad a^4 - 81b^4 &= (a^2)^2 - (9b^2)^2 = (a^2 + 9b^2)(a^2 - 9b^2) \\ &= (a^2 + 9b^2)\{a^2 - (3b)^2\} = (a^2 + 9b^2) \\ &\quad (a+3b)(a-3b) \end{aligned}$$

$$\begin{aligned} 42 \quad (1-y)^2 - (a-b)^2 \\ &= \{(1-y) + (a-b)\}\{(1-y) - (a-b)\} \\ &= (1-y+a-b)(1-y-a+b) \end{aligned}$$

$$\begin{aligned} 43 \quad 81x^4 - 625y^4 &= (9x^2)^2 - (25y^2)^2 \\ &= (9x^2 + 25y^2)(9x^2 - 25y^2) \\ &= (9x^2 + 25y^2)\{(3x)^2 - (5y)^2\} \\ &= (9x^2 + 25y^2)(3x+5y)(3x-5y) \end{aligned}$$

$$\begin{aligned} 44 \quad (2a-5x)^2 - 36x^2 &= (2a-5x)^2 - (6x)^2 \\ &= (2a-5x+6x)(2a-5x-6x) = (2a+x)(2a-11x) \end{aligned}$$

$$\begin{aligned} 45 \quad 25x^2 - (4x + 1)^2 &= (5x)^2 - (4x + 1)^2 \\ &= (5x + 4x + 1)\{5x - (4x + 1)\} = (9x + 1)(x - 1) \end{aligned}$$

$$\begin{aligned} 46 \quad (4a + 7b)^2 - (3a - 8b)^2 \\ &= \{(4a + 7b) + (3a - 8b)\}\{(4a + 7b) - (3a - 8b)\} \\ &= (7a - b)(a + 15b) \end{aligned}$$

$$\begin{aligned} 47 \quad (3x + 5y)^2 - (2x - 7y)^2 \\ &= \{(3x + 5y) + (2x - 7y)\}\{(3x + 5y) - (2x - 7y)\} \\ &= (5x - 2y)(x + 12y) \end{aligned}$$

$$\begin{aligned} 48 \quad (a + 2b - 3c)^2 - (a + b - c)^2 \\ &= \{(a + 2b - 3c) + (a + b - c)\}\{(a + 2b - 3c) - (a + b - c)\} \\ &= (2a + 3b - 4c)(b - 2c) \end{aligned}$$

$$\begin{aligned} 49 \quad (2m + 3n - 5p)^2 - (2n + 3p)^2 \\ &= \{(2m + 3n - 5p) + (2n + 3p)\} \\ &\quad \times \{(2m + 3n - 5p) - (2n + 3p)\} \\ &= (2m + 5n - 2p)(2m + n - 8p) \end{aligned}$$

$$\begin{aligned} 50 \quad (3x - 4y + 7z)^2 - (2x - 3y + 5z)^2 \\ &= \{(3x - 4y + 7z) + (2x - 3y + 5z)\} \\ &\quad \times \{(3x - 4y + 7z) - (2x - 3y + 5z)\} \\ &= (5x - 7y + 12z)(x - y + 2z) \end{aligned}$$

Exercise 25

$$\begin{aligned} 1 \quad (2x + 1)^3 &= (2x)^3 + 3(2x)^2 \cdot 1 + 3(2x) \cdot 1^2 + 1^3 \\ &= 8x^3 + 12x^2 + 6x + 1 \end{aligned}$$

$$\begin{aligned} 2 \quad (3x + y)^3 &= (3x)^3 + 3(3x)^2 y + 3(3x) y^2 + y^3 \\ &= 27x^3 + 27x^2 y + 9xy^2 + y^3 \end{aligned}$$

$$\begin{aligned} 3 \quad (2x + 3a)^3 &= (2x)^3 + 3(2x)^2 (3a) + 3 \cdot 2x (3a)^2 + (3a)^3 \\ &= 8x^3 + 36x^2 a + 54xa^2 + 27a^3 \end{aligned}$$

$$\begin{aligned} 4 \quad (a^2 + 2b)^3 &= (a^2)^3 + 3(a^2)^2 \cdot 2b + 3a^2 (2b)^2 + (2b)^3 \\ &= a^6 + 6a^4 b + 12a^2 b^2 + 8b^3 \end{aligned}$$

$$\begin{aligned} 5 \quad (ab + bc)^3 &= (ab)^3 + 3(ab)^2(bc) + 3(ab)(bc)^2 + (bc)^3 \\ &= a^3 b^3 + 3a^2 b^2 c + 3ab^3 c^2 + b^3 c^3 \end{aligned}$$

$$\begin{aligned}
 6 \quad (a+2x+b)^3 &= \{a+(2x+b)\}^3 \\
 &= a^3 + 3a^2(2x+b) + 3a(2x+b)^2 + (2x+b)^3 \\
 &= a^3 + 6a^2x + 3a^2b + 3a(4x^2 + 4xb + b^2) + (8x^3 + 12x^2b + 6xb^2 + b^3) \\
 &= a^3 + 8x^3 + b^3 + 6a^2x + 3a^2b + 12ax^2 + 3ab^2 \\
 &\quad + 12x^2b + 6xb^2 + 12abx
 \end{aligned}$$

$$\begin{aligned}
 7 \quad (2m+3n+p)^3 &= \{2m+(3n+p)\}^3 \\
 &= (2m)^3 + 3(2m)^2(3n+p) + 3(2m)(3n+p)^2 + (3n+p)^3 \\
 &= 8m^3 + 3 \cdot 4m^2(3n+p) + 6m(9n^2 + 6np + p^2) \\
 &\quad + (27n^3 + 27n^2p + 9np^2 + p^3) \\
 &= 8m^3 + 36m^2n + 12m^2p + 54mn^2 + 36mnp \\
 &\quad + 6mp^2 + 27n^3 + 27n^2p + 9np^2 + p^3 \\
 &= 8m^3 + 27n^3 + p^3 + 12m^2p + 6mp^2 \\
 &\quad + 36m^2n + 54mn^2 + 9np^2 + 27n^2p + 36mnp
 \end{aligned}$$

$$\begin{aligned}
 8 \quad (xy+yz+zx)^3 &= \{xy+(yz+zx)\}^3 \\
 &= (xy)^3 + 3(xy+yz)^2(zx) + 3(xy+yz)(zx)^2 + (zx)^3 \\
 &= (x^3y^3 + 3x^2y^3z + 3xy^3z^2 + y^3z^3) + \\
 &\quad 3(x^2y^2 + 2xy^2z + y^2z^2)(zx) + 3(xy+yz)z^2x^2 + z^3x^3 \\
 &= x^3y^3 + 3x^2y^3z + 3xy^3z^2 + y^3z^3 + 3x^3y^2z \\
 &\quad + 6x^2y^2z^2 + 3xy^2z^3 + 3x^2yz^2 + 3x^2yz^3 + 3x^2y^2z^2 \\
 &= x^3y^3 + y^3z^3 + z^3x^3 + 3x^2y^3z + 3xy^3z^2 \\
 &\quad + 3x^3y^2z + 3xy^2z^3 + 3x^2yz^2 + 3x^2yz^3 + 6x^2y^2z^2
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \text{Putting } x \text{ for } (3m+5n) \text{ and } y \text{ for } (2m-5n), \text{ we have the} \\
 \text{given expression} &= x^3 + 3x^2y + 3xy^2 + y^3 \\
 &= (x+y)^3 = \{(3m+5n) + (2m-5n)\}^3 \\
 &= (5m)^3 = \underline{125m^3}
 \end{aligned}$$

$$\begin{aligned}
 10 \quad \text{Putting } a \text{ for } (3x-8y) \text{ and } b \text{ for } (9y-2x), \\
 \text{we have } a+b &= (3x-8y) + (9y-2x) = x+y \\
 \text{We have the given expression} \\
 &= a^3 + b^3 + 3ab(a+b) = (a+b)^3 = (x+y)^3 \\
 &= \underline{x^3 + 3x^2y + 3xy^2 + y^3}
 \end{aligned}$$

$$\begin{aligned}
 11 \quad \text{Putting } x \text{ for } (3a-7b) \text{ and } y \text{ for } (10b-3a), \\
 \text{we have } x+y &= (3a-7b) + (10b-3a) = 3b \\
 9b &= 3(x+y), \text{ and therefore we have the given expression} \\
 &= x^3 + y^3 + 3(1+y)xy = (x+y)^3 = (3b)^3 = \underline{27b^3}
 \end{aligned}$$

- 12 Putting
- a
- for
- $(5x-2)$
- and
- b
- for
- $(3-4x)$
- ,

$$\text{we have } a+b=(5x-2)+(3-4x)=x+1$$

We have the given expression

$$\begin{aligned} &=a^3+b^3+3ab(a+b)=(a+b)^3=(x+1)^3 \\ &=x^3+3x^2+3x+1. \end{aligned}$$

- 13 Putting
- a
- for
- $(3-7x)$
- and
- b
- for
- $(8x-1)$
- ,

$$\text{we have } a+b=(3-7x)+(8x-1)=x+2$$

We have the given expression

$$\begin{aligned} &=a^3+b^3+3ab(a+b)=(a+b)^3 \\ &=(x+2)^3=x^3+6x^2+12x+8 \end{aligned}$$

- 14 Putting
- x
- for
- $(a-b+c)$
- and
- y
- for
- $(a+b-c)$
- ,

$$\text{we have } x+y=(a-b+c)+(a+b-c)=2a$$

$$3(x+y)=6a$$

$$\begin{aligned} \text{Also } xy &= (a-b+c)(a-b-c) = \{a-(b-c)\}\{a+(b-c)\} \\ &= \{a^2-(b-c)^2\} \end{aligned}$$

We have the given expression

$$=x^3+y^3+3(x+y)xy=(x+y)^3=(2a)^3=8a^3$$

$$15 \quad a^3+b^3=(a+b)^3-3ab(a+b)=(6)^3-3 \cdot 7 \cdot 6=216-126=90$$

$$16 \quad a^3+b^3=(a+b)^3-3ab(a+b)=7^3-3 \cdot 8 \cdot 7=343-168=175$$

$$\begin{aligned} 17 \quad a^3+\left(\frac{1}{a}\right)^3 &= \left(a+\frac{1}{a}\right)^3-3a\frac{1}{a}\left(a+\frac{1}{a}\right)=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right) \\ &=(3)^3-3 \cdot 3=27-9=18 \end{aligned}$$

$$\begin{aligned} 18 \quad z^3+\frac{1}{z^3} &= \left(z+\frac{1}{z}\right)^3-3z\frac{1}{z}\left(z+\frac{1}{z}\right)=\left(z+\frac{1}{z}\right)^3-3\left(z+\frac{1}{z}\right) \\ &=4^3-3 \cdot 4=64-12=52 \end{aligned}$$

- 19 The given expression

$$\begin{aligned} &=(4)^3+3 \cdot 4^2 \cdot 1+3 \cdot 4 \cdot 1^2+1^3=(4+1)^3 \\ &=(4+1)^3=5^3=125 \end{aligned}$$

- 20 The given expression

$$\begin{aligned} &=(2m)^3+3(2m)^2 \cdot 3n+3(2m)(3n)^2+(3n)^3+64 \\ &=(2m+3n)^3+64=(-3)^3+64=-27+64=37. \end{aligned}$$

21 The given expression

$$\begin{aligned}
 &= r^3 + 3(r^2)(5) + 3(r)(5)^2 + 125 + 64 \\
 &= r^3 + 3(r^2)(5) + 3(r)(5)^2 + (5)^3 + 64 \\
 &= (r+5)^3 + 64 = (-9+5)^3 + 64 \\
 &= (-4)^3 + 64 = -64 + 64 = 0
 \end{aligned}$$

22 The given expression

$$\begin{aligned}
 &= r^3 + 3r^2(6) + 3r(6)^2 + 216 + 135 \\
 &= r^3 + 3(r^2)(6) + 3(r)(6)^2 + (6)^3 + 135 \\
 &= (r+6)^3 + 135 = (-11+6)^3 + 135 \\
 &= (-5)^3 + 135 = -125 + 135 = 10.
 \end{aligned}$$

23 $r^3 + y^3 + 15ry = r^3 + y^3 + 3(5)(r)y$

$$= x^3 + y^3 + 3(x+y)xy = (x+y)^3 = (5)^3 = 125$$

24 $a^3 + b^3 + 3a^2b^2c^2 = (a^2)^3 + (b^2)^3 + 3a^2b^2c^2$

$$\begin{aligned}
 &= (a^2)^3 + (b^2)^3 + 3a^2b^2(a^2 + b^2) \\
 &= (a^2 + b^2)^3 = (c^2)^3 = \underline{c^6}
 \end{aligned}$$

25 $p^3 + q^3 + 6pq = p^3 + q^3 + 3(2)pq$

$$= p^3 + q^3 + 3(p+q)pq = (p+q)^3 = (2)^3 = 8$$

Exercise 26

$$\begin{aligned}
 1 \quad (1-2a)^3 &= (1)^3 - 3(1)^2(2a) + 3(1)(2a)^2 - (2a)^3 \\
 &= 1 - 6a + 12a^2 - 8a^3
 \end{aligned}$$

$$\begin{aligned}
 2 \quad (2-3x)^3 &= (2)^3 - 3(2)^2(3x) + 3(2)(3x)^2 - (3x)^3 \\
 &= 8 - 3(4)(3x) + 3(2)(9x^2) - 27x^3 \\
 &= 8 - 36x + 54x^2 - 27x^3
 \end{aligned}$$

$$\begin{aligned}
 3 \quad (3-4x)^3 &= (3)^3 - 3(3)^2(4x) + 3(3)(4x)^2 - (4x)^3 \\
 &= 27 - 3(9)(4x) + 3(3)(16x^2) - 64x^3 \\
 &= 27 - 108x + 144x^2 - 64x^3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad (5m-4n)^3 &= (5m)^3 - 3(5m)^2(4n) + 3(5m)(4n)^2 - (4n)^3 \\
 &= 125m^3 - 3(25m^2)(4n) + 3(5m)(16n^2) - 64n^3 \\
 &= 125m^3 - 300m^2n + 240mn^2 - 64n^3
 \end{aligned}$$

$$\begin{aligned}
 5 \quad (2p-5q)^3 &= (2p)^3 - 3(2p)^2(5q) + 3(2p)(5q)^2 - (5q)^3 \\
 &= 8p^3 - 3(4p^2)(5q) + 3(2p)(25q^2) - 125q^3 \\
 &= 8p^3 - 60p^2q + 150pq^2 - 125q^3
 \end{aligned}$$

$$\begin{aligned}
 6 \quad (2x - y - z)^2 &= \{2x - (y + z)\}^2 \\
 &= (2x)^2 - 2(2x)(y + z) + (y + z)^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2 \\
 &= 4x^2 - 4xy - 4xz + y^2 + 2yz + z^2
 \end{aligned}$$

$$\begin{aligned}
 7 \quad (2m - 3n - p)^2 &= \{2m - (3n + p)\}^2 \\
 &= (2m)^2 - 2(2m)(3n + p) + (3n + p)^2 \\
 &= 4m^2 - 12mn - 2mp + 9n^2 + 6np + p^2 \\
 &= 4m^2 - 12mn - 2mp + 9n^2 + 6np + p^2 \\
 &= 4m^2 - 12mn - 2mp + 9n^2 + 6np + p^2 \\
 &= 4m^2 - 12mn - 2mp + 9n^2 + 6np + p^2 \\
 &= 4m^2 - 12mn - 2mp + 9n^2 + 6np + p^2
 \end{aligned}$$

$$\begin{aligned}
 8 \quad (l^2 - m^2 - n^2)^2 &= \{l^2 - (m^2 + n^2)\}^2 \\
 &= (l^2)^2 - 2(l^2)(m^2 + n^2) + (m^2 + n^2)^2 \\
 &= l^4 - 2l^2m^2 - 2l^2n^2 + m^4 + 2m^2n^2 + n^4 \\
 &= l^4 - 2l^2m^2 - 2l^2n^2 + m^4 + 2m^2n^2 + n^4 \\
 &= l^4 - 2l^2m^2 - 2l^2n^2 + m^4 + 2m^2n^2 + n^4 \\
 &= l^4 - 2l^2m^2 - 2l^2n^2 + m^4 + 2m^2n^2 + n^4 \\
 &= l^4 - 2l^2m^2 - 2l^2n^2 + m^4 + 2m^2n^2 + n^4
 \end{aligned}$$

9 Putting x for $(a + 2b)$ and y for $(a - 2b)$,
we have the given expression

$$\begin{aligned}
 &= x^2 - 3x^2y + 3xy^2 - y^2 \\
 &= (x - y)^2 = \{(a + 2b) - (a - 2b)\}^2 = (4b)^2 = \underline{64b^2}
 \end{aligned}$$

10 Putting a for $(3x - 8y)$ and b for $(2x - 7y)$,
we have $a - b = (3x - 8y) - (2x - 7y) = x - y$
We have the given expression

$$\begin{aligned}
 &= a^3 - b^3 - 3ab(a - b) \\
 &= (a - b)^3 = (x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3
 \end{aligned}$$

11 Putting a for $(5x-8)$ and b for $(3x-8)$ we have,
 $a-b=(5x-8)-(3x-8)=2x$ we have the
 given expression $=a^3-b^3-3(a-b)ab$
 $=(a-b)^3=(2x)^3=8x^3$

12 The given expression
 $=(m)^3-3(m)^2(4n)+3(m)(4n)^2-(4n)^3$
 $=(m-4n)^3=(12-12)^3=0$

13 The given expression
 $=(3a)^3-3(3a)^2(5)+3(3a)(5)^2-(5)^3$
 $=(3a-5)^3=(12-5)^3=7^3=343$

14. The given expression
 $=7+1-9a+27a^2-27a^3$
 $=7+(1)^3-3(1)^2 3a+3(1)(3a)^2-(3a)^3$
 $=7+(1-3a)^3=7+(1-9)^3=7+(-8)^3$
 $=7-512=-505$

15 $216-144x+108x^2-27x^3$
 $=(216-64)+(64-144x+108x^2-27x^3)$
 $=152+(4)^3-3(4)^2 3x+3(4)(3x)^2-(3x)^3$
 $=152+(4-3x)^3=152+(4-9)^3=152+(-5)^3$
 $=152-125=27$

16. $a^3-\left(\frac{1}{a}\right)^3=\left(a-\frac{1}{a}\right)^3+3a\frac{1}{a}\left(a-\frac{1}{a}\right)$
 $=\left(a-\frac{1}{a}\right)^3+3a\left(a-\frac{1}{a}\right)$
 $=(3)^3+3 \cdot 3=27+9=36$

17. $c^3-\left(\frac{1}{c}\right)^3=\left(c-\frac{1}{c}\right)^3+3c\frac{1}{c}\left(c-\frac{1}{c}\right)$
 $=\left(c-\frac{1}{c}\right)^3+3\left(c-\frac{1}{c}\right)$
 $=(5)^3+3 \cdot 5=125+15=140.$

18 x^3-y^3-9xy
 $=x^3-y^3-3xy \cdot 3$
 $=x^3-y^3-3xy(x-y)$
 $=(x-y)^3=(3)^3=27$

19 p^3-8q^3-24pq
 $=(p)^3-(2q)^3-3 \cdot 4(p)(2q)$
 $=p^3-(2q)^3-3(p-2q)p \cdot 2q$
 $=(p-2q)^3=(4)^3=64$

$$\begin{aligned}
 20 \quad & 8a^3 - 27b^3 - 90ab \\
 & = (2a)^3 - (3b)^3 - 3(2a)(3b) \\
 & = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b) \\
 & = (2a - 3b)^3 = (5)^3 = 125
 \end{aligned}$$

Exercise 27

- 1 Putting b for 1, we have,

$$a^3 - a + 1 = a^3 - a \cdot 1 + (1)^3 = a^3 - ab + b^3$$

$$\begin{aligned}
 \text{Hence } (a+1)(a^3 - a + 1) &= (a+b)(a^3 - ab + b^3) \\
 &= (a^3 + b^3) = a^3 + 1
 \end{aligned}$$

- 2 Putting a for $(2x)$ and b for 1, we have,

$$4x^3 - 2x + 1 = (2x)^3 - (2x) \cdot 1 + 1^3 = a^3 - ab + b^3$$

$$\begin{aligned}
 (2x+1)(4x^3 - 2x + 1) &= (a+b)(a^3 - ab + b^3) \\
 &= a^3 + b^3 = \{(2x)^3 + 1^3\} = 8x^3 + 1
 \end{aligned}$$

- 3 Putting a for $5m$ and b for 1, we have,

$$25m^3 - 5m + 1 = (5m)^3 - 5m \cdot 1 + 1^3 = a^3 - ab + b^3$$

$$\begin{aligned}
 \text{Hence, } (5m+1)(25m^3 - 5m + 1) & \\
 &= (a+b)(a^3 - ab + b^3) \\
 &= a^3 + b^3 = \{(5m)^3 + 1^3\} \\
 &= 125m^3 + 1
 \end{aligned}$$

- 4 Putting a for $4x$ and b for $7y$, we have,

$$16x^3 - 28xy + 49y^3 = (4x)^3 - (4x)(7y) + (7y)^3 = a^3 - ab + b^3$$

$$\begin{aligned}
 \text{Hence } (4x+7y)(16x^3 - 28xy + 49y^3) & \\
 &= (a+b)(a^3 - ab + b^3) \\
 &= a^3 + b^3 = \{(4x)^3 + (7y)^3\} \\
 &= 64x^3 + 343y^3
 \end{aligned}$$

- 5 Putting a for $3m$ and b for $8n$, we have $9m^3 - 24mn + 64n^3$

$$= (3m)^3 - (3m)(8n) + (8n)^3 = a^3 - ab + b^3$$

$$\begin{aligned}
 & (3m+8n)(9m^3 - 24mn + 64n^3) \\
 &= (a+b)(a^3 - ab + b^3) = (a^3 + b^3) = (3m)^3 + (8n)^3 \\
 &= 27m^3 + 512n^3
 \end{aligned}$$

- 6 Putting x for ab and y for $4c$, we have,

$$\begin{aligned}(ab+4c)(a^2b^2-4abc+16c^2) &= (x+y)(x^2-xy+y^2) \\ &= x^2+y^2=(ab)^2+(4c)^2 \\ &= a^2b^2+16c^2\end{aligned}$$

- 7 Putting m for ax and n for $5b$, we have,

$$\begin{aligned}a^2x^2-5abx+25b^2 &= (ax)^2-(ax)(5b)+(5b)^2=m^2-mn+n^2 \\ (ax+5b)(a^2x^2-5abx+25b^2) &= (m+n)(m^2-mn+n^2) \\ &= m^3+n^3=(ax)^3+(5b)^3 \\ &= a^3x^3+125b^3\end{aligned}$$

- 8 Putting x for $5a$ and y for $9b$, we have,

$$\begin{aligned}25a^2-45ab+81b^2 &= (5a)^2-(5a)(9b)+(9b)^2=x^2-xy+y^2 \\ \text{Hence } (5a+9b)(25a^2-45ab+81b^2) & \\ &= (x+y)(x^2-xy+y^2)=x^3+y^3 \\ &= (5a)^3+(9b)^3=125a^3+729b^3\end{aligned}$$

- 9 $(x^3+8)=(x^3+(2)^3)=(x+2)(x^2-2x+4)$
 $=(x+2)(x^2-2x+4)$

- 10 $8a^3+1=(2a)^3+1=(2a+1)\{(2a)^2-2a+1\}$
 $= (2a+1)(4a^2-2a+1)$

- 11 $m^3+27=m^3+(3)^3=(m+3)(m^2-m+3)$
 $= (m+3)(m^2-3m+9)$

- 12 $27x^3+1=(3x)^3+1=(3x+1)\{(3x)^2+(3x)+1\}$
 $= (3x+1)(9x^2+3x+1)$

- 13 $x^3+64=x^3+(4)^3=(x+4)\{x^2-x+4\}$
 $= (x+4)(x^2-4x+16)$

- 14 $125m^3+1=(5m)^3+1=\{(5m)+1\}\{(5m)^2-(5m)+1\}$
 $= (5m+1)(25m^2-5m+1)$

- 15 $8a^3+343x^3=(2a)^3+(7x)^3$
 $= (2a+7x)(4a^2-14ax+49x^2)$

- 16 $64a^3x^3+27y^3=(4ax)^3+(3y)^3$
 $= (4ax+3y)\{(4ax)^2-(4ax)(3y)+(3y)^2\}$
 $= (4ax+3y)(16a^2x^2-12a^2xy+9y^2)$

Exercise 28

- 1 $(1-3a)(1+3a+9a^2)$
 $= (1-3a)\{1^2 + 1(3a) + (3a)^2\}$
 $= 1^3 - (3a)^3 = 1 - 27a^3$
- 2 $(4x-1)(16x^2+4x+1)$
 $= (4x-1)\{(4x)^2 + (4x)1 + 1^2\}$
 $= (4x)^3 - 1 = 64x^3 - 1$
3. $(5m-3n)(25m^2+15mn+9n^2)$
 $= (5m-3n)\{(5m)^2 + (5m)(3n) + (3n)^2\}$
 $= (5m)^3 - (3n)^3 = 125m^3 - 27n^3$
- 4 $(x-2yz)(x^2+2xyz+4y^2z^2)$
 $= (x-2yz)\{x^2 + x(2yz) + (2yz)^2\}$
 $= (x)^3 - (2yz)^3 = x^3 - 8y^3z^3$
- 5 $(3a-bc)(9a^2+3abc+b^2c^2)$
 $= (3a-bc)\{(3a)^2 + (3a)(bc) + (bc)^2\}$
 $= (3a)^3 - (bc)^3 = 27a^3 - b^3c^3$
- 6 $125a^3-1=(5a)^3-1$
 $= \{(5a)-1\}\{(5a)^2+(5a)+1\}$
 $= (5a-1)(25a^2+5a+1)$
- 7 $343x^3-8y^6=(7x)^3-(2y^2)^3$
 $= (7x-2y^2)\{(7x)^2+(7x)(2y^2)+(2y^2)^2\}$
 $= (7x-2y^2)(49x^2+14xy^2+4y^4)$
- 8 $216k^3-125l^3=(6k)^3-(5l)^3$
 $= (6k-5l)\{(6k)^2+(6k)(5l)+(5l)^2\}$
 $= (6k-5l)(36k^2+30kl+25l^2)$
- 9 $1-512k^3=1-(8k)^3=(1-8k)\{1^2+1(8k)+(8k)^2\}$
 $= (1-8k)(1+8k+64k^2)$
- 10 $729m^3-64a^3n^6=(9m)^3-(4an^2)^3$
 $= (9m-4an^2)\{(9m)^2+(9m)(4an^2)+(4an^2)^2\}$
 $= (9m-4an^2)(81m^2+36am^2n^2+16a^2n^4)$

Exercise 29.

- | | | |
|-----|---|---------------------------------------|
| 1 | $\left. \begin{array}{l} 2+8=10 \\ \text{and } 2 \times 8=16 \end{array} \right\},$ | the required product
$=x^2+10x+16$ |
| 2 | $\left. \begin{array}{l} 3+5=8 \\ \text{and } 3 \times 5=15 \end{array} \right\},$ | the required product
$=x^2+8x+15$ |
| 3 | $\left. \begin{array}{l} 6+11=17 \\ \text{and } 6 \times 11=66 \end{array} \right\},$ | the required product
$=x^2+17x+66$ |
| 4 | $\left. \begin{array}{l} 7+9=16 \\ \text{and } 7 \times 9=63 \end{array} \right\},$ | the required product
$=x^2-4x-12$ |
| 5 | $\left. \begin{array}{l} -6+2=-4 \\ \text{and } (-6) \times 2=-12 \end{array} \right\},$ | the required product
$=x^2-4x-12$ |
| 6 | $\left. \begin{array}{l} -2+8=6 \\ \text{and } (-2) \times 8=-16 \end{array} \right\},$ | the required product
$=m^2+6m-16$ |
| 7 | $\left. \begin{array}{l} -3-4=-7 \\ \text{and } (-3) \times (-4)=12 \end{array} \right\},$ | the required product
$=a^2-7a+12$ |
| 8 | $\left. \begin{array}{l} 5-3=2 \\ \text{and } 5 \times (-3)=-15 \end{array} \right\},$ | the required product
$=x^2+2x-15$ |
| 9 | $\left. \begin{array}{l} -4+9=5 \\ \text{and } (-4) \times 9=-36 \end{array} \right\},$ | the required product
$=x^2+5x-36$ |
| 10. | $\left. \begin{array}{l} -5-10=-15 \\ \text{and } (-5) \times (-10)=50 \end{array} \right\},$ | the required product
$=x^2-15x+50$ |
| 11 | $\left. \begin{array}{l} -12+5=-7 \\ \text{and } (-12) \times 5=-60 \end{array} \right\},$ | the required product
$=x^2-7x-60$ |
| 12 | $\left. \begin{array}{l} -13+2=-11 \\ \text{and } (-13) \times 2=-26 \end{array} \right\},$ | the required product
$=k^2-11k-26$ |
| 13 | $\left. \begin{array}{l} 5+14=19 \\ \text{and } 5 \times 14=70 \end{array} \right\},$ | the required product
$=a^2+19a+70$ |
| 14 | $\left. \begin{array}{l} -14+6=-8 \\ \text{and } -14 \times 6=-84 \end{array} \right\},$ | the required product
$=m^2-8m-84$ |
| 15 | $\left. \begin{array}{l} -5-13=-18 \\ \text{and } (-5) \times (-13)=65 \end{array} \right\},$ | the required product
$=x^2-18x+65$ |
| 16 | $\left. \begin{array}{l} 7+12=19 \\ \text{and } 7 \times 12=84 \end{array} \right\},$ | the required product
$=x^2+19x+84$ |
| 17 | $\left. \begin{array}{l} -3-11=-14 \\ \text{and } (-3) \times (-11)=33 \end{array} \right\},$ | the required product
$=a^2-14a+33$ |

- 18 $\left. \begin{array}{l} 4-13=-9 \\ \text{and } 4 \times (-13)=-52 \end{array} \right\}$, the required product
 $=x^2-9x-52$
- 19 $\left. \begin{array}{l} 5-16=-11 \\ \text{and } 5 \times (-16)=-80 \end{array} \right\}$, the required product
 $=m^2-11m-80$
- 20 $\left. \begin{array}{l} -8-10=-18 \\ \text{and } (-8) \times (-10)=80 \end{array} \right\}$, the required product
 $=x^2-18x+80$
- 21 $\left. \begin{array}{l} 6-12=-6 \\ \text{and } 6 \times (-12)=-72 \end{array} \right\}$, the required product
 $=a^2-6a-72$
- 22 $\left. \begin{array}{l} -7+13=6 \\ \text{and } (-7) \times (13)=-91 \end{array} \right\}$, the required product
 $=m^2+6m-91$
- 23 $\left. \begin{array}{l} -10-16=-26 \\ \text{and } (-10) \times (-16)=160 \end{array} \right\}$, the required product
 $=x^2-26x+160$
- 24 $\left. \begin{array}{l} 5-18=-13 \\ \text{and } (5) \times (-18)=-90 \end{array} \right\}$, the required product
 $=x^2-13x-90$
- 25 $\left. \begin{array}{l} -16+10=-6 \\ \text{and } (-16) \times 10=-160 \end{array} \right\}$, the required product
 $=x^2-6x-160$

Exercise 30

- 1 $1+2+3=6$,
 $1 \times 2 + 1 \times 3 + 2 \times 3 = 2+3+6=11$,
 $1 \times 2 \times 3=6$
 The required product $=x^3+6x^2+11x+6$
- 2 $2+5+7=14$,
 $2 \times 5 + 2 \times 7 + 5 \times 7 = 10+14+35=59$,
 $2 \times 5 \times 7=70$
 The required product $=x^3+14x^2+59x+70$
- 3 $3-6+2=-1$,
 $(3) \times (-6) + (3) \times (2) + (-6) \times (2) = -18+6-12=-24$,
 $(3) \times (-6) \times (2)=-36$
 The required product $=x^3-x^2-24x-36$

$$4 - 4 + 5 - 10 = -1,$$

$$(4 \times 5) + (4) \times (-10) + (5) \times (-10) = 20 - 40 - 50 = -70,$$

$$(4) \times (5) \times (-10) = -200$$

$$\text{The required product} = x^3 - x^2 - 70x - 200$$

$$5 - 8 + 3 + 1 = -4,$$

$$(-8) \times (3) + (-8) \times (1) + (3) \times (1) = -24 - 8 + 3 = -29,$$

$$(-8) \times (3) \times (1) = -24$$

$$\text{The required product} = x^3 - 4x^2 - 29x - 24$$

$$6 - 5 - 2 + 8 = 1, (-5) \times (-2) + (-5) \times (8) + (-2) \times (8)$$

$$= 10 - 40 - 16 = -46, (-5) \times (-2) \times 8 = 80$$

$$\text{The required product} = x^3 + x^2 - 46x + 80$$

$$7 - 3 + 7 - 4 = 0, (-3) \times (7) + (-3) \times (-4) + (7) \times (-4)$$

$$= -21 + 12 - 28 = -37, (-3) \times (7) \times (-4) = 84$$

$$\text{The required product} = x^3 + 0x^2 - 37x + 84$$

$$= x^3 - 37x + 84$$

$$8 - 6 - 5 - 7 = -6, (6)(-5) + (6)(-7) + (-5)(-7)$$

$$= -30 - 42 + 35 = -37, (6)(-5)(-7) = 210$$

$$\text{The required product} = x^3 - 6x^2 - 37x + 210$$

$$9 - 5 - 7 - 11 = -23, (-5)(-7) + (-5)(-11) + (-7)(-11)$$

$$= 35 + 55 + 77 = 167, (-5)(-7)(-11) = -385$$

$$\text{The required product} = x^3 - 23x^2 + 167x - 385$$

$$10 - 3 - 6 - 9 = -18, (-3)(-6) + (-6)(-9) + (-3)(-9)$$

$$= 18 + 54 + 27 = 99, (-3)(-6)(-9) = -162$$

$$\text{The required product} = x^3 - 18x^2 + 99x - 162$$

$$11 - 4 - 5 - 12 = -13,$$

$$(4)(-5) + (4)(-12) + (-5)(-12) = -20 - 48 + 60 = -8,$$

$$(4)(-5)(-12) = 240$$

$$\text{The required product} = x^3 - 13x^2 - 8x + 240$$

$$12 \quad 5+9+11=25, \quad 5 \times 9 + 5 \times 11 + 9 \times 11 = 45 + 55 + 99 = 199 \\ 5 \times 9 \times 11 = 495$$

$$\text{The required product} = x^3 + 25x^2 + 199x + 495$$

$$13 \quad -6+8-2=0, \quad (-6)(8)+(-6)(-2)+(8)(-2) \\ = -48+12-16 = -52, \quad (-6) \times 8 \times (-2) = 96$$

$$\text{The required product} = x^3 - 0.x^2 - 52x + 96 \\ = x^3 - 52x + 96$$

$$14 \quad -3-7-13=-23, \quad (-3)(-7)+(-3)(-13)+(-7)(-13) \\ = 21+39+91=151, \quad (-3) \times (-7) \times (-13) = -273$$

$$\text{The required product} = x^3 - 23x^2 + 151x - 273$$

$$15 \quad -3+12+4=13, \quad (-3)(12)+(-3)(4)+(12)(4) \\ = -36-12+48=0, \quad (-3)(12)(4) = -144$$

$$\text{The required product} = x^3 + 13x^2 - 0.x - 144 \\ = x^3 + 13x^2 - 144$$

$$16 \quad -9-10+12=-7, \quad (-9)(-10)+(-9)(12)+(-10)(12) \\ = 90-108-120 = -138, \quad (-9)(-10)(12) = 1080$$

$$\text{The required product} = x^3 - 7x^2 - 138x + 1080$$

$$17 \quad 9-5-7=-3, \quad (9)(-5)+(9)(-7)+(-5)(-7) \\ = -45-63+35 = -73, \quad (9)(-5)(-7) = 315$$

$$\text{The required product} = x^3 - 3x^2 - 73x + 315$$

$$18 \quad 8+12+15=35, \quad 8 \times 12 + 8 \times 15 + 12 \times 15 \\ = 96 + 120 + 180 = 396, \quad 8 \times 12 \times 15 = 1440$$

$$\text{The required product} = x^3 + 35x^2 + 396x + 1440$$

$$19 \quad -14+8+6=0, \quad (-14)(8)+(-14)(6)+(8)(6) \\ = -112-84+48 = -148, \quad (-14)(8)(6) = -672$$

$$\text{The required product} = x^3 - 0.x^2 - 148x - 672 \\ = x^3 - 148x - 672$$

20 $-5-10-16=-31,$

$$(-5)(-10)+(-5)(-16)+(-10)(-16)$$

$$=50+80+160=290, (-5)(-10)(-16)=-800$$

The required product $=1^3-311^2+2901-800$

Exercise 31.

$$\begin{aligned} 1 \quad (1+j-z)^2 &= x^2 + j^2 + z^2 + 2x(j-z) + 2j(-z) \\ &= x^2 + j^2 + z^2 + 2xj - 2xz - 2jz \end{aligned}$$

$$\begin{aligned} 2 \quad (1-j+z)^2 &= x^2 + j^2 + z^2 + 2x(-j+z) + 2(-j)(z) \\ &= x^2 + j^2 + z^2 - 2xj + 2xz - 2jz \end{aligned}$$

$$\begin{aligned} 3 \quad (-1+j+z)^2 &= x^2 + j^2 + z^2 + 2(-1)(j+z) + 2jz \\ &= x^2 + j^2 + z^2 - 2xj - 2xz + 2jz \end{aligned}$$

$$\begin{aligned} 4 \quad (-1-j+z)^2 &= x^2 + j^2 + z^2 + 2(-1)(-j+z) + 2(-j)(z) \\ &= x^2 + j^2 + z^2 + 2xj - 2xz - 2jz \end{aligned}$$

$$\begin{aligned} 5 \quad (x-y-z)^2 &= x^2 + y^2 + z^2 + 2x(-y-z) + 2(-y)(-z) \\ &= x^2 + y^2 + z^2 - 2xy - 2xz + 2yz \end{aligned}$$

$$\begin{aligned} 6 \quad (a-x+j-z)^2 &= a^2 + x^2 + j^2 + z^2 + 2a(-x+y-z) \\ &\quad + 2(-x)(y-z) + 2y(-z) \\ &= a^2 + x^2 + j^2 + z^2 - 2ax + 2ay - 2az - 2xy \\ &\quad + 2xz - 2yz \end{aligned}$$

$$\begin{aligned} 7 \quad (a-x-y-z)^2 &= a^2 + x^2 + y^2 + z^2 + 2a(-x-y-z) \\ &\quad + 2(-x)(-y-z) + 2(-y)(-z) \\ &= a^2 + x^2 + y^2 + z^2 - 2ax - 2ay - 2az + 2xy \\ &\quad + 2xz + 2yz \end{aligned}$$

$$\begin{aligned} 8 \quad (m+n+p+q+r)^2 &= m^2 + n^2 + p^2 + q^2 + r^2 + 2m(n+p+q+r) \\ &\quad + 2n(p+q+r) + 2p(q+r) + 2qr \\ &= m^2 + n^2 + p^2 + q^2 + r^2 + 2mn + 2mp + 2mq + 2mr \\ &\quad + 2np + 2nq + 2nr + 2pq + 2pr + 2qr \end{aligned}$$

$$\begin{aligned}
 9 \quad (p-q+r-x-y)^2 &= p^2+q^2+r^2+x^2+y^2+2p(-q+r-x-y)+2 \\
 &\quad (-q)(r-x-y)+2r(-x-y)+2(-x)(-y) \\
 &= p^2+q^2+r^2+x^2+y^2-2pq+2pr-2px \\
 &\quad -2py-2qr+2qz+2qy-2rx-2ry+2xy
 \end{aligned}$$

$$\begin{aligned}
 10 \quad (-a+b-c+x-y-z)^2 &= a^2+b^2+c^2+x^2+y^2+z^2+2(-a)(b-c+x-y-z)+2b(-c \\
 &\quad +x-y-z)+2(-c)(x-y-z)+2x(-y-z)+2(-y)(-z) \\
 &= a^2+b^2+c^2+x^2+y^2+z^2-2ab+2ac-2ax+2ay+2az \\
 &\quad -2bc+2bx-2by-2bz-2cx+2cy+2cz-2xy-2xz \\
 &\quad +2yz
 \end{aligned}$$

$$\begin{aligned}
 11 \quad (a-2x-3y-4z)^2 &= a^2+(-2x)^2+(-3y)^2+(-4z)^2+2a(-2x-3y-4z) \\
 &\quad +2(-2x)(-3y-4z)+2(-3y)(-4z) \\
 &= a^2+4x^2+9y^2+16z^2-4ax-6ay-8az+12xy+16xz+24yz
 \end{aligned}$$

$$\begin{aligned}
 12 \quad (2a-b+2c-d)^2 &= (2a)^2+(-b)^2+(2c)^2+(-d)^2+2(2a)(-b+2c \\
 &\quad -d)+2(-b)(2c-d)+2(2c)(-d) \\
 &= 4a^2+b^2+4c^2+d^2-4ab+8ac-4ad-4bc \\
 &\quad +2bd-4cd
 \end{aligned}$$

$$\begin{aligned}
 13 \quad \text{The given expression} &= l^2+(-m)^2+n^2+2l(-m+n)+2(-m)(n) \\
 &= (l-m+n)^2=(17-23+13)^2=(7)^2=49
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \text{The given expression} &= p^2+q^2+(-r)^2+2p(b-r)+2q(-r) \\
 &= (p+q-r)^2=(16+12-25)^2=(3)^2=9
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \text{The given expression} &= (-a)^2+b^2+c^2+2(-a)(b+c)+2bc \\
 &= (-a+b+c)^2=(-28+13+15)^2=0
 \end{aligned}$$

$$\begin{aligned}
 16 \quad \text{The given expression} &= x^2+y^2+(-1)^2+2x(y-1)+2y(-1) \\
 &= (x+y-1)^2=(6+7-1)^2=(12)^2=144
 \end{aligned}$$

17. The given expression

$$\begin{aligned}
 &= x^2 + y^2 + 1 + 2xy - 2x - 2y + 35 \\
 &= x^2 + y^2 + (-1)^2 + 2x(-1) + 2(-1)(-1) + 35 \\
 &= (x + y - 1)^2 + 35 = (23 + 18 - 1)^2 + 35 \\
 &= (40)^2 + 35 = 1600 + 35 = 1635
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= x^2 + (-2y)^2 + (-1)^2 + 2x(-2y - 1) + 2(-y)(-1) \\
 &= (x - 2y - 1)^2 = (26 - 2 \cdot 12 - 1)^2 \\
 &= (26 - 24 - 1)^2 = (1)^2 = 1
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= x^2 + 9y^2 + 1 - 6xy - 2x + 6y + 63 \\
 &= x^2 + (-3y)^2 + (-1)^2 + 2x(-3y - 1) + 2(-3y)(-1) + 63 \\
 &= (x - 3y - 1)^2 + 63 = (49 - 48 - 1)^2 + 63 = 0 + 63 = 63
 \end{aligned}$$

20. The given expression

$$\begin{aligned}
 &= 9x^2 + y^2 + 1 - 6xy + 6x - 2y - 25 \\
 &= (3x)^2 + (-y)^2 + 1^2 + 2(3x)(-y + 1) + 2 \times (-y)(1) - 25 \\
 &= (3x - y + 1)^2 - 25 = (42 - 38 + 1)^2 - 25 \\
 &= (5)^2 - 25 = 25 - 25 = 0
 \end{aligned}$$

21 We have $2(ab + ac + bc) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

$$= (12)^2 - 50 = 144 - 50 = 94$$

$$ab + ac + bc = 94 \div 2 = 47$$

22 We have $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + ac + bc)$

$$= (13)^2 - 2 \cdot 50 = 169 - 100 = 69$$

Exercise 32.

1 The 1st term $= x^5$, 2nd term $= 5x^4 \cdot 1 = 5x^4$, 3rd term $= 5 \times 4 x^3 \cdot 1^2 = 10x^3$, 4th term $= 10 \times 3 x^2 \cdot 1^3 = 10x^2$, 5th term $= 10 \times 2 x \cdot 1^4 = 20x$, 6th term $= 5 \cdot 1^5 = 1$

$$(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 20x + 1$$

- 2 The 1st term $=x^6$, 2nd term $=6x^5$, 3rd term $=\frac{6 \times 5}{2}x^4 = 15x^4$,
4th term $=\frac{15 \times 4}{3}x^3 = 20x^3$

The remaining terms are respectively $15x^2$, $6x$ and 1
 $(x+1)^6 = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

- 3 The 1st term $=a^8$, 2nd term $=8a^7b$, 3rd term $=\frac{8 \times 7}{2}a^6b^2$
 $=28a^6b^2$, 4th term $=\frac{28 \times 6}{3}a^5b^3 = 56a^5b^3$, 5th term
 $=\frac{56 \times 5}{4}a^4b^4 = 70a^4b^4$

The remaining terms are respectively

$$56a^3b^5, 28a^2b^6, 8ab^7 \text{ and } b^8,$$

$$(a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 \\ + 28a^2b^6 + 8ab^7 + b^8$$

- 4 The 1st term $=a^9$, 2nd term $=9a^8b$, 3rd term $=\frac{9 \times 8}{2}a^7b^2$
 $=36a^7b^2$, 4th term $=\frac{36 \times 7}{3}a^6b^3 = 84a^6b^3$, 5th term
 $=\frac{84 \times 6}{4}a^5b^4 = 126a^5b^4$

The remaining 5 terms are respectively $126a^4b^5$,

$$84a^3b^6, 36a^2b^7, 9ab^8 \text{ and } b^9$$

$$(a+b)^9 = a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 \\ + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9$$

- 5 The 1st term $=x^5$, 2nd term $=-5x^4y$, 3rd term
 $=\frac{5 \times 4}{2}x^3y^2 = 10x^3y^2$

The remaining 3 terms are respectively

$$-10x^2y^3, 5xy^4 \text{ and } -y^5$$

$$(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$$

- 6 The 1st term $=m^7$, 2nd term $=-7m^6n$, 3rd term
 $=\frac{7 \times 6}{2}m^5n^2 = 21m^5n^2$, 4th term $=-\frac{21 \times 5}{3}m^4n^3 = -35m^4n^3$

The remaining 4 terms are respectively

$$35m^3n^4, -21m^2n^5, 7mn^6 \text{ and } -n^7$$

$$(m-n)^7 = m^7 - 7m^6n + 21m^5n^2 - 35m^4n^3 \\ + 35m^3n^4 - 21m^2n^5 + 7mn^6 - n^7$$

- 7 The 1st term = x^4 , 2nd term = $4x^3 \cdot 2$, 3rd term
 $= \frac{4 \times 3}{2} x^2 \cdot 2^2 = 6x^2 \cdot 4$

The remaining 2 terms are respectively $4x \cdot 2^3$ and 2^4

$$(x+2)^4 = x^4 + (4x^3 \cdot 2) + (6x^2 \cdot 4) + (4x \cdot 2^3) + (2^4) \\ = x^4 + 8x^3 + 24x^2 + 32x + 16$$

- 8 The 1st term = x^5 , 2nd term = $5x^4 \cdot 2$, 3rd term
 $= \frac{5 \times 4}{2} x^3 \cdot (2)^2 = 10x^3 \cdot 4$

The remaining 3 terms are respectively

$$10x^2 (2)^3, 5x(2)^4 \text{ and } (2)^5$$

$$(x+2)^5 = x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 4 + 10x^2 (2)^3 + 5x(2)^4 + (2)^5 \\ = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

- 9 The 1st term = x^6 , 2nd term = $6x^5 \cdot 1 = 6x^5$, 3rd term
 $= \frac{6 \times 5}{2} x^4 \cdot 1^2 = 15x^4$, 4th term = $\frac{6 \times 5 \times 4}{6} x^3 \cdot 1^3 = 20x^3$,
 5th term = $\frac{6 \times 5 \times 4 \times 3}{24} x^2 \cdot 1^4 = 5x^2$

The remaining 4 terms are respectively

$$5x, 1, 1 \text{ and } 1$$

$$(x+1)^6 = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

10. The 1st term = x^4 , 2nd term = $4x^3 \cdot 3$, 3rd term
 $= \frac{4 \times 3}{2} x^2 (3)^2 = 6x^2 \cdot 9$

The remaining two terms are respectively $4x(3)^3$ and $(3)^4$

$$(x+3)^4 = x^4 + 4x^3 \cdot 3 + 6x^2 \cdot 9 + 4x(3)^3 + (3)^4 \\ = x^4 + 12x^3 + 54x^2 + 108x + 81$$

- 11 The 1st term = x^5 , 2nd term = $-5x^4$, 3rd term = $10x^3$

The remaining 3 terms are respectively $-10x^2$, $5x$ and -1

$$(x-1)^5 = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

- 12 The 1st term = 2^6 , 2nd term = $-6 \cdot 2^5 \cdot x$, 3rd term = $\frac{6 \times 5}{2} 2^4 x^2$
 $= 15 \cdot 2^4 x^2$, 4th term = $-\frac{6 \times 5 \times 4}{6} 2^3 x^3 = -20 \cdot 2^3 x^3$

The remaining 3 terms are respectively

$$15 \cdot 2^2 x^4, -6 \cdot 2 x^5 \text{ and } x^6$$

$$(2-x)^6 = 2^6 - 6 \cdot 2^5 x + 15 \cdot 2^4 x^2 - 20 \cdot 2^3 x^3 + 15 \cdot 2^2 x^4 - 6 \cdot 2 x^5 + x^6 \\ = 64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$$

- 13 The 1st term $= (2x)^4$, 2nd term $= -4(2x)^3$, 3rd term
 $= \frac{4 \times 3}{2} (2x)^2 = 6(2x)^2$

The remaining two terms are respectively $-4(2x)$ and 1

$$(2x-4)^2 = (2x)^4 - 4(2x)^3 + 6(2x)^2 - 4(2x) + 1 \\ = 16x^4 - 32x^3 + 24x^2 - 8x + 1$$

- 14 The 1st term $= x^3$, 2nd term $= -9x^2y$, 3rd term
 $= \frac{9 \times 8}{2} x^2y^2 = 36x^2y^2$, 4th term $= -\frac{3 \times 8 \times 7}{1} x^2y^3 = -84x^2y^3$,
 5th term $= \frac{8 \times 7 \times 6}{4} x^2y^4 = 126x^2y^4$

The remaining 5 terms are respectively $-126x^2y^5$, $84x^2y^6$,
 $-36x^2y^7$, $9x^2y^8$ and $-y^9$

$$(1-y)^9 = x^9 - 9x^8y + 36x^7y^2 - 84x^6y^3 + 126x^5y^4 - 126x^4y^5 \\ + 84x^3y^6 - 36x^2y^7 + 9xy^8 - y^9$$

- 15 The 1st term $= (3x)^2$, 2nd term $= -5(3x)^2(2)$, 3rd term
 $= \frac{5 \times 4}{2} (3x)^2(2)^2 = 10(3x)^2(2)^2$

The remaining 3 terms are respectively

$$-10(3x)^2(2)^3, 5(3x)(2)^4 \text{ and } -(2)^5$$

$$(3x-2)^5 = (3x)^5 - 5(3x)^4(2) + 10(3x)^3(2)^2 - 10(3x)^2(2)^3 \\ + 5(3x)(2)^4 - (2)^5 \\ = 243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$$

- 16 The 1st term $= 1^8 = 1$, 2nd term $= -81^7a = -8a$, 3rd term
 $= \frac{8 \times 7}{2} 1^6a^2 = 28a^2$, 4th term $= -\frac{8 \times 7 \times 6}{3} 1^5a^3 = -56a^3$,
 5th term $= \frac{8 \times 7 \times 6 \times 5}{4} 1^4a^4 = 701^4a^4 = 70a^4$

The remaining 4 terms are respectively

$$-56a^5, 28a^6, -8a^7 \text{ and } a^8$$

$$(1-a)^8 = 1 - 8a + 28a^2 - 56a^3 + 70a^4 - 56a^5 + 28a^6 - 8a^7 + a^8$$

- 17 The 1st term $= 1^7$, 2nd term $= -71^6c = -7c$, 3rd term
 $= \frac{7 \times 6}{2} 1^5c^2 = 21c^2$, 4th term $= -\frac{7 \times 6 \times 5}{1} 1^4c^3 = -351^4c^3 \\ = -35c^3$

The remaining 4 terms are respectively

$$35c^4, -21c^5, 7c^6 \text{ and } -c^7$$

$$(1-c)^7 = 1 - 7c + 21c^2 - 35c^3 + 35c^4 - 21c^5 + 7c^6 - c^7$$

18. The 1st term = 1^6 the 2nd term = $-6 \cdot 1^5 \cdot 3x$, 3rd term
 $= \frac{6 \cdot 5}{2} 1^4 (3x)^2 = 15 \cdot 1^4 (3x)^2$, 4th term
 $= -\frac{6 \cdot 5 \cdot 4}{3!} 1^3 (3x)^3 = -20 \cdot 1^3 (3x)^3$

The remaining 3 terms are respectively

$$15 \cdot 1^2 (3x)^4, -6 \cdot 1 (3x)^5 \text{ and } (3x)^6$$

$$(1 - 3x)^6 = 1 - 6(3x) + 15(3x)^2 - 20(3x)^3 + 15(3x)^4$$

$$- 6(3x)^5 + (3x)^6$$

$$= 1 - 18x + 135x^2 - 540x^3 + 1215x^4 - 1458x^5 + 729x^6$$

19 The 1st term = 1^7 , 2nd term = $-7 \cdot 1^6 (2x)$, 3rd term
 $= \frac{7 \cdot 6}{2} 1^5 (2x)^2 = 21 \cdot 1^5 (2x)^2$, 4th term
 $= -\frac{7 \cdot 6 \cdot 5}{3!} 1^4 (2x)^3 = -35 \cdot 1^4 (2x)^3$

The remaining 4 terms are respectively

$$35 \cdot 1^3 (2x)^4, -21 \cdot 1^2 (2x)^5, 7 \cdot 1 (2x)^6 \text{ and } -(2x)^7$$

$$(1 - 2x)^7 = 1 - 7(2x) + 21(2x)^2 - 35(2x)^3 + 35(2x)^4 - 21(2x)^5$$

$$+ 7(2x)^6 - (2x)^7$$

$$= 1 - 14x + 84x^2 - 280x^3 + 560x^4 - 672x^5$$

$$+ 448x^6 - 128x^7$$

20 The 1st term = $(2x)^5$, 2nd term = $-8(2x)^4 a$, 3rd term
 $= \frac{8 \cdot 7}{2} (2x)^3 a^2 = 28(2x)^3 a^2$, 4th term
 $= -\frac{8 \cdot 7 \cdot 6}{3!} (2x)^2 a^3 = -56(2x)^2 a^3$, 5th term
 $= \frac{8 \cdot 7 \cdot 6 \cdot 5}{4!} (2x)^1 a^4 = 70(2x)^1 a^4$

The remaining 4 terms are respectively $-56(2x)^0 a^5$,

$$28(2x)^1 a^6, -8(2x)^2 a^7 \text{ and } a^8$$

$$(2x - a)^5 = (2x)^5 - 8(2x)^4 a + 28(2x)^3 a^2 - 56(2x)^2 a^3 + 70(2x)^1 a^4$$

$$- 56(2x)^0 a^5 + 28(2x)^1 a^6 - 8(2x)^2 a^7 + a^8$$

$$= 256x^5 - 1024x^4 a + 1792x^3 a^2 - 1792x^2 a^3 + 1120x a^4$$

$$- 448x^0 a^5 + 112x^1 a^6 - 16x^2 a^7 + a^8$$

21 The 1st term = 1^{10} , 2nd term = $-10x^1 a$, 3rd term
 $= \frac{10 \cdot 9}{2} x^2 a^2 = 45x^2 a^2$, 4th term
 $= -\frac{10 \cdot 9 \cdot 8}{3!} x^3 a^3 = -120x^3 a^3$, 5th term
 $= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} x^4 a^4 = 210x^4 a^4$, 6th term
 $= -\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5!} x^5 a^5 = -252x^5 a^5$

The remaining 5 terms are $210x^6 a^6, -120x^7 a^7, 45x^8 a^8$
 $-10x^9 a^9 \text{ and } x^{10}$

$$\therefore (r-a)^{10} = r^{10} - 10r^9a + 45x^8a^2 - 120x^7a^3 + 210x^6a^4 - 252x^5a^5 \\ + 210x^4a^6 - 120r^3a^7 + 45x^2a^8 - 10xa^9 + a^{10}$$

22 The 1st term = $(3x)^5$, 2nd term = $-5(3x)^4(2a)$, 3rd term = $\frac{5 \times 4}{2} (3x)^3 (2a)^2 = 10(3x)^3 (2a)^2$

the remaining 3 terms are respectively

$$-10(3x)^2 (2a)^3, 5(3x)(2a)^4 \text{ and } -(2a)^5 \\ (3x-2a)^5 = (3x)^5 - 5(3x)^4(2a) + 10(3x)^3 (2a)^2 - 10(3x)^2 (2a)^3 \\ + 5(3x)(2a)^4 - (2a)^5 \\ = 243x^5 - 810x^4a + 1080x^3a^2 - 720x^2a^3 + 240xa^4 - 32a^5$$

23 $(x+1)^5 - (x-1)^5 = (x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) \\ - (x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1) \\ = 10x^4 + 20x^2 + 2$

24 $(r-1)^6 + (r+1)^6 = (x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1) \\ + (r^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1) \\ = 2x^6 + 30x^4 + 30x^2 + 2$

25 $(x+a)^7 - (x-a)^7 = (x^7 + 7x^6a + 21x^5a^2 + 35x^4a^3 + 35x^3a^4 + 21x^2a^5 \\ + 7xa^6 + a^7) - (x^7 - 7x^6a + 21x^5a^2 - 35x^4a^3 \\ + 35x^3a^4 - 21x^2a^5 + 7xa^6 - a^7) \\ = 14x^6a + 70x^4a^3 + 42x^2a^5 + 2a^7$

26 $(r+a)^4 = r^4 + 4r^3a + 6r^2a^2 + 4ra^3 + a^4$, the co efficient
are respectively 1, 4, 6, 4 and 1
the required sum = $1 + 4 + 6 + 4 + 1 = 16$

27 $(x+a)^5 = x^5 + 5x^4a + 10x^3a^2 + 10x^2a^3 + 5xa^4 + a^5$, the
co efficient are respectively 1, 5, 10, 10, 5 and 1
the required sum = $1 + 5 + 10 + 10 + 5 + 1 = 32$

28 $(x+a)^6 = r^6 + 6x^5a + 15x^4a^2 + 20x^3a^3 + 15x^2a^4 + 6xa^5 + a^6$,
the respective co efficient are 1, 6, 15, 20, 15, 6 and 1
the required sum = $1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$

29 $(r+a)^7 = x^7 + 7x^6a + 21x^5a^2 + 35x^4a^3 + 35x^3a^4 + 21x^2a^5 + 7xa^6 + a^7$,
the respective co efficient are 1, 7, 21, 35, 35, 21, 7 and 1
the required sum = $1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128$

$$30 \quad (1+a)^8 = 1^8 + 8 \cdot 1^7 a + 28 \cdot 1^6 a^2 + 56 \cdot 1^5 a^3 + 70 \cdot 1^4 a^4 + 56 \cdot 1^3 a^5 + 28 \cdot 1^2 a^6 + 8 \cdot 1 a^7 + a^8, \text{ the respective coefficients are } 1, 8, 28, 56, 70, 56, 28, 8 \text{ and } 1$$

$$\text{the req sum} = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256$$

31 The given expression

$$\begin{aligned} &= (1^5 + 5 \cdot 1^4 + 10 \cdot 1^3 + 10 \cdot 1^2 + 5 \cdot 1 + 1) + 31 \\ &= (1+1)^5 + 31 = (-2+1)^5 + 31 = (-1)^5 + 31 \\ &= -1 + 31 = 30 \end{aligned}$$

32 The given expression

$$\begin{aligned} &= (1^6 - 6 \cdot 1^5 + 15 \cdot 1^4 - 20 \cdot 1^3 + 15 \cdot 1^2 - 6 \cdot 1 + 1) - 1 \\ &= (1-1)^6 - 1 = (\sqrt{2})^6 - 1 = 2^3 - 1 = 4 - 1 = 3 \end{aligned}$$

33 The given expression

$$\begin{aligned} &= (2x)^4 - 4(2x)^3 + 6(2x)^2 - 4(2x) + 1 - 81 \\ &= (2x-1)^4 - 81 = (4-1)^4 - 81 = (3)^4 - 81 \\ &= 81 - 81 = 0 \end{aligned}$$

34 The given expression

$$\begin{aligned} &= (1)^4 + 4(1)^3 \cdot 3 + 6(1)^2 (3)^2 + 4(1)(3)^3 + (3)^4 \\ &= (1+3)^4 = (-5+3)^4 = (-2)^4 = 16 \end{aligned}$$

35 The given expression

$$\begin{aligned} &= 1^4 + 4x^3 \cdot 2 + 6x^2 (2)^2 + 6x (2)^3 + (2)^4 - 625 \\ &= (1+2)^4 - 625 = (-7+2)^4 - 625 \\ &= (-5)^4 - 625 = 625 - 625 = 0 \end{aligned}$$

Exercise 33

1 Putting a for x , b for y and c for $(-z)$, we have the given expression

$$\begin{aligned} &= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\ &= a^3+b^3+c^3-3abc = x^3+y^3-z^3+3xyz \end{aligned}$$

2 Putting a for p , b for $-2q$ and c for $-r$, we have the given expression

$$\begin{aligned} &= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\ &= a^3+b^3+c^3-3abc = p^3-8q^3-r^3-6pqr \end{aligned}$$

3 Putting a for $2x$, b for $-3y$ and c for $-z$, we have the given expression

$$\begin{aligned} &= (2x-3y-z)\{(2x)^2+(-3y)^2+(-z)^2-(2x) \\ &\quad (-3y)-(2x)(-z)-(-3y)(-z)\} \end{aligned}$$

$$\begin{aligned}
&= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\
&= a^3+b^3+c^3-3abc \\
&= (2x)^3+(-3y)^3+(-z)^3-3(2x)(-3y)(-z) \\
&= 8x^3-27y^3-z^3-18xyz
\end{aligned}$$

- 4 Putting x for a , y for $-2b$ and z for 3 , we have the given expression $= (a-2b+3)\{a^2+(-2b)^2+(3)^2-(a)(-2b)-(a)(3)-(-2b)(3)\}$
- $$\begin{aligned}
&= (x+y+z)(x^2+y^2+z^2-xy-xz-yz) \\
&= x^3+y^3+z^3-3xyz \\
&= (a)^3+(-2b)^3+(3)^3-3(a)(-2b)(3) \\
&= a^3-8b^3+27+18ab
\end{aligned}$$

- 5 Putting x for $3a$, y for $(-5b)$ and z for (-4) , we have the given expression $= (3a-5b-4)\{(3a)^2+(-5b)^2+(-4)^2-(3a)(-5b)-(3a)(-4)-(-5b)(-4)\}$
- $$\begin{aligned}
&= (x+y+z)(x^2+y^2+z^2-xy-xz-yz) \\
&= x^3+y^3+z^3-3xyz \\
&= (3a)^3+(-5b)^3+(-4)^3-3(3a)(-5b)(-4) \\
&= 27a^3-125b^3-64-180ab
\end{aligned}$$

- 6 Putting a for x , b for $(-y)$ and c for (-1) , we have
- $$\begin{aligned}
x^3-y^3-1-3xy &= x^3+(-y)^3+(-1)^3-3(x)(-y)(-1) \\
&= a^3+b^3+c^3-3abc \\
&= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\
&= (x-y-1)(x^2+y^2+1+xy+yx-y)
\end{aligned}$$

- 7 Putting a for x , b for $(-y)$ and c for 2 , we have
- $$\begin{aligned}
x^3-y^3+6xy+8 &= x^3+(-y)^3+(2)^3-3(x)(-y)(2) \\
&= a^3+b^3+c^3-3abc \\
&= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\
&= (x-y+2)(x^2+y^2+4+xy-2x+2y)
\end{aligned}$$

- 8 Putting a for x , b for $(-2y)$ and c for $(-3z)$, we have
- $$\begin{aligned}
x^3-8y^3-27z^3-18xyz &= x^3+(-2y)^3+(-3z)^3-3(x)(-2y)(-3z) \\
&= a^3+b^3+c^3-3abc \\
&= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\
&= (x-2y-3z)(x^2+4y^2+9z^2+2xy+3xz-6yz)
\end{aligned}$$

9 $x^3 + y^3 + 18xy - 216$

$$\begin{aligned} &= x^3 + y^3 + (-6)^3 - 3(x)(y)(-6) \\ &= (x+y-6)(x^2+y^2+36-xy+6x+6y) \\ &= (6-6)(x^2+y^2+36-xy+6x+6y) = 0 \end{aligned}$$

10 $a^3 - 8b^3 - 24ab - 64$

$$\begin{aligned} &= a^3 + (-2b)^3 + (-4)^3 - 3a(-2b)(-4) \\ &= (a-2b-4)(a^2+4b^2+16+2ab+4a-8b) \\ &= (4-4)(a^2+4b^2+16+2ab+4a-8b) \\ &= 0 \times (a^2+4b^2+16+2ab+4a-8b) \\ &= 0 \end{aligned}$$

11 The given expression

$$\begin{aligned} &= \{(s-a) + (s-b) + (s-c)\} \{(s-a)^2 + (s-b)^2 + (s-c)^2 \\ &\quad - (s-a)(s-b) - (s-a)(s-c) - (s-b)(s-c)\} \\ &= \{3s - (a+b+c)\} \{(s-a)^2 + (s-b)^2 + (s-c)^2 - (s-a)(s-b) \\ &\quad - (s-a)(s-c) - (s-b)(s-c)\} \\ &= 0 \times \{(s-a)^2 + (s-b)^2 + (s-c)^2 - (s-a)(s-b) \\ &\quad - (s-a)(s-c) - (s-b)(s-c)\} \end{aligned}$$

12 Putting x for $(a-2b)$, y for $(2b-c)$ and z for $(3c-a)$, we have

$$x+y+z = (a-2b) + (2b-c) + (3c-a) = 0$$

Hence $(a-2b)^3 + (2b-3c)^3 + (3c-a)^3 - 3(a-2b)(2b-3c)(3c-a)$

$$\begin{aligned} &= x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-xz-yz) \\ &= 0 \times (x^2+y^2+z^2-xy-xz-yz) = 0 \end{aligned}$$

$$(a-2b)^3 + (2b-3c)^3 + (3c-a)^3 = 3(a-2b)(2b-3c)(3c-a)$$

13 Putting a for $(x+y-2z)$, b for $(y+z-2x)$ and c for

$$(z+x-2y), \text{ we have } a+b+c = (x+y-2z) +$$

$$(y+z-2x) + (z+x-2y) = 0$$

$$\begin{aligned} &(x+y-2z)^3 + (y+z-2x)^3 + (z+x-2y)^3 \\ &\quad - 3(x+y-2z)(y+z-2x)(z+x-2y) \\ &= a^3 + b^3 + c^3 - 3abc \\ &= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\ &= 0 \times (a^2+b^2+c^2-ab-ac-bc) = 0 \end{aligned}$$

$$\begin{aligned} &(x+y-2z)^3 + (y+z-2x)^3 + (z+x-2y)^3 \\ &= 3(x+y-2z)(y+z-2x)(z+x-2y) \end{aligned}$$

- 14 Putting x for $(a+2b-3c)$, y for $(b+2c-3a)$ and z for $(c+2a-3b)$, we have $x+y+z=(a+2b-3c)+(b+2c-3a)+(c+2a-3b)=0$

$$\begin{aligned} \text{Hence } (a+2b-3c)^3 + (b+2c-3a)^3 + (c+2a-3b)^3 \\ - 3(a+2b-3c)(b+2c-3a)(c+2a-3b) \\ = x^3 + y^3 + z^3 - 3xyz \\ = (x+y+z)(x^2+y^2+z^2-xy-yz-xz) \\ = 0 \times (x^2+y^2+z^2-xy-yz-xz) = 0 \\ (a+2b-3c)^3 + (b+2c-3a)^3 + (c+2a-3b)^3 \\ = 3(a+2b-3c)(b+2c-3a)(c+2a-3b) \end{aligned}$$

15. Putting x for $(2p-5q+3r)$, y for $(2q-5r+3p)$ and z for $(2r-5p+3q)$ we have $x+y+z$

$$= (2p-5q+3r) + (2q-5r+3p) + (2r-5p+3q) = 0$$

$$\begin{aligned} \text{Hence } (2p-5q+3r)^3 + (2q-5r+3p)^3 + (2r-5p+3q)^3 \\ - 3(2p-5q+3r)(2q-5r+3p)(2r-5p+3q) \\ = x^3 + y^3 + z^3 - 3xyz \\ = (x+y+z)(x^2+y^2+z^2-xy-yz-xz) \\ = 0 \times (x^2+y^2+z^2-xy-yz-xz) = 0 \\ (2p-5q+3r)^3 + (2q-5r+3p)^3 + (2r-5p+3q)^3 \\ = 3(2p-5q+3r)(2q-5r+3p)(2r-5p+3q) \end{aligned}$$

Exercise 34

- 1 Putting a for $(x-y)$, b for $(y-z)$ and c for $(z-x)$, we have $a-b=x-2y+z$, $a-c=2x-y-z$ and $b-c=y-2z+x$

The given expression

$$\begin{aligned} = (a-b)(a-c)(b-c) &= a^2(b-c) + b^2(c-a) + c^2(a-b) \\ &= (x-y)^2(y-2z+x) + (y-z)^2 \\ &\quad (z-2x+y) + (z-x)^2(x-2y+z) \end{aligned}$$

- 2 Putting x for $(a+b)$, y for $(b+c)$ and z for $(c+a)$, we have $x-y=a-c$, $y-z=b-a$ and $z-x=c-b$

The given expression

$$\begin{aligned} = x^2(y-z) + y^2(z-x) + z^2(x-y) + (y-z)(z-x)(x-y) \\ = -(y-z)(z-x)(x-y) + (y-z)(z-x)(x-y) = 0 \end{aligned}$$

- 3 Putting x for $(a-b+c)$, y for $(b-c+a)$ and z for $(c-a+b)$, we have $x-y=2(c-b)$, $y-z=2(a-c)$ and $z-x=2(b-a)$

The given expression

$$\begin{aligned} &= x^2(y-z) + y^2(z-x) + z^2(x-y) \\ &= (x-y)(y-z)(y-z) \\ &= 2(c-b) \cdot 2(a-b) \cdot 2(a-c) \\ &= 8(c-b)(a-b)(a-c) \end{aligned}$$

- 4 Putting x for $(x+y)$, y for $(y+z)$ and z for $(z+x)$, we have $x-y=z$, $y-z=x$ and $z-x=y$

The given expression

$$\begin{aligned} &= x^2(y-z) + y^2(z-x) + z^2(x-y) \\ &= (x-y)(y-z)(y-z) \end{aligned}$$

- 5 Putting x for $(a-b-c)$, y for $(b-c-a)$ and z for $(c-a-b)$, we have $x-y=2(a-b)$, $y-z=2(b-c)$ and $z-x=2(c-a)$
 $(x-y)(y-z)(z-x)=8(a-b)(b-c)(c-a)$

The given expression

$$\begin{aligned} &= x^2(y-z) + y^2(z-x) + z^2(x-y) + (x-y)(y-z)(z-x) \\ &= -(x-y)(y-z)(z-x) + (x-y)(y-z)(z-x) = 0 \end{aligned}$$

- 6 Putting x for $(x-y)$, y for $(y-z)$ and z for $(z-x)$, we have $x-y=z$, $y-z=x$ and $z-x=y$

The given expression

$$\begin{aligned} &= ab(a-b) + bc(b-c) + ca(c-a) + (a-b)(b-c)(c-a) \\ &= -(a-b)(b-c)(c-a) + (a-b)(b-c)(c-a) = 0 \end{aligned}$$

Exercise 35.

- 1 $(a^3+ax-x^2)(a^2-ax+x^2) = \{a^3+(ax-x^2)\}\{a^3-(ax-x^2)\}$
 $= a^6 - (ax-x^2)^2$
 $= a^6 - \{a^2x^2 - 2ax^3 + x^4\}$
 $= a^6 - a^2x^2 + 2ax^3 - x^4$

- 2 $(a^2-ax+x^2)(a^2-a^2+x^2)$
 $= \{x^2+(a^2-ax)\}\{x^2-(a^2-ax)\}$
 $= x^4 - (a^2-ax)^2 = x^4 - \{a^4 - 2a^2x + a^2x^2\}$
 $= x^4 - a^4 + 2a^2x - a^2x^2$

- 3 $(a+b+c)(a-b-c)+(b+c-a)(a-b+c)$
 $=\{a+(b+c)\}\{a-(b+c)\}+\{c+(b-a)\}\{c-(b-a)\}$
 $=\{a^2-(b+c)^2\}+\{c^2-(b-a)^2\}$
 $=\{a^2-b^2-2bc-c^2+c^2-b^2+2ab-a^2\}$
 $=2ab-2b^2-2bc=2b(a-b-c)$
- 4 $2(a^2+b^2+c^2-ab-ac-bc)$
 $=(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)$
 $=(a-b)^2+(b-c)^2+(c-a)^2$
- 5 $\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$
 $=\frac{1}{2}(a+b+c)\{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac\}$
 $=\frac{1}{2}(a+b+c)(2a^2+2b^2+2c^2-2ab-2bc-2ac)$
 $=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$
 $=a^3+b^3+c^3-3abc$
- 6 $(a+b)(a+c)(b+c)=\{a^2+(b+c)a+bc\}(b+c)$
 $=a^2(b+c)+a(b+c)^2+bc(b+c)$
 $=a^2(b+c)+a(b^2+2bc+c^2)+b^2c+bc^2$
 $=a^2(b+c)+ab^2+2abc+ac^2+b^2c+bc^2$
 $=a^2(b+c)+b^2(a+c)+c^2(a+b)+2abc$
- 7 $2(x^3-x)+3x(x+1)=2x(x^2-1)+3x(x+1)$
 $=2x(x+1)(x-1)+3x(x+1)$
 $=x(x+1)\{2(x-1)+3\}$
 $=x(x+1)(2x-2+3)=x(x+1)(2x+1)$
- 8 $x^3+x+x(x+1)(2x+1)-2x(x+1)$
 $=x(x^2+1)+x(x+1)(2x+1)-2x(x+1)$
 $=x(x+1)(x^2-x+1)+x(x+1)(2x+1)-2x(x+1)$
 $=x(x+1)\{(x^2-x+1)+(2x+1)-2\}$
 $=x(x+1)(x^2-x+1+2x+1-2)$
 $=x(x+1)(x^2+1)=x(x+1)x(x+1)=x^2(x+1)^2$
- 9 $(a^2+b^2)(c^2+d^2)=a^2c^2+b^2c^2+a^2d^2+b^2d^2$
 $=a^2c^2+b^2d^2-2acbd+b^2c^2+a^2d^2+2acbd$
 $=(ac-bd)^2+(ad+bc)^2$
- 10 $(a+b)^2-(c+d)^2+(a+c)^2-(b+d)^2$
 $=(a+b+c+d)(a+b-c-d)+(a+c+b+d)(a+c-b-d)$

$$\begin{aligned}
 &= (a+b+c+d)\{(a+b-c-d)+(a+c-b-d)\} \\
 &= (a+b+c+d)(a+b-c-d+a+c-b-d) \\
 &= 2(a+b+c+d)(a-d)
 \end{aligned}$$

$$\begin{aligned}
 11. \quad &(9x^2+4y^2+z^2-2yz-3xz-6xy)(3x+2y+z) \\
 &= \{(3x)^2+(2y)^2+z^2-(2y)z-(3x)z-(2y)3x\}(3x+2y+z) \\
 &= (3x)^2+(2y)^2+z^2-3(3x)(2y)z \\
 &= 27x^2+8y^2+z^2-18xyz
 \end{aligned}$$

$$\begin{aligned}
 12 \quad &(a+b+c-d)(d-a-b+c) = \{c+(a+b-d)\}\{c-(a+b-d)\} \\
 &= c^2-(a+b-d)^2
 \end{aligned}$$

$$\begin{aligned}
 13 \quad &\{(b+c)^2-a^2\}\{a^2-b^2-c^2+2bc\} \\
 &= \{(b+c)^2-a^2\}\{a^2-(b^2+c^2-2bc)\} \\
 &= \{-a^2+(b+c)^2\}\{a^2-(b-c)^2\} \\
 &= -a^4+a^2\{(b+c)^2+(b-c)^2\}-(b^2-c^2)^2 \\
 &= -a^4+a^2(2b^2+2c^2)-b^4-c^4+2b^2c^2 \\
 &= -a^4+2a^2b^2+2a^2c^2-b^4-c^4+2b^2c^2 \\
 &= 2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4
 \end{aligned}$$

$$\begin{aligned}
 14. \quad &(a+b+c)(a+b-c)(a-b+c)(b+c-a) \\
 &= \{(a+b)+c\}\{(a+b)-c\}\{c+(a-b)\}\{c-(a-b)\} \\
 &= \{(a+b)^2-c^2\}\{c^2-(a-b)^2\} \\
 &= \{-c^2+(a+b)^2\}\{c^2-(a-b)^2\} \\
 &= -c^4+c^2\{(a+b)^2+(a-b)^2\}-(a^2-b^2)^2 \\
 &= -c^4+c^2(2a^2+2b^2)-a^4-b^4+2a^2b^2 \\
 &= -c^4+2a^2c^2+2b^2c^2-a^4-b^4+2a^2b^2 \\
 &= 2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4
 \end{aligned}$$

15 We have $(a^2+b^2+c^2)^2 = a^4+b^4+c^4+2a^2b^2+2a^2c^2+2b^2c^2$ and $(a+b+c)(a+b-c)(a+c-b)(b+c-a) = 2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4$ (as in the previous example),

The given expression $= 4(a^2b^2+a^2c^2+b^2c^2)$

16 It follows evidently from the previous example that the given expression $= 2(a^4+b^4+c^4)$

$$\begin{aligned}
 17. \quad &ax = a^3 - abc, \quad by = b^3 - abc \text{ and } cz = c^3 - abc \\
 &\text{Hence } ax + by + cz = a^3 + b^3 + c^3 - 3abc
 \end{aligned}$$

$$\begin{aligned}
 &= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) \\
 &= (a+b+c)\{(a^2-bc)+(b^2-ac)+(c^2-ab)\} \\
 &= (a+b+c)(x+y+z)
 \end{aligned}$$

18 $a^2x = x^3 - xyz$, $b^2y = y^3 - xyz$ and $c^2z = z^3 - xyz$

Hence $a^2x + b^2y + c^2z$

$$\begin{aligned}
 &= x^3 + y^3 + z^3 - 3xyz \\
 &= (x+y+z)(x^2+y^2+z^2-xy-xz-yz) \\
 &= (x+y+z)\{(x^2-yz)+(y^2-xz)+(z^2-xy)\} \\
 &= (x+y+z)(a^2+b^2+c^2)
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= \{(a+b+c)^2 - (a+b-c)^2\} + \{(a+c-b)^2 - (b+c-a)^2\} \\
 &= \{(a+b+c) + (a+b-c)\}\{(a+b+c) - (a+b-c)\} \\
 &\quad + \{(a+c-b) + (b+c-a)\}\{(a+c-b) - (b+c-a)\} \\
 &= (2a+2b)2c + (2c)(2a-2b) = 2c\{(2a+2b) + (2a-2b)\} \\
 &= 2c \quad 4a = 8ac
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= \{(a^2+b^2+c^2)^2 - (b^2+c^2-a^2)^2\} - \{(a^2-b^2+c^2)^2 - (a^2+b^2-c^2)^2\} \\
 &= \{(a^2+b^2+c^2) + (b^2+c^2-a^2)\}\{(a^2+b^2+c^2) - (b^2+c^2-a^2)\} \\
 &\quad - \{(a^2-b^2+c^2) + (a^2+b^2-c^2)\}\{(a^2-b^2+c^2) - (a^2+b^2-c^2)\} \\
 &= (2b^2+2c^2)(2a^2) - (2a^2)(2c^2-2b^2) = 2a^2\{2b^2+2c^2-2c^2+2b^2\} \\
 &= 2a^2 \quad 4b^2 = 8a^2b^2
 \end{aligned}$$

21 The left-hand expression

$$\begin{aligned}
 &= \{(a+d) + (b-c)\}\{(a+d) - (b-c)\} + \{(b+c) \\
 &\quad + (a-d)\}\{(b+c) - (a-d)\} \\
 &= \{(a+d)^2 - (b-c)^2\} + \{(b+c)^2 - (a-d)^2\} \\
 &= \{(a+d)^2 - (a-d)^2\} + \{(b+c)^2 - (b-c)^2\} \\
 &= 4ad + 4bc = 4(ad+bc)
 \end{aligned}$$

22 $(b+c+a-d)(b+c-a+d)$

$$\begin{aligned}
 &= \{(b+c) + (a-d)\}\{(b+c) - (a-d)\} \\
 &= (b+c)^2 - (a-d)^2 = (b^2+c^2+2bc) - (a^2+d^2-2ad) \\
 &= 2(ad+bc) - (a^2-b^2-c^2+d^2)
 \end{aligned}$$

23 The given expression

$$= (2ad+2bc)^2 - (a^2-b^2-c^2+d^2)^2$$

$$\begin{aligned}
&= (2ad + 2bc + a^2 - b^2 - c^2 + d^2)(2ad + 2bc \\
&\quad - a^2 + b^2 + c^2 - d^2) \\
&= \{(a^2 + d^2 + 2ad) - (b^2 + c^2 - 2bc)\} \{(b^2 + c^2 + 2bc) \\
&\quad - (a^2 + d^2 - 2ad)\} \\
&= \{(a+d)^2 - (b-c)^2\} \{(b+c)^2 - (a-d)^2\} \\
&= (a+d+b-c)(a+d-b+c)(b+c+a-d)(b+c-a+d)
\end{aligned}$$

$$24 \quad (x^2 + 1 - 2x)(x^2 + 1 + 2x) = (x^2 + 1)^2 - (2x)^2 = x^4 - 2x^2 + 1$$

Hence the given expression

$$\begin{aligned}
&= (x^4 + 1 + 2x^2)(x^4 + 1 - 2x^2) = (x^4 + 1)^2 - 4x^4 \\
&= x^8 - 2x^4 + 1
\end{aligned}$$

$$\begin{aligned}
25 \quad &(a^2 + 2ab + b^2 - c^2)(a^2 - 2ab + b^2 + c^2) \\
&= \{(a+b)^2 - c^2\} \{(a-b)^2 + c^2\} \\
&= (a^2 - b^2)^2 + c^2 \{(a+b)^2 - (a-b)^2\} - c^4 \\
&= a^4 - 2a^2b^2 + b^4 + c^2 4ab - c^4 \\
&= a^4 - 2a^2b^2 + b^4 + 4ab c^2 - c^4
\end{aligned}$$

26 Putting a for $(x-y+z)$, b for $(y-z+x)$ and c for $(z-x+y)$,
we have $a+b+c=x+y+z$ and

$$\begin{aligned}
\text{the given expression} &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\
&= (a+b+c)^2 = (x+y+z)^2
\end{aligned}$$

27 The given expression

$$\begin{aligned}
&= -\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} - a^2(c-b) \\
&\quad + \{c^2(a-b) - b^2(c+a)\} \\
&= -a^2(b-c) - b^2(c-a) - c^2(a-b) + a^2(b-c) + c^2(a-b) \\
&\quad - b^2(c+a) \\
&= -b^2(c-a+c+a) = -2b^2c
\end{aligned}$$

28 The given expression

$$\begin{aligned}
&= -\{ab(a-b) + bc(b-c) + ca(c-a)\} - ab(b-a) \\
&\quad - bc(c-b) + ca(c+a) \\
&= -ab(a-b) - bc(b-c) - ca(c-a) + ab(a-b) + bc(b-c) \\
&\quad + ca(c+a) \\
&= ca\{(c+a) - (c-a)\} = ca 2a = 2a^2c
\end{aligned}$$

29 The left-hand expression

$$\begin{aligned}
&= (x^2 - 2ax + a^2)(b-c) + (x^2 - 2bx + b^2)(c-a) \\
&\quad + (x^2 - 2cx + c^2)(a-b) + (b-c)(c-a)(a-b)
\end{aligned}$$

$$\begin{aligned}
&= x^2\{(b-c) + (c-a) + (a-b)\} - 2x\{a(b-c) + b(c-a) \\
&\quad + c(a-b)\} + \{a^2(b-c) + b^2(c-a) + c^2(a-b) \\
&\quad + (b-c)(c-a)(a-b)\} \\
&= -(a-b)(b-c)(c-a) + (b-c)(c-a)(a-b) = 0
\end{aligned}$$

30 The left-hand expression

$$\begin{aligned}
&= a^2x^4 + a^2y^2 + a^2z^2 + b^2x^2 + b^2y^2 + b^2z^2 + c^2x^2 + c^2y^2 + c^2z^2 \\
&\quad - (a^2x^2 + b^2y^2 + c^2z^2 + 2abxy + 2acxz + 2bcyz) \\
&= (a^2y^2 - 2abxy + b^2x^2) + (c^2x^2 - 2acxz + a^2z^2) \\
&\quad + (b^2z^2 - 2bcyz + c^2y^2) \\
&= (ay - bx)^2 + (cx - az)^2 + (bz - cy)^2
\end{aligned}$$

31 $(as + bc)(bs + ca)(cs + ab)$

$$\begin{aligned}
&= (a^2 + ab + ac + bc)(ab + b^2 + bc + ca)(ac + bc + c^2 + ab) \\
&= \{a(a+b) + c(a+b)\}\{b(a+b) + c(a+b)\}\{c(a+c) + b(a+c)\} \\
&= (a+b)(a+c)(a+b)(b+c)(a+c)(b+c) \\
&= (a+b)^2(a+c)^2(b+c)^2
\end{aligned}$$

32 $(a+b)(a+c)(b+c) - 2abc$

$$\begin{aligned}
&= \{a^2 + a(b+c) + bc\}(b+c) - 2abc \\
&= a^2(b+c) + a(b+c)^2 + bc(b+c) - 2abc \\
&= a^2(b+c) + a(b^2 + c^2 + 2bc) + b^2c + bc^2 - 2abc \\
&= a^2(b+c) + b^2(a+c) + c^2(a+b)
\end{aligned}$$

33 Putting x for $(a+c)$ and y for $(b+c)$ we have

$$\begin{aligned}
x-y &= (a+c) - (b+c) = a-b, \text{ and the left-hand} \\
\text{expression} &= x^3 - y^3 - 3xy(x-y) = (x-y)^3 = (a-b)^3
\end{aligned}$$

34 Putting a for $(x-2y+3z)$ and b for $(x+2y-3z)$,

$$\begin{aligned}
\text{we have } a+b &= x-2y+3z + x+2y-3z = 2x, \text{ and} \\
\text{the given expression} &= a^3 + b^3 + 3(a+b)ab \\
&= (a+b)^3 = (2x)^3 = 8x^3
\end{aligned}$$

35 $4(a+b+c)^2 = (2a+2b+2c)^2$

$$\begin{aligned}
&= \{(a+b) + (b+c) + (c+a)\}^2 \\
&= (a+b)^2 + (b+c)^2 + (c+a)^2 + 2(a+b)(b+c) \\
&\quad + 2(a+b)(c+a) + 2(b+c)(c+a)
\end{aligned}$$

36 $8(a+b+c)^3 = (2a+2b+2c)^3$

$$= \{(a+b) + (b+c) + (c+a)\}^3$$

$$\begin{aligned}
 &= (a+b)^3 + (b+2c+a)^3 + 3(a+b)(b+2c+a) \\
 &\quad \{2(a+b+c)\} \\
 &= (a+b)^3 + (b+2c+a^3) + 6(a+b)(b+2c+a)(a+b+c)
 \end{aligned}$$

$$\begin{aligned}
 37 \quad 27(a+b+c)^3 &= \{3(a+b+c)\}^3 \\
 &= (3a+3b+3c)^3 \\
 &= \{(a+3b+2c) + (2a+c)\}^3 \\
 &= (a+3b+2c)^3 + (2a+c)^3 \\
 &\quad + 3(a+3b+2c)(2a+c)\{3(a+b+c)\} \\
 &= (a+3b+2c)^3 + (2a+c)^3 + 9(a+3b+2c)(2a+c)(a+b+c)
 \end{aligned}$$

$$\begin{aligned}
 38 \quad 8(a+b+c)^3 &= (2a+2b+2c)^3 = \{(a+b) + (b+c) + (c+a)\}^3 \\
 &= (a+b)^3 + (b+c)^3 + (c+a)^3 + 3\{(a+b) + (b+c)\}\{(b+c) \\
 &\quad + (c+a)\}\{(c+a) + (a+b)\} \\
 &\quad \text{(See Ex. 6, Page 91)} \\
 &= (a+b)^3 + (b+c)^3 + (c+a)^3 + 3(a+2b+c)(a+2c+b) \\
 &\quad (2a+b+c) \\
 &= 8(a+b+c)^3 - (a+b)^3 - (b+c)^3 - (c+a)^3 \\
 &= 3(2a+b+c)(a+2b+c)(a+b+2c)
 \end{aligned}$$

$$\begin{aligned}
 39 \quad 27(a+b+c)^3 &= (3a+3b+3c)^3 \\
 &= \{(a+2b) + (b+2c) + (c+2a)\}^3 \\
 &= (a+2b)^3 + (b+2c)^3 + (c+2a)^3 + 3\{(a+2b) \\
 &\quad + (b+2c)\}\{(b+2c) + (c+2a)\}\{(c+2a) + (a+2b)\} \\
 &= (a+2b)^3 + (b+2c)^3 + (c+2a)^3 + 3(a+3b+2c) \\
 &\quad (b+3c+2a)(c+3a+2b) \\
 &= 27(a+b+c)^3 - (a+2b)^3 - (b+2c)^3 - (c+2a)^3 \\
 &= 3(a+3b+2c)(b+3c+2a)(c+3a+2b)
 \end{aligned}$$

$$\begin{aligned}
 40 \quad 2(s-a)(s-b)(s-c) &+ a(s-b)(s-c) + b(s-c)(s-a) + c(s-a)(s-b) \\
 &= (s-b)(s-c)\{2s-2a+a\} + b(s-c)(s-a) + c(s-a)(s-b) \\
 &= (s-b)(s-c)(2s-a) + b(s-c)(s-a) + c(s-a)(s-b) \\
 &= (s-b)(s-c)(b+c) + b(s-c)(s-a) + c(s-a)(s-b) \\
 &= b(s-b)(s-c) + b(s-c)(s-a) + c(s-a)(s-b) + c(s-b) \\
 &\quad (s-c) \\
 &= b(s-c)(s-b+s-a) + c(s-b)(s-a+s-c) \\
 &= b(s-c)(2s-b-a) + c(s-b)(2s-a-c)
 \end{aligned}$$

$$\begin{aligned}
 &= b(s-c)c + c(s-b)b \\
 &= bc(s-c) + bc(s-b) = bc(s-c+s-b) \\
 &= bc(2s-b-c) = bc(a) = abc
 \end{aligned}$$

$$\begin{aligned}
 41 \quad & s(s-a)(s-b) + s(s-a)(s-c) + s(s+a)(s-c) + c(s+a)(s+b) \\
 &= s(s-a)(s-b+s-c) + s(s+a)(s-c) + c(s+a)(s+b) \\
 &= s(s-a)(s+a) + s(s+a)(s-c) + c(s+a)(s+b) \\
 & \qquad \qquad \qquad [\quad s-b-c=a] \\
 &= s(s+a)(s-a+s-c) + c(s+a)(s+b) \\
 &= s(s+a)(s+b) + c(s+a)(s+b) \quad [\quad s-a-c=b] \\
 &= (s+a)(s+b)(s+c)
 \end{aligned}$$

42 The left-hand expression

$$\begin{aligned}
 &= \frac{1}{2} \{ (s-a) + (s-b) + (s-c) \} [\{ (s-a) - (s-b) \}^2 \\
 & \qquad \qquad \qquad + \{ (s-b) - (s-c) \}^2 + \{ (s-c) - (s-a) \}^2] \\
 &= \frac{1}{2} \{ 3s - (a+b+c) \} [(b-a)^2 + (c-b)^2 + (a-c)^2] \\
 &= \frac{1}{2} (s) \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \} \\
 &= \frac{1}{2} \{ \frac{1}{2} (a+b+c) \} \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \} \\
 &= \frac{1}{2} \{ a^3 + b^3 + c^3 - 3abc \}
 \end{aligned}$$

$$\begin{aligned}
 43 \quad & (s-3a)^2 + (s-3b)^2 + (s-3c)^2 \\
 &= (s^2 - 6as + 9a^2) + (s^2 - 6bs + 9b^2) + (s^2 - 6cs + 9c^2) \\
 &= 3s^2 - 6s(a+b+c) + 9(a^2 + b^2 + c^2) \\
 &= 3 \{ s^2 - 2s^2 + 3(a^2 + b^2 + c^2) \} \\
 &= 3 \{ 3(a^2 + b^2 + c^2) - s^2 \} \\
 &= 3 \{ 3(a^2 + b^2 + c^2) - (a+b+c)^2 \} \\
 &= 3 \{ 3(a^2 + b^2 + c^2) - (a^2 + b^2 + c^2 + 2ab + 2ac + 2bc) \} \\
 &= 3 \{ 2(a^2 + b^2 + c^2) - 2(ab + bc + ca) \} \\
 &= 3 \{ (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) \} \\
 &= 3 \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \}
 \end{aligned}$$

44 We have $2(ab + bc + ca)$

$$\begin{aligned}
 &= (a+b+c)^2 - (a^2 + b^2 + c^2) \\
 &= (13)^2 - 69 = 169 - 69 = 100 \qquad (ab + bc + ca) = 50
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } a^3 + b^3 + c^3 - 3abc &= (a+b+c) \{ (a^2 + b^2 + c^2) - (ab + bc + ca) \} \\
 &= 13 \{ 69 - 50 \} = 13 \times 19 = 247
 \end{aligned}$$

$$\begin{aligned}
 45 \quad \text{We have } a^2 + b^2 + c^2 &= (a+b+c)^2 - 2(ab+bc+ca) \\
 &= (12)^2 - 2 \cdot 47 = 144 - 94 = 50 \\
 a^2 + b^2 + c^2 - 3abc &= (a+b+c)\{(a^2+b^2+c^2) \\
 &\quad - (ab+bc+ca)\} = 12\{50 - 47\} = 12 \times 3 = 36
 \end{aligned}$$

$$\begin{aligned}
 46 \quad \text{We have } 2(x^2 + y^2 + z^2) \\
 &= (x+y+z)^2 - (x^2 + y^2 + z^2) \\
 &= (14)^2 - 74 = 196 - 74 = 122 \qquad xy + yz + zx = 61
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } x^2 + y^2 + z^2 - 3xyz &= (x+y+z)\{(x^2 + y^2 + z^2) \\
 &\quad - (xy + yz + zx)\} = 14\{74 - 61\} = 14 \times 13 = 182
 \end{aligned}$$

$$\begin{aligned}
 \therefore 3xyz &= (x^2 + y^2 + z^2) - (x^2 + y^2 + z^2 - 3xyz) \\
 &= 434 - 182 = 252 \qquad xyz = 84
 \end{aligned}$$

$$\begin{aligned}
 47 \quad x^2 + y^2 + z^2 &= (x+y+z)^2 - 2(xy+yz+zx) \\
 &= (13)^2 - 2(52) = 169 - 104 = 65 \qquad x^3 + y^3 + z^3 \\
 &= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz \\
 &= 13(65 - 52) + 180 = 13 \cdot 13 + 180 = 169 + 180 = 349
 \end{aligned}$$

Exercise 36.

The examples are to be worked out mentally by the application of the rule given in Art 4, Page 99

Exercise 37.

$$1 \quad \frac{a^2b^2 - 2a^2b^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2} + \frac{-2a^2b^2}{a^2b^2} = \underline{a - 2b}$$

$$2 \quad \frac{3x^4 - 6x^3a}{-3x^2} = \frac{3x^4}{-3x^2} + \frac{-6x^3a}{-3x^2} = -x^2 + \underline{2xa}$$

$$3 \quad \frac{4x^2a^2 - 8x^2a^2}{4x^2a^2} = \frac{4x^2a^2}{4x^2a^2} + \frac{-8x^2a^2}{4x^2a^2} = \underline{xa - 2x^2}$$

$$4 \quad \frac{-9a^5 + 12a^3b^2}{-3a^3} = \frac{-9a^5}{-3a^3} + \frac{12a^3b^2}{-3a^3} = \underline{3a^2 - 4b^2}$$

$$5 \quad \frac{14a^6b^4 - 21a^7b^3}{-7a^5b^2} = \frac{14a^6b^4}{-7a^5b^2} + \frac{-21a^7b^3}{-7a^5b^2} = \underline{-2ab^2 + 3a^2b}$$

$$6 \quad \frac{2ax^3 - 4a^2x^2 + 6a^3x}{2ax} = \frac{2ax^3}{2ax} + \frac{-4a^2x^2}{2ax} + \frac{6a^3x}{2ax} \\ = \underline{x^2 - 2ax + 3a^2}$$

$$7 \quad \frac{-3a^3x^4 + 6a^2x^5 - 9a^4x^3}{-3a^2x^3} = \frac{-3a^3x^4}{-3a^2x^3} + \frac{6a^2x^5}{-3a^2x^3} + \frac{-9a^4x^3}{-3a^2x^3} \\ = \underline{ax - 2x^2 + 3a^2}$$

$$8 \quad \frac{12x^5 - 8x^3a^3 + 20ax^4}{-4x^3} = \frac{12x^5}{-4x^3} + \frac{-8x^3a^3}{-4x^3} + \frac{20ax^4}{-4x^3} \\ = \underline{-3x^2 + 2a^3 - 5ax}$$

$$9 \quad \frac{10m^5n^4 - 15m^7n^2 - 20m^3n^6}{5m^3n^3} = \frac{10m^5n^4}{5m^3n^3} + \frac{-15m^7n^2}{5m^3n^3} + \frac{-20m^3n^6}{5m^3n^3} \\ = \underline{2m^2n^2 - 3m^4 - 4n^3}$$

$$10 \quad \frac{8p^4q^2 - 5p^3q^3 - 3p^2q^4}{-8p^3q^4} = \frac{8p^4q^2}{-8p^3q^4} + \frac{-5p^3q^3}{-8p^3q^4} + \frac{-3p^2q^4}{-8p^3q^4} \\ = \underline{-\frac{p}{2q} + \frac{5}{8} + \frac{3}{8p}}$$

$$11 \quad \frac{-14x^8y^5 + 21x^{10}y^3 - 28x^7y^0}{7x^7y^3} = \frac{-14x^8y^5}{7x^7y^3} + \frac{21x^{10}y^3}{7x^7y^3} + \frac{-28x^7y^0}{7x^7y^3} \\ = \underline{-2xy^2 + 3x^3 - 4y^3}$$

$$12 \quad \frac{15a^4x^3 - 30a^7x^5 - 45a^6x^0}{20a^4x^5} = \frac{15a^4x^3}{20a^4x^5} + \frac{-30a^7x^5}{20a^4x^5} + \frac{-45a^6x^0}{20a^4x^5} \\ = \underline{\frac{3}{4}x^{-2} - \frac{3}{2}a^3 - \frac{9}{4}a^2x^{-5}}$$

$$13 \quad \frac{-60x^4a^5 - 75x^3a^6 + 80x^6a^4}{-20x^3a^4} = \frac{-60x^4a^5}{-20x^3a^4} + \frac{-75x^3a^6}{-20x^3a^4} + \frac{80x^6a^4}{-20x^3a^4} \\ = \underline{3xa + \frac{15}{4}a^2 - 4x^3}$$

$$14 \quad \frac{25m^3n^2p - 35m^2n^3p - 40mnp^3}{5mnp} = \frac{25m^3n^2p}{5mnp} + \frac{-35m^2n^3p}{5mnp} \\ + \frac{-40mnp^3}{5mnp} = \underline{5m^2n - 7mn^2 - 8p^2}$$

$$15 \quad \frac{-a^2x^3y^2 + a^4x^2y - ax^3y^3 + a^5xy}{-axy} = \frac{-a^2x^3y^2}{-axy} + \frac{a^4x^2y}{-axy} + \frac{ax^3y^3}{-axy} \\ + \frac{a^5xy}{-axy} = \underline{ax^2y - a^3x + x^2y^3 - a^4}$$

Exercise 38

- 1
$$\begin{array}{r} r-3 \overline{) x^2 - 5x + 6} \left(\underline{x-2} \text{ (Quotient)} \right. \\ \underline{x^2 - 3x} \\ -2x + 6 \\ \underline{-2r + 6} \end{array}$$
- 2
$$\begin{array}{r} 2x-1 \overline{) 2x^2 - 11x + 5} \left(\underline{x-5} \text{ (Quotient)} \right. \\ \underline{2x^2 - 2x} \\ -10x + 5 \\ \underline{-10x + 5} \end{array}$$
- 3
$$\begin{array}{r} 2x-3 \overline{) 6x^2 - x - 12} \left(\underline{3x+4} \text{ (Quotient)} \right. \\ \underline{6x^2 - 9x} \\ 8x - 12 \\ \underline{8x - 12} \end{array}$$
- 4
$$\begin{array}{r} 3x+4 \overline{) 15x^2 - x - 28} \left(\underline{5x-7} \text{ (Quotient)} \right. \\ \underline{15x^2 + 20x} \\ -21x - 28 \\ \underline{-21x - 28} \end{array}$$
- 5
$$\begin{array}{r} a-2b \overline{) 2a^2 - 7ab + 6b^2} \left(\underline{2a-3b} \text{ (Quotient)} \right. \\ \underline{2a^2 - 4ab} \\ -3ab + 6b^2 \\ \underline{-3ab + 6b^2} \end{array}$$
- 6
$$\begin{array}{r} x^2+xy+y^2 \overline{) x^4+x^3y^2+y^4} \left(\underline{x^2-xy+y^2} \text{ (Quotient)} \right. \\ \underline{x^4+x^3y+x^2y^2} \\ -x^2y+y^4 \\ \underline{-x^2y-x^2y^2-xy^3} \\ x^2y^2+xy^3+y^4 \\ \underline{x^2y^2+xy^3+y^4} \end{array}$$
- 7
$$\begin{array}{r} 2x+3a \overline{) 4x^2-9a^2} \left(\underline{2x-3a} \text{ (Quotient)} \right. \\ \underline{4x^2+6ax} \\ -6ax-9a^2 \\ \underline{-6ax-9a^2} \end{array}$$
- 8
$$\begin{array}{r} x+a \overline{) x^3+a^3} \left(\underline{x^2-ax+a^2} \text{ (Quotient)} \right. \\ \underline{x^3+ax^2} \\ -ax^2+a^3 \\ \underline{-ax^2-a^2x} \\ a^2x+a^3 \\ \underline{a^2x+a^3} \end{array}$$

- 9 $a-3b \left) \begin{array}{r} a^3 - a^2b - 7ab^2 + 3b^3 \\ a^3 - 3a^2b \\ \hline 2a^2b - 7ab^2 + 3b^3 \\ 2a^2b - 6ab^2 \\ \hline -ab^2 + 3b^3 \\ -ab^2 + 3b^3 \\ \hline \end{array} \right. \left(\frac{a^2 + 2ab - b^2}{a^2 + 2ab - b^2} \right) \text{ (Quotient)}$
- 10 $2m-3n \left) \begin{array}{r} 2m^3 - 9m^2n + 13mn^2 - 6n^3 \\ 2m^3 - 3m^2n \\ \hline -6m^2n + 13mn^2 - 6n^3 \\ -6m^2n + 9mn^2 \\ \hline 4mn^2 - 6n^3 \\ 4mn^2 - 6n^3 \\ \hline \end{array} \right. \left(\frac{m^2 - 3mn + 2n^2}{m^2 - 3mn + 2n^2} \right) \text{ (Quotient)}$
- 11 $a^2-b^3 \left) \begin{array}{r} a^4 - 3a^3b + 3ab^3 - b^4 \\ a^4 - a^2b^3 \\ \hline -3a^3b + a^2b^3 + 3ab^3 - b^4 \\ -3a^3b + 3ab^3 \\ \hline a^2b^3 - b^4 \\ a^2b^3 - b^4 \\ \hline \end{array} \right. \left(\frac{a^2 - 3ab + b^2}{a^2 - 3ab + b^2} \right) \text{ (Quotient)}$
- 12 $x^2+y^2 \left) \begin{array}{r} 2x^4 - 3x^3y + 3xy^3 - 2y^4 \\ 2x^4 + 2x^2y^2 \\ \hline -3x^3y - 2x^2y^2 - 3xy^3 - 2y^4 \\ -3x^3y - 3xy^3 \\ \hline -2x^2y^2 - 2y^4 \\ -2x^2y^2 - 2y^4 \\ \hline \end{array} \right. \left(\frac{2x^2 - 3xy - 2y^2}{2x^2 - 3xy - 2y^2} \right) \text{ (Quotient)}$
- 13 $2a^2+8ax \left) \begin{array}{r} 2a^4 - 36a^3x + 16ax^3 \\ 2a^4 + 8a^3x \\ \hline -8a^3x - 36a^2x^2 - 16ax^3 \\ -8a^3x - 32a^2x^2 \\ \hline -4a^2x^2 - 16ax^3 \\ -4a^2x^2 - 16ax^3 \\ \hline \end{array} \right. \left(\frac{a^2 - 4ax - 2x^2}{a^2 - 4ax - 2x^2} \right) \text{ (Quotient)}$
- 14 $1+2x^2 \left) \begin{array}{r} 3+2x+4x^2+5x^3-4x^4+2x^5 \\ 3 + 6x^2 \\ \hline 2x - 2x^3 + 5x^3 - 4x^4 + 2x^5 \\ 2x + 4x^3 \\ \hline -2x^2 + x^3 - 4x^4 + 2x^5 \\ -2x^2 - 4x^4 \\ \hline x^3 + 2x^5 \\ x^3 + 2x^5 \\ \hline \end{array} \right. \left(\frac{3+2x-2x^2+x^2}{3+2x-2x^2+x^2} \right) \text{ (Quotient)}$

$$\begin{array}{r}
 15 \quad x^2 + 2x - 3 \bigg) \frac{x^4 - 4x^2 + 12x - 9}{x^4 + 2x^3 - 3x^2} \left(x^2 - 2x + 3 \text{ (Quotient)} \right) \\
 \underline{-2x^3 - x^2 + 12x - 9} \\
 3x^2 + 6x - 9 \\
 \underline{3x^2 + 6x - 9} \\
 0
 \end{array}$$

$$\begin{array}{r}
 16 \quad 2a^2 - 3ab + 4b^2 \bigg) \frac{4a^4 - 9a^2b^2 + 24ab^3 - 16b^4}{4a^4 - 6a^2b + 8a^2b^2} \left(2a^2 + 3ab - 4b^2 \text{ (Quotient)} \right) \\
 \underline{6a^2b - 17a^2b^2 + 24ab^3 - 16b^4} \\
 6a^2b - 9a^2b^2 + 12ab^3 \\
 \underline{-8a^2b^2 + 12ab^3 - 16b^4} \\
 8a^2b^2 + 12ab^3 - 16b^4 \\
 \underline{8a^2b^2 + 12ab^3 - 16b^4} \\
 0
 \end{array}$$

$$\begin{array}{r}
 17 \quad a^2 + 2ax + 4x^2 \bigg) \frac{x^4 + 2a^2x + 4a^2x^2}{x^4 + 2a^2x + 4a^2x^2} + 16x^4 \left(a^2 - 2ax + 4x^2 \text{ (Quotient)} \right) \\
 \underline{-2a^2x} \qquad \qquad \qquad + 16x^4 \\
 -2a^2x - 4a^2x^2 - 8ax^2 \\
 \underline{4a^2x^2 + 8ax^2 + 16x^4} \\
 4a^2x^2 + 8ax^2 + 16x^4 \\
 \underline{4a^2x^2 + 8ax^2 + 16x^4} \\
 0
 \end{array}$$

$$\begin{array}{r}
 18 \quad a^2 + 2ab + 2b^2 \bigg) \frac{a^4 + 2a^2b + 2a^2b^2}{a^4 + 2a^2b + 2a^2b^2} + 4b^4 \left(a^2 - 2ab + 2b^2 \text{ (Quotient)} \right) \\
 \underline{-2a^2b - 2a^2b^2} \qquad \qquad \qquad + 4b^4 \\
 -2a^2b - 4a^2b^2 - 4ab^3 \\
 \underline{2a^2b^2 + 4ab^3 + 4b^4} \\
 2a^2b^2 + 4ab^3 + 4b^4 \\
 \underline{2a^2b^2 + 4ab^3 + 4b^4} \\
 0
 \end{array}$$

$$\begin{array}{r}
 19 \quad x^2 - 2x^2 + 1 \bigg) \frac{2x^6 - 7x^4 - 2x^3 + 18x^2 - 3x - 8}{2x^6 - 4x^4 + 2x^2} \left(2x^2 - 3x - 8 \text{ (Quotient)} \right) \\
 \underline{-3x^4 - 2x^3 + 16x^2 - 3x - 8} \\
 -3x^4 + 6x^3 - 3x \\
 \underline{-8x^3 + 16x^2 - 8} \\
 -8x^3 + 16x^2 - 8 \\
 \underline{-8x^3 + 16x^2 - 8} \\
 0
 \end{array}$$

$$\begin{array}{r}
 20 \quad x - 3 \bigg) \frac{x^4 - 81}{x^4 - 3x^3} \left(x^3 + 3x^2 + 9x + 27 \text{ (Quotient)} \right) \\
 \underline{3x^3} \qquad \qquad \qquad - 81 \\
 3x^3 - 9x^2 \\
 \underline{9x^2} \qquad \qquad \qquad - 81 \\
 9x^2 - 27x \\
 \underline{27x} \qquad \qquad \qquad - 81 \\
 27x - 81 \\
 \underline{27x - 81} \\
 0
 \end{array}$$

$$\begin{array}{r}
 21. \quad a-2 \overline{) a^5 - 2a^4} \qquad -32 \left(a^4 + 2a^3 + 4a^2 + 8a + 16 \right. \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \left. \text{(Quotient)} \right) \\
 \underline{2a^4} \qquad \qquad \qquad -32 \\
 2a^4 - 4a^3 \qquad \qquad \qquad \\
 \underline{4a^3} \qquad \qquad \qquad -32 \\
 4a^3 - 8a^2 \qquad \qquad \qquad \\
 \underline{8a^2} \qquad \qquad \qquad -32 \\
 8a^2 - 16a \qquad \qquad \qquad \\
 \underline{16a} \qquad \qquad \qquad -32 \\
 16a - 32 \qquad \qquad \qquad \\
 \underline{16a - 32}
 \end{array}$$

$$\begin{array}{r}
 22 \quad 1-3x+x^3 \overline{) 3-9x+2x^3+5x^5-7x^4+2x^6} \left(3-x^2+2x^5 \text{ (Quotient)} \right) \\
 \underline{3-9x+3x^3} \qquad \qquad \qquad \\
 -x^2+5x^5-7x^4+2x^6 \\
 \underline{-x^3+3x^3-x^4} \qquad \qquad \qquad \\
 2x^3-6x^4+2x^6 \\
 \underline{2x^3-6x^4+2x^6}
 \end{array}$$

$$\begin{array}{r}
 23 \quad 6x^3-7x+8 \overline{) 18x^4-45x^3+82x^2-67x+40} \left(3x^2-4x+5 \text{ (Quotient)} \right) \\
 \underline{18x^4-21x^3+24x^2} \qquad \qquad \qquad \\
 -24x^3+58x^2-67x+40 \\
 \underline{-24x^3+28x^2-32x} \qquad \qquad \qquad \\
 30x^2-35x+40 \\
 \underline{30x^2-35x+40}
 \end{array}$$

$$\begin{array}{r}
 24 \quad 2-x \overline{) 64} \qquad \qquad \qquad -x^0 \left(32+16x+8x^2+4x^3 \right. \\
 \underline{64-32x} \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \left. +2x^4+x^5 \text{ (Quotient)} \right) \\
 32x \qquad \qquad \qquad -x^1 \\
 \underline{32x-16x^2} \qquad \qquad \qquad \\
 16x^2 \qquad \qquad \qquad -x^2 \\
 \underline{16x^2-8x^3} \qquad \qquad \qquad \\
 8x^3 \qquad \qquad \qquad -x^3 \\
 \underline{8x^3-4x^4} \qquad \qquad \qquad \\
 4x^4 \qquad \qquad \qquad -x^4 \\
 \underline{4x^4-2x^5} \qquad \qquad \qquad \\
 2x^5 \qquad \qquad \qquad -x^5 \\
 \underline{2x^5-x^6}
 \end{array}$$

$$\begin{array}{r}
 25 \quad x^6 - 2x^5 + x^4 - 2x^3 + 1 \Big) x^6 - 2x^5 + x^4 - 2x^3 + 1 \quad \text{(Quotient)} \\
 \underline{2x^5 - x^4 - 2x^3 + 1} \\
 2x^5 - 4x^4 + 2x^3 \\
 \underline{3x^4 - 4x^3 + 1} \\
 3x^4 - 6x^3 + 3x^2 \\
 \underline{2x^3 - 3x^2 + 1} \\
 2x^3 - 4x^2 + 2x \\
 \underline{x - 2x + 1} \\
 x^2 - 2x + 1
 \end{array}$$

$$\begin{array}{r}
 26 \quad 3a^3 + 4ab + b^3 \Big) 6a^4 - a^3b + 2a^2b^2 + 13ab^3 + 4b^4 \quad \text{(Quotient)} \\
 \underline{-9a^3b + 13ab^3 + 4b^4} \\
 -9a^3b - 12a^2b^2 - 3ab^3 \\
 \underline{12a^2b^2 + 16ab^3 + 4b^4} \\
 12a^2b^2 + 16ab^3 + 4b^4
 \end{array}$$

$$\begin{array}{r}
 27 \quad a^3 - 2ab + 3b^3 \Big) a^4 + a^3b - 8a^2b^2 + 19ab^3 - 15b^4 \quad \text{(Quotient)} \\
 \underline{3a^3b - 11a^2b^2 + 19ab^3 - 15b^4} \\
 3a^3b - 6a^2b^2 + 9ab^3 \\
 \underline{-5a^2b^2 + 10ab^3 - 15b^4} \\
 -5a^2b^2 + 10ab^3 - 15b^4
 \end{array}$$

$$\begin{array}{r}
 28 \quad x^5 - 2x^4a + 2x^3a^2 - a^5 \Big) x^6 - 2x^5a + 2x^4a^2 - x^3a^3 \quad \text{(Quotient)} \\
 \underline{2x^5a - 2x^4a^2 + x^3a^3 - a^5} \\
 2x^5a - 4x^4a^2 + 4x^3a^3 - 2x^2a^4 \\
 \underline{2x^4a^2 - 3x^3a^3 + 2x^2a^4 - a^5} \\
 2x^4a^2 - 4x^3a^3 + 4x^2a^4 - 2xa^5 \\
 \underline{x^3a^3 - 2x^2a^4 + 2xa^5 - a^5} \\
 x^3a^3 - 2x^2a^4 + 2xa^5 - a^5
 \end{array}$$

$$\begin{array}{r}
 29 \quad a^3 + 2ab - 3b^3 \Big) a^6 - a^4b - 9a^3b^2 + 8a^2b^3 - 2ab^4 + 3b^5 \quad \text{(Quotient)} \\
 \underline{-3a^4b - 6a^3b^2 + 8a^2b^3 - 2ab^4 + 3b^5} \\
 -3a^4b - 6a^3b^2 + 9a^2b^3 \\
 \underline{-a^2b^3 - 2ab^4 + 3b^5} \\
 -a^2b^3 - 2ab^4 + 3b^5
 \end{array}$$

$$xy'(x^2 - 2xy + 3x^2y^2 - 2xy^3 + y^4) \text{ (Quotient)}$$

$$\begin{array}{r} 32 \quad \frac{x^6 + 2x^3y^3 + 2x^2y^4}{x^6 + 2x^3y^3 + x^2y^4} \\ \quad \frac{-2x^3y^3 - 2x^2y^4}{-2x^3y^3 - 2x^2y^4} \\ \quad \frac{3x^4y^3 + 4x^3y^4}{3x^4y^3 + 6x^2y^4 + 3x^2y^4} \\ \quad \frac{-2x^3y^3 - 3x^2y^4}{-2x^3y^3 - 4x^2y^4 - 2x^2y^4} \\ \quad \frac{x^2y^4 + 2x^2y^4 + y^6}{x^2y^4 + 2x^2y^4 + y^6} \end{array}$$

$$33 \quad \frac{x^7 + (a+b+c)x^5 + (ab+ac+bc)x + abc}{x^7 + (a+b+c)x^5 + (ab+ac+bc)x + abc} \text{ (Quotient)}$$

$$34 \quad \frac{x^3 + (b-a)x^2 + (b-c-a)x^2 + (ca-ab-bc)x + abc}{x^3 + (b-a)x^2 - abx} \text{ (Quotient)}$$

$$35 \quad \frac{a^3 - bc}{a^3} \frac{a^3 + a^2b + a^2c - abc - b^3c - bc^2}{a^3} (a+b+c) \text{ (Quotient)}$$

$$36 \quad a-(b-c) \left(\frac{a^2(b+c) - a(b^2 - bc - c^2) - bc(b-c)}{a^2(b+c) - a(b^2 - c^2)} \right) \frac{abc}{abc} \quad \frac{-bc(b-c)}{-bc(b-c)} \\ a(b+c) + bc = ab + ac + bc \text{ (Quotient)}$$

$$37 \quad a+(b+c) \left(\frac{a^2(b+c) + a(b^2 + c^2 + bc) - bc(b+c)}{a^2(b+c) + a(b^2 + c^2 + 2bc)} \right) \frac{-abc - bac(b+c)}{-abc - bac(b+c)} \\ a(b+c) - bc = ab + ac - bc \text{ (Quotient)}$$

$$38 \quad r-(a+b) \left(\frac{r^3 - 2ar^2}{r^3 - (a+b)r^2} + \frac{(a^2 - ab - b^2)r + ab(a+b)}{(a-b)x^2 + (a' - ab - b^2)x + ab(a+b)} \right) \frac{-abx + ab(a+b)}{-abx + ab(a+b)} \\ r^3 - (a-b)r = r^3 - (a-b)r - ab \text{ (Quotient)}$$

$$39 \quad a+b+c \left(\frac{a^3}{a^3 + a^2b + a^2c} - \frac{a^2b - a^2c}{a^2b - ab^2} - \frac{a^2c + ab^2 - 2abc}{a^2c - ab^2 - ac^2} + \frac{b^3 + c^3}{b^3 + c^3} \right) \\ \frac{ab^3 - abd + ac^3 + b^3}{ab^3} + \frac{b^3 + c^3}{b^3 + c^3} \\ - \frac{a^2b - a^2c}{a^2b - ab^2} - \frac{a^2c + ab^2 - 2abc}{a^2c - ab^2 - ac^2} + \frac{b^3 + c^3}{b^3 + c^3} \\ - \frac{abd + ac^3}{abd} - \frac{b^3c - bc^2}{b^3c - bc^2} + \frac{bc^3 + c^3}{bc^3 + c^3}$$

Or, Otherwise —

$$\begin{aligned}\text{The Dividend} &= (x)^3 + (-2y)^3 + (-3z)^3 - 3(x)(-2y)(-3z) \\ &= (x - 2y - 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 3xz - 6yz),\end{aligned}$$

the Divisor $= (x - 2y - 3z)$, and the Quotient

$$= x^2 + 4y^2 + 9z^2 + 2xy + 3xz - 6yz$$

$$\begin{array}{r} 42 \quad x - y + z \overline{) x^3 - x^2y + x^2z} \qquad + 3xy^2 - y^3 + z^3 \qquad - y^3 + z^3 \text{ (Quotient)} \\ \underline{x^3y - x^2z} \qquad - xy^2 \qquad + xy^2 - xz^2 \qquad - y^3 + z^3 \\ \underline{-x^2z + xy^2} \qquad - x^2z \qquad + xy^2 - xz^2 \qquad - y^3 + z^3 \\ \underline{-x^2z} \qquad \qquad \qquad xy^2 + xz^2 - y^3 \qquad - y^3 + yz^2 \qquad + z^3 \\ \underline{xy^2 + xz^2 - y^2z} \qquad + z^3 \qquad \qquad \qquad xy^2 + yz^2 \qquad + z^3 \\ \underline{yz^2} \qquad \qquad \qquad - y^2z + z^3 \qquad \qquad \qquad xz^2 \qquad \qquad \qquad - y^2z + z^3 \\ \underline{xz^2} \qquad \qquad \qquad - y^2z + z^3 \end{array}$$

Or, Otherwise —

$$\begin{aligned}\text{The Dividend} &= (x)^3 + (-y)^3 + (z)^3 - 3(x)(-y)(z) \\ &= (x - y + z)(x^2 + y^2 + z^2 + xy - xz + yz),\end{aligned}$$

the Divisor $= x - y + z$, and the Quotient —

$$= x^2 + y^2 + z^2 + xy - xz + yz$$

$$\begin{array}{r}
 43 \quad 4x^2 + 6xy + 2xz + 9y^2 - 3yz + z^2 \quad 81x^3 \\
 \hline
 81x^3 + 121xy + 4x^2z + 18xy^2 - 6xz + 21z^2 - 27y^3 - z^3 \quad (2x - 3y - z \text{ (Quo-)} \\
 \hline
 -121xy - 1x^2z - 181y^2 - 121yz - 21xz - 27y^3 - z^3 \\
 \hline
 -121xy - 181y^2 - 61yz - 27y^3 + 9y^2z - 3yz^2 \\
 \hline
 -4x^2z \\
 \hline
 -4x^2z \\
 \hline
 \end{array}$$

Or Otherwise —

The Dividend = $(2x)^3 + (-3y)^3 + (-z)^3 - 3(2x)(-3y)(-z)$
 $= (2x - 3y - z)(4x^2 + 9y^2 + z^2 + 6xy + 2xz - 3yz)$,
the Divisor = $4x^2 + 6xy + 2xz + 9y^2 - 3yz + z^2$, and the Quotient
 $= 2x - 3y - z$

$$\begin{array}{r}
 44 \quad a - b \quad a^2(b - c) - a(b^2 - c^2) + bc(b - c) \quad (a(b - c) - (bc - c^2) = ab - ac - bc + c^2 \text{ (Quotient)}) \\
 \hline
 a^2(b - c) - a(b^2 - c^2) - a(b^2 - bc) \\
 \hline
 -a(bc - c^2) + bc(b - c) \\
 \hline
 -a(bc - c^2) + bc(b - c)
 \end{array}$$

$$\begin{array}{r}
 45 \quad x^2 + x(a - b) - ab \quad x^3 + x^2(a - b + c) - x(ab - ac + bc) - abc \quad (x + c \text{ (Quotient)}) \\
 \hline
 x^3 + x^2(a - b) - xab \\
 \hline
 x^2c + xc(a - b) - abc \\
 \hline
 x^2c + xc(a - b) - abc
 \end{array}$$

$$\begin{array}{r}
 46 \quad x^2 - x(b + c) + bc \quad x^3 + x^2(a - b - c) - x(ab + ac - bc) + abc \quad (x + a \text{ (Quotient)}) \\
 \hline
 x^3 + x^2(-b - c) - x(-bc) \\
 \hline
 x^2a - xa(b + c) + abc \\
 \hline
 x^2a - xa(b + c) + abc
 \end{array}$$

$$47. \frac{a(b-c)-b(b-c)}{a^2(b-c)-a^2b(b-c)} \left(\frac{a^2(b-c)-a(b^3-c^3)}{a^2b(b-c)-a^2b^2(b-c)} + bc(b^2-c^2) \right) (a^3+ab-c(b+c)=a^3+ab-bc-c^3) \text{ (Quotient)}$$

$$48. \frac{a'(b+c)-ac^3-bc^3}{a^2(b^3-c^3)-a^2bc^2(b-c)} \left(\frac{a^2(b^3-c^3)-a'(b^3-c^3)+b^2c^2(b-c)}{a^2(b^3-c^3)-a^2bc^2(b-c)} + \frac{bc(b^3-c^3)}{bc(b^2-c^2)} \right) (a(b-c)-b(b-c) \text{ (Quotient)})$$

$$49. \frac{y-(1-z)}{y^2(1+2z)} \left(\frac{y^2(1+2z)-y^2(1-z)-y^2(1-z^2)+1^2z-1z^2}{y^2(1^3-2z^3)-y^2(1^2+1z-2z^2)} + \frac{y^2(1+2z)+y^2(1-z^2)-1^2z^2-1z^2}{y^2(1^3-2z^3)-y^2(1^2+1z-2z^2)} \right) (y^2x+2y^2z \text{ (Quotient)})$$

$$50. \frac{r(a+b)+a(a+b)}{r^2(a+b)+a^2(a+b)} \left(\frac{r^2(a+b)+a^2(a+b)}{r^2(a+b)+a^2(a+b)} + \frac{a^2(a+b)}{a^2(a+b)} \right) (r^2-ar+a^2 \text{ (Quotient)})$$

$$51. \frac{c^2(a-b)-cb^3+ab^3}{c^2(a-b)-c^2b^3+ab^3} \left(\frac{c^2(a-b)+c^2(a^2-2ab)+cb^3+ab^3}{c^2(a-b)-c^2b^3+ab^3} + \frac{c^2(a-b)+c^2(a-b)+ab^3(a-b)}{c^2(a-b)-c^2b^3+ab^3} \right) (c+(a-b)=c+a-b \text{ (Quotient)})$$

- 52 $x^2(a^2 - ab + b^2) + 3aby^4 \bigg/ \frac{2x^2(a^2 + b^2) + 6abxy^4(a+b)}{2x^2(a^2 + b^2) + 6abxy^4(a+b)} = 2(a+b)x \text{ (Quotient)}$
- 53 $x(y+z) + (yz+1) \bigg/ \frac{x^2(y+z)(yz+1) + x\{(y+z)^2 + (yz+1)^2\} + \frac{x(y+z)^2 + (y+z)(yz+1)}{x(y+z)^2 + (y+z)(yz+1)}}{x^2(y+z)(yz+1) + x\{(y+z)^2 + (yz+1)^2\} + \frac{x(y+z)^2 + (y+z)(yz+1)}{x(y+z)^2 + (y+z)(yz+1)}} = x(y+z) + yz \text{ (Quotient)}$
- 54 $x^2 + y^2 - a^2 \bigg/ \frac{16x^{10} - 24x^8y^2 + 9a^2x^6 + 16y^6 - 24y^4a^2 + 9a^2y^2 - a^6}{16x^{10} - 16x^8y^2 + 16x^6y^4 - 8x^4y^6 + 16y^8 - 8x^2y^4a^2 + 9a^2x^2 + 16y^6 - 24y^4a^2 + 9a^2y^2 - a^6} = \frac{x^2(16y^4 - 8y^2a^2 + a^2) + 16y^6 - 24y^4a^2 + 9a^2y^2 - a^6}{x^2(16y^4 - 8y^2a^2 + a^2) + 16y^6 - 24y^4a^2 + 9a^2y^2 - a^6} \text{ (Quotient)}$
- 55 and 56 Worked out in the book
- 57 $x^5 - x^3 = x^3 - x^3 = 0$
- 58 $x^2 - x - 1 = x^2 - x - 1 = x^2 - x - 1 = 0$
- 59 $a^2b^3 - a^{-1}b^{-1} = a^2 + 1b^3 + 1 = a^2b$
60. $a^{-1}b^{\frac{1}{2}}c^{\frac{5}{2}} + a^{-1}b^{\frac{1}{2}}c^{\frac{3}{2}} = a^{-2+3}b^{\frac{1}{2}+\frac{1}{2}}c^{\frac{5}{2}+\frac{3}{2}} = ab^{-1}c^4$
- 61 $15xy - (-5x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}) = -3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} = -3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}$

$$\begin{array}{r}
 62 \quad 3x^{\frac{4}{3}} + 4y^{\frac{1}{3}} \Big) 9x^{\frac{4}{3}} - 16y^{\frac{2}{3}} \left(3x^{\frac{4}{3}} - 4y^{\frac{1}{3}} \right. \text{ (Quotient)} \\
 \underline{9x^{\frac{4}{3}} + 12x^{\frac{1}{3}}y^{\frac{1}{3}}} \\
 -12x^{\frac{1}{3}}y^{\frac{1}{3}} - 16y^{\frac{2}{3}} \\
 \underline{-12x^{\frac{2}{3}}y^{\frac{1}{3}} - 16y^{\frac{1}{3}}}
 \end{array}$$

$$\begin{array}{r}
 63 \quad a^{\frac{1}{3}} + b^{\frac{1}{3}} \Big) a + a^{\frac{2}{3}}b^{\frac{1}{3}} + b \left(a^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right. \text{ (Quotient)} \\
 \underline{-a^{\frac{1}{3}}b^{\frac{1}{3}}} \quad +b \\
 -a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} \\
 \underline{\phantom{-a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}}}} \\
 a^{\frac{1}{3}}b^{\frac{2}{3}} + b \\
 \underline{a^{\frac{1}{3}}b^{\frac{2}{3}} + b}
 \end{array}$$

$$\begin{array}{r}
 64 \quad a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{4}} \Big) a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{4}} \left(a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{4}} \right. \text{ (Quotient)} \\
 \underline{-a^{\frac{1}{4}}b^{\frac{1}{4}}} \quad +b^{\frac{1}{2}} \\
 -a^{\frac{1}{4}}b^{\frac{1}{4}} - a^{\frac{1}{4}}b^{\frac{1}{4}} - a^{\frac{1}{4}}b^{\frac{1}{4}} \\
 \underline{\phantom{-a^{\frac{1}{4}}b^{\frac{1}{4}} - a^{\frac{1}{4}}b^{\frac{1}{4}} - a^{\frac{1}{4}}b^{\frac{1}{4}}}} \\
 a^{\frac{1}{4}}b^{\frac{1}{4}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}} \\
 \underline{a^{\frac{1}{4}}b^{\frac{1}{4}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}}
 \end{array}$$

$$\begin{array}{r}
 65 \quad 2x^{\frac{4}{3}} + 5x^{\frac{1}{3}}y^{\frac{2}{3}} - 3y^{\frac{4}{3}} \Big) 4x^{\frac{8}{3}} - 37x^{\frac{4}{3}}y^{\frac{4}{3}} + 9y^{\frac{8}{3}} \left(2x^{\frac{4}{3}} - 5x^{\frac{2}{3}}y^{\frac{2}{3}} - 3y^{\frac{4}{3}} \right. \text{ (Quotient)} \\
 \underline{4x^{\frac{8}{3}} + 10x^{\frac{2}{3}}y^{\frac{2}{3}} - 6x^{\frac{1}{3}}y^{\frac{4}{3}}} \\
 -10x^{\frac{2}{3}}y^{\frac{2}{3}} - 31x^{\frac{1}{3}}y^{\frac{4}{3}} + 9y^{\frac{8}{3}} \\
 \underline{-10x^{\frac{2}{3}}y^{\frac{2}{3}} - 25x^{\frac{1}{3}}y^{\frac{4}{3}} + 15x^{\frac{2}{3}}y^{\frac{2}{3}}} \\
 -6x^{\frac{1}{3}}y^{\frac{4}{3}} - 15x^{\frac{2}{3}}y^{\frac{2}{3}} + 9y^{\frac{8}{3}} \\
 \underline{-6x^{\frac{1}{3}}y^{\frac{4}{3}} - 15x^{\frac{2}{3}}y^{\frac{2}{3}} + 9y^{\frac{8}{3}}}
 \end{array}$$

$$66 \quad (a^{\frac{1}{2}} - b^{\frac{1}{2}})^2 = a^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} + b^{\frac{1}{2}} = -b^{\frac{1}{2}}(a^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} + b^{\frac{1}{2}}) \quad (\text{Quotient})$$

$$\begin{array}{r} a - a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}}} \\ a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}}} \\ a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}}} \\ a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}}} \\ a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}} \end{array}$$

$$67 \quad (2a^{-\frac{5}{2}} + 3a^{-\frac{5}{2}}b^{-\frac{1}{2}} - 5b^{-\frac{1}{2}})(4a^{-10} + 12a^{-\frac{15}{2}}b^{-\frac{1}{2}} + 9a^{-5}b^{-\frac{3}{2}} - 25b^{-6}) = 2a^{-5} + 3a^{-\frac{5}{2}}b^{-\frac{1}{2}} + 5b^{-1} \quad (\text{Quotient})$$

$$\begin{array}{r} 4a^{-10} + 12a^{-\frac{15}{2}}b^{-\frac{1}{2}} + 9a^{-5}b^{-\frac{3}{2}} - 25b^{-6} \\ \underline{4a^{-10} + 6a^{-\frac{15}{2}}b^{-\frac{1}{2}} - 10a^5b^{-3}} \\ 6a^{-\frac{15}{2}}b^{-\frac{1}{2}} + 19a^{-5}b^{-\frac{3}{2}} - 25b^{-6} \\ \underline{6a^{-\frac{15}{2}}b^{-\frac{1}{2}} + 9a^{-5}b^{-\frac{3}{2}} - 15a^{-\frac{5}{2}}b^{-\frac{5}{2}}} \\ 10a^{-5}b^{-\frac{3}{2}} + 15a^{-\frac{5}{2}}b^{-\frac{5}{2}} - 25b^{-6} \\ \underline{10a^{-5}b^{-\frac{3}{2}} + 15a^{-\frac{5}{2}}b^{-\frac{5}{2}} - 25b^{-6}} \end{array}$$

$$70 \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}} \Big) x \qquad -3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + y + z \Big(x^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}} + z^{\frac{1}{2}} \Big) \quad (\text{Quotient})$$

$$\begin{array}{r} x + x^{\frac{1}{2}}y^{\frac{1}{2}} + x^{\frac{1}{2}}z^{\frac{1}{2}} \\ \hline -x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} - 3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + y + z \\ \hline -x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} \\ \hline -x^{\frac{1}{2}}z^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + y + z \\ -x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} \\ \hline x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + x^{\frac{1}{2}}z^{\frac{1}{2}} + y \\ x^{\frac{1}{2}}y^{\frac{1}{2}} \qquad \qquad \qquad + y + y^{\frac{1}{2}}z^{\frac{1}{2}} \qquad + z \\ \hline -x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + x^{\frac{1}{2}}z^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}} \qquad + z \\ -x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}} \qquad \qquad \qquad + y^{\frac{1}{2}}z^{\frac{1}{2}} + z \\ \hline x^{\frac{1}{2}}z^{\frac{1}{2}} \qquad \qquad \qquad + y^{\frac{1}{2}}z^{\frac{1}{2}} + z \end{array}$$

Or Otherwise —

$$\begin{aligned} \text{The Dividend} &= (x^{\frac{1}{2}})^3 + (y^{\frac{1}{2}})^3 + (z^{\frac{1}{2}})^3 - 3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} \\ &= (x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}})(x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}}), \end{aligned}$$

the Divisor $= x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}}$, and the quotient
 $= x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}}$

EXERCISE 39.

$$\begin{array}{r}
 1 \quad r+a \left) \begin{array}{l} x^3 \\ x^3+ax^2 \end{array} \right. \begin{array}{l} +a^3(x^2-ax+a^2) \text{ (Quotient)} \\ -ax^2 \\ -ar^2-a^2x \\ \hline a^2x+a^3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2 \quad x+a \left) \begin{array}{l} r^4 \\ r^4+ax^3 \end{array} \right. \begin{array}{l} -a^4(x^3-ar^2+a^2r-a^2r) \text{ (Quotient)} \\ -ax^3 \\ -ax^3-a^2x^2 \\ \hline a^2x^2-a^4 \\ a^2x^2+a^3x \\ \hline -a^3x-a^4 \\ -a^3x-a^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 3 \quad r+a \left) \begin{array}{l} r^5 \\ x^5+ar^4 \end{array} \right. \begin{array}{l} +a^5 \left(r^4-ax^3+a^2x^2-a^2x+a^4 \right) \text{ (Quotient)} \\ -ar^4+a^5 \\ -ar^4-a^2x^3 \\ \hline a^2x^3+a^5 \\ a^2x^3+a^3x^3 \\ \hline -a^3x^2+a^5 \\ -a^3x^2-a^4x \\ \hline a^4x+a^5 \\ a^4x+a^5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 4 \quad r+a \left) \begin{array}{l} r^6 \\ x^6+ax^5 \end{array} \right. \begin{array}{l} -a^6 \left(x^5-ar^4+a^2x^3-a^3x^2+a^4x-a^6 \right) \text{ (Quotient)} \\ -ar^5-a^6 \\ -ar^5-a^2x^4 \\ \hline a^2x^4-a^6 \\ a^2x^4+a^3x^3 \\ \hline -a^3x^3 \\ -a^3x^3-a^4x^2 \\ \hline a^4x^2-a^6 \\ a^4x^2+a^5x \\ \hline -a^5x-a^6 \\ -a^5x-a^6 \end{array}
 \end{array}$$

$$\begin{array}{r}
 5 \quad x+a \quad \left) \frac{x^7}{x^7+ax^4} + a^7 \left(\begin{array}{l} x^6 - ax^5 + a^2x^4 - a^3x^3 + a^4x^2 - a^5x + a^6 \\ \text{(Quotient)} \end{array} \right. \\
 \underline{-ax^4 + a^7} \\
 -ax^6 - a^2x^7 \\
 \underline{a^2x^5 + a^7} \\
 a^2x^5 + a^2x^4 \\
 \underline{-a^3x^4 + a^7} \\
 -a^3x^4 - a^4x \\
 \underline{a^4x^3 + a^7} \\
 a^4x^3 + a^5x^2 \\
 \underline{-a^5x^2 + a^7} \\
 -a^5x^2 - a^6x \\
 \underline{a^6x + a^7} \\
 a^6x + a^7
 \end{array}$$

$$\begin{array}{r}
 6 \quad 1+a \quad \left) \frac{x^8}{x^8+ax^7} - a^5 \left(\begin{array}{l} x^7 - ax^6 + a^2x^5 - a^3x^4 + a^4x^3 - a^5x^2 + \\ a^6x - a^7 \text{ (Quotient)} \end{array} \right. \\
 \underline{-ax^7 - a^5} \\
 -ax^7 - a^2x^6 \\
 \underline{a^2x^6 - a^5} \\
 a^2x^6 + a^2x^5 \\
 \underline{-a^3x^5 - a^5} \\
 -a^3x^5 - a^2x^4 \\
 \underline{a^4x^4 - a^5} \\
 a^4x^4 + a^5x^3 \\
 \underline{-a^5x^3 - a^5} \\
 -a^5x^3 - a^6x^2 \\
 \underline{a^6x^2 - a^5} \\
 a^6x^2 + a^7x \\
 \underline{-a^7x - a^5} \\
 -a^7x - a^5
 \end{array}$$

$$\begin{array}{r}
 7 \quad x+a \quad \left) \frac{x^3}{x^3+ax^2} - a^3 \left(\begin{array}{l} x^2 - ax + a^2 \text{ (Quotient)} \\ \underline{-ax^2 - a^3} \\
 -ax^2 - a^2x \\
 \underline{a^2x - a^3} \\
 a^2x + a^3 \\
 \underline{-2a^3}
 \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 8 \quad 1+a \left) \frac{x^4}{x^4+ax^3} + a^4 \left(\begin{array}{l} 1^4 - a1^3 + a^21^2 - a^31 + a^4 \end{array} \right. \text{(Quotient)} \\
 \underline{-ax^3+a^4} \\
 -ax^3-a^21^2 \\
 \underline{a^2x^2+a^4} \\
 a^2x^2+a^31 \\
 \underline{-a^31+a^4} \\
 -a^31-a^4 \\
 \underline{2a^4}
 \end{array}$$

$$\begin{array}{r}
 9 \quad 1+a \left) \frac{1^7}{1^5+ax^4} - a^6 \left(\begin{array}{l} 1^7 - a1^6 + a^21^5 - a^31^4 + a^41^3 - a^51^2 + a^61 \end{array} \right. \text{(Quotient)} \\
 \underline{-a1^6-a^6} \\
 -ax^4-a^2x^3 \\
 \underline{a^21^5-a^6} \\
 a^21^5+a^2x^2 \\
 \underline{-a^3x^2-a^6} \\
 -a^3x^2-a^41 \\
 \underline{a^4x-a^6} \\
 a^4x+a^5 \\
 \underline{-2a^6}
 \end{array}$$

$$\begin{array}{r}
 10 \quad 1+a \left) \frac{x^6}{x^6+ax^5} + a^6 \left(\begin{array}{l} 1^6 - a1^5 + a^21^4 - a^31^3 + a^41^2 - a^51 + a^6 \end{array} \right. \text{(Quotient)} \\
 \underline{-ax^5+a^6} \\
 -ax^5-a^2x^4 \\
 \underline{a^2x^4+a^6} \\
 a^2x^4+a^3x^3 \\
 \underline{-a^3x^3+a^6} \\
 -a^3x^3-a^4x^2 \\
 \underline{a^4x^2+a^6} \\
 a^4x^2+a^5x \\
 \underline{-a^5x+a^6} \\
 -a^5x-a^6 \\
 \underline{2a^6}
 \end{array}$$

$$\begin{array}{r}
 11 \quad 1+a) \frac{x^7}{x^7+ax^6} - a^7 \left(\begin{array}{l} x^6 - ax^5 + a^2x^4 - a^3x^3 + a^4x^2 - a^5x + a^6 \\ \text{(Quotient)} \end{array} \right) \\
 \underline{-ax^6 - a^7} \\
 -a^7 - a^2x^4 \\
 \underline{a^2x^4 - a^7} \\
 a^2x^4 + a^7x^4 \\
 \underline{-a^3x^4 - a^7} \\
 -a^3x^4 - a^4x^3 \\
 \underline{a^4x^3 - a^7} \\
 a^4x^3 + a^6x^2 \\
 \underline{-a^5x^2 - a^7} \\
 -a^5x^2 - a^6x \\
 \underline{a^6x - a^7} \\
 a^6x + a^7 \\
 \underline{-2a^7}
 \end{array}$$

$$\begin{array}{r}
 12 \quad 1+a) \frac{x^8}{x^8+ax^7} + a^8 \left(\begin{array}{l} x^7 - ax^6 + a^2x^5 - a^3x^4 + a^4x^3 - a^5x^2 \\ + a^6x - a^7 \text{ (Quotient)} \end{array} \right) \\
 \underline{-ax^7 + a^8} \\
 -a^8 - a^2x^5 \\
 \underline{a^2x^5 + a^8} \\
 a^2x^5 + a^3x^5 \\
 \underline{-a^3x^5 + a^8} \\
 -a^3x^5 - a^4x^4 \\
 \underline{a^4x^4 + a^8} \\
 a^4x^4 + a^6x^3 \\
 \underline{-a^5x^3 + a^8} \\
 -a^5x^3 - a^6x^2 \\
 \underline{a^6x^2 + a^8} \\
 a^6x^2 + a^7x \\
 \underline{-a^7x + a^8} \\
 -a^7x - a^8 \\
 \underline{2a^8}
 \end{array}$$

$$\begin{aligned}
 13 \quad x^4 - 1 &= x^3(x-1) + x^2(x-1) + x(x-1) + (x-1), \\
 \frac{x^4 - 1}{x - 1} &= \underline{x^3 + x^2 + x + 1}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad x^4 - y^4 &= x^3(x+y) - x^2y(x+y) + xy^2(x+y) - y^3(x+y) \\
 \frac{x^4 - y^4}{x+y} &= \underline{x^3 - x^2y + xy^2 - y^3}
 \end{aligned}$$

- 15 $x^4 - 1 = x^4(1-1) + x^3(1-1) + x^2(1-1) + x(1-1) + (1-1)$
 $\frac{x^4-1}{x-1} = \underline{x^3+x^2+x+1+1}$
- 16 $x^5 + y^5 = x^4(x+y) - x^3y(x+y) + x^2y^2(x+y) - xy^3(x+y) + y^4(x+y),$
 $\frac{x^5+y^5}{x+y} = \underline{x^4-x^3y+x^2y^2-xy^3+y^4}$
- 17 $x^5 - 1 = x^4(1-1) + x^3(1-1) + x^2(1-1) + x(1-1) + (1-1),$
 $\frac{x^5-1}{x-1} = \underline{x^4+x^3+x^2+x+1}$
- 18 $x^4 - y^4 = x^3(x+y) - x^2y(x+y) + x^2y^2(x+y) - xy^3(x+y) + y^4(x+y),$
 $\frac{x^4-y^4}{x-y} = \underline{x^3+xy^2+x^2y+y^3}$
- 19 $x^5 - 1 = x^4(1-1) + x^3(1-1) + x^2(1-1) + x(1-1) + (1-1)$
 $+ x(1-1) + (1+1),$
 $\frac{x^5-1}{x-1} = \underline{x^4+x^3+x^2+x+1}$
- 20 $x^5 + y^5 = x^4(x+y) - x^3y(x+y) + x^2y^2(x+y) - xy^3(x+y) + y^4(x+y),$
 $\frac{x^5+y^5}{x+y} = \underline{x^4-x^3y+x^2y^2-xy^3+y^4}$

Exercise 40

- 1 $ab+ac=a(b+c)$ 2 $a^2b^2+a^2b^2=a^2b^2(b+a)$
- 3 $x^3y^4-2x^4y^5=x^3y^4(y-2x)$
- 4 $2x^2yz+4xy^2z-6xyz^2=2xyz(x+2y-3z)$
- 5 $4a^5b-6a^4b^2-8a^3b^3=2a^3b(2a^2-3ab-4b^2)$
- 6 $ax^2y-5a^2x^3y^2+3ax^4=a^2x^2(y-5axy^2+3x^2)$
- 7 $3x^4y^3z^2-12x^3y^4z^3+21x^2y^5z^4=3x^2y^3z^2(x^2-4y^2z+7yz^2)$
- 8 $28a^8b^5-42a^6b^3=14a^6b^3(2a^2-3b^2)$
- 9 $72x^{10}y^8+108x^8y^{10}=36x^8y^8(2x+3y^2)$
- 10 $39a^2b^7c^7-65b^5c^7a^7-91c^6a^7b^7=13a^7b^7c^6(3b^2c^2-5c^2a^2-7a^2b^2).$

Exercise 41

- 1 $9a^2 - 16b^2 = (3a)^2 - (4b)^2 = (3a + 4b)(3a - 4b)$
- 2 $4a^3 - 25a^2 = a(4a^2 - 25a) = a\{(2a)^2 - (5a)^2\}$
 $= a(2a + 5a)(2a - 5a)$
- 3 $36x^4 - 1 = (6x^2)^2 - 1 = (6x^2 + 1)(6x^2 - 1)$
- 4 $16x^4 - 1 = (4x^2)^2 - 1 = (4x^2 + 1)(4x^2 - 1) = (4x^2 + 1)\{(2x)^2 - 1\}$
 $= (4x^2 + 1)(2x + 1)(2x - 1)$
- 5 $16x^5 - 9x = x(16x^4 - 9) = x\{(4x^2)^2 - (3)^2\} = x(4x^2 + 3)(4x^2 - 3)$
- 6 $16x^5 - 81x = x(16x^4 - 81) = x\{(4x^2)^2 - (9)^2\}$
 $= x(4x^2 + 9)(4x^2 - 9)$
 $= x(4x^2 + 9)\{(2x)^2 - (3)^2\}$
 $= x(4x^2 + 9)(2x + 3)(2x - 3)$
- 7 $1 - 16a^4 = 1 - (4a^2)^2 = (1 + 4a^2)(1 - 4a^2) = (1 + 4a^2)\{1 - (2a)^2\}$
 $= (1 + 4a^2)(1 + 2a)(1 - 2a)$
- 8 $x^3 - 81x^2 = x^2(x - 81x) = x^2\{1 - (9x^2)^2\}$
 $= x^2(1 + 9x^2)(1 - 9x^2)$
 $= x^2(1 + 9x^2)\{1 - (3x)^2\}$
 $= x^2(1 + 9x^2)(1 + 3x)(1 - 3x)$
- 9 $36 - x^4a^2 = (6)^2 - (x^2a)^2 = (6 + x^2a)(6 - x^2a)$
- 10 $64a^4 - 49a^6 = (8a^2)^2 - (7a^3)^2 = (8a^2 + 7a^3)(8a^2 - 7a^3)$
- 11 $121 - m^4 = (11)^2 - (m^2)^2 = (11 + m^2)(11 - m^2)$
- 12 $49x^4a^{10} - 81 = (7x^2a^5)^2 - (9)^2 = (7x^2a^5 + 9)(7x^2a^5 - 9)$
- 13 $a^2b^2 - 25c^2d^2 = (ab)^2 - (5cd)^2 = (ab + 5cd)(ab - 5cd)$
- 14 $81x^{12} - 64a^{10} = (9x^6)^2 - (8a^5)^2 = (9x^6 + 8a^5)(9x^6 - 8a^5)$
- 15 $p^3q^4 - 100p^2 = p^2\{(q^2)^2 - (10)^2\} = p^2(q^2 + 10)(q^2 - 10)$
- 16 $144x^7 - 25x^7a^4 = x^7\{(12x^2)^2 - (5a^2)^2\} = x^7(12x^2 + 5a^2)(12x^2 - 5a^2)$
- 17 $192a^6 - 243a^5x^4 = 3a^5(64a - 81x^4) = 3a^5\{(8a)^2 - (9x^2)^2\}$
 $= 3a^5(8a + 9x^2)(8a - 9x^2)$
- 18 $98a^7x^6 - 128a^7 = 2a^7(49a^2x^4 - 64) = 2a^7\{(7ax^2)^2 - (8)^2\}$
 $= 2a^7(7ax^2 + 8)(7ax^2 - 8)$

$$\begin{aligned}
 19 \quad 324x^{17}a^6 - 484x^{17}a^6 &= 4x^{16}a^6(511^{17}a^6 - 121) = 1x^{16}a^6\{(9x^6a^6)^2 - (11)^2\} \\
 &= 4x^{16}a^6(9x^6a^6 + 11) \\
 &\quad (9x^6a^6 - 11)
 \end{aligned}$$

$$\begin{aligned}
 20 \quad 245m^{17}n^{17} - 605m^{17}n^{17} &= 5m^{16}n^{17}(49m^3n^3 - 121) \\
 &= 5m^{16}n^{17}\{(7m^3n^3)^2 - (11)^2\} \\
 &= 5m^{16}n^{17}(7m^3n^3 + 11)(7m^3n^3 - 11)
 \end{aligned}$$

$$\begin{aligned}
 21 \quad (a+3b)^2 - 25c^2 &= (a+3b) - (5c)^2 \\
 &= (a+3b+5c)(a+3b-5c)
 \end{aligned}$$

$$\begin{aligned}
 22 \quad a^2 - (3b-5c)^2 &= \{a+(3b-5c)\}\{a-(3b-5c)\} \\
 &= (a+3b-5c)(a-3b+5c)
 \end{aligned}$$

$$\begin{aligned}
 23 \quad (1+j)^2 - (1-i)^2 &= \{(1+j)+(1-i)\}\{(1+j)-(1-i)\} \\
 &= 2(1+j) \cdot 2i
 \end{aligned}$$

$$\begin{aligned}
 24 \quad (3a+2i)^2 - (2a+1)^2 &= \{(3a+2i)+(2a+1)\}\{(3a+2i)-(2a+1)\} \\
 &= (5a+3i)(a+1)
 \end{aligned}$$

$$\begin{aligned}
 25 \quad 4(a-b)^2 - 9(c-d)^2 &= \{2(a-b)\}^2 - \{3(c-d)\}^2 \\
 &= \{2(a-b)+3(c-d)\}\{2(a-b)-3(c-d)\} \\
 &= (2a-2b+3c-3d)(2a-2b-3c+3d)
 \end{aligned}$$

$$\begin{aligned}
 26 \quad 49x^2 - (5y-3z)^2 &= (7x)^2 - (5y-3z)^2 \\
 &= (7x+5y-3z)(7x-5y+3z)
 \end{aligned}$$

$$\begin{aligned}
 27 \quad (8x+5)^2 - (2x-7)^2 &= \{(8x+5)+(2x-7)\}\{(8x+5)-(2x-7)\} \\
 &= (10x+2)(6x+12) \\
 &= 2(5x+1)6(x+2) = 12(5x+1)(x+2)
 \end{aligned}$$

$$\begin{aligned}
 28 \quad (a+b-c)^2 - (a-b+c)^2 \\
 &= \{(a+b-c)+(a-b+c)\}\{(a+b-c)-(a-b+c)\} \\
 &= 2a(2b-2c) = 4a(b-c)
 \end{aligned}$$

$$\begin{aligned}
 29 \quad (2a-3b+4c)^2 - (a+4b-5c)^2 \\
 &= \{(2a-3b+4c)+(a+4b-5c)\}\{(2a-3b+4c)-(a+4b-5c)\} \\
 &= (3a+b-c)(a-7b+9c)
 \end{aligned}$$

$$\begin{aligned}
 30 \quad 64(a+3x-4y)^2 - 9(2a-x+3y)^2 \\
 &= \{8(a+3x-4y)\}^2 - \{3(2a-x+3y)\}^2 \\
 &= \{8(a+3x-4y)+3(2a-x+3y)\}\{8(a+3x-4y)-3(2a-x+3y)\} \\
 &= (14a+21x-23y)(2a+27x-41y)
 \end{aligned}$$

- 31 $(4x^2 - 5a^2)^2 - (5x^2 - 4a^2)^2$
 $= \{(4x^2 - 5a^2) + (5x^2 - 4a^2)\} \{(4x^2 - 5a^2) - (5x^2 - 4a^2)\}$
 $= (9x^2 - 9a^2)(-x^2 - a^2) = -9(x^2 - a^2)(x^2 + a^2)$
 $= -9(x + a)(x - a)(x^2 + a^2)$
- 32 $(5a^2 - 3a + 7)^2 - (5a^2 - 3a - 7)^2$
 $= \{(5a^2 - 3a + 7) + (5a^2 - 3a - 7)\} \{(5a^2 - 3a + 7) - (5a^2 - 3a - 7)\}$
 $= (10a^2 - 6a)14 = 2a(5a - 3)14 = 28a(5a - 3)$

Exercise 42

- 1 $x^4 + x^2 + 1 = x^4 + x^2 + 1 - x^2$
 $= (x^2 + 1)^2 - x^2 = (x^2 + 1 + x)(x^2 + 1 - x)$
 $= (x^2 + x + 1)(x^2 - x + 1)$
- 2 $x^6 + x^4 + 1 = x^6 + 2x^4 + 1 - x^4$
 $= (x^4 + 1)^2 - (x^2)^2$
 $= (x^4 + 1 + x^2)(x^4 + 1 - x^2)$
 $= (x^4 - x^2 + 1)(x^2 + x + 1)(x^2 - x + 1)$
- 3 $a^4 + a^2x^2 - x^4 = a^4 + 2a^2x^2 + x^4 - a^2x^4$
 $= (a^2 + x^2)^2 - (ax)^2 = (a^2 + x^2 + ax)(a^2 + x^2 - ax)$
 $= (a^2 + ax + x^2)(a^2 - ax + x^2)$
- 4 $a^5 + a^4x + x^5 = a^5 + 2a^4x + x^5 - a^4x$
 $= (a^4 + x^4)^2 - (a^2x^2)^2$
 $= (a^4 + x^4 + a^2x^2)(a^4 + x^4 - a^2x^2)$
 $= (a^4 + ax + x^2)(a^4 - ax + x^2)(a^4 - a^2x^2 + x^4)$
- 5 $x^4 + 64 = x^4 + 16x^2 + 64 - 16x^2$
 $= (x^2 + 8)^2 - (4x)^2 = (x^2 + 8 + 4x)(x^2 + 8 - 4x)$
 $= (x^2 + 4x + 8)(x^2 - 4x + 8)$
- 6 $4x^4 + 81 = 4x^4 + 36x^2 + 81 - 36x^2$
 $= (2x^2 + 9)^2 - (6x)^2$
 $= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$
 $= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$
- 7 $9x^4 + 36 = 9(x^4 + 4) = 9(x^4 + 4x^2 + 4 - 4x^2)$
 $= 9\{(x^2 + 2)^2 - (2x)^2\}$
 $= 9(x^2 + 2x + 2)(x^2 - 2x + 2)$
- 8 $a^4 + 2a^2 + 9 = a^4 + 6a^2 + 9 - 4a^2$
 $= (a^2 + 3)^2 - (2a)^2 = (a^2 + 2a + 3)(a^2 - 2a + 3)$

- 9 $x^4 - 7x^2 + 9 = x^4 - 6x^2 + 9 - x^2$
 $= (x^2 - 3)^2 - (x)^2 = (x^2 + x - 3)(x^2 - x - 3)$
- 10 $4x^4 + 8x^2 + 9 = 4x^4 + 12x^2 + 9 - 4x^2$
 $= (2x^2 + 3)^2 - (2x)^2 = (2x^2 + 2x + 3)(2x^2 - 2x + 3)$
- 11 $4x^4 - 16x^2 + 9 = 4x^4 - 12x^2 + 9 - 4x^2$
 $= (2x^2 - 3)^2 - (2x)^2 = (2x^2 + 2x - 3)(2x^2 - 2x - 3)$
- 12 $4x^4 + 3x^2 + 9 = 4x^4 + 12x^2 + 9 - 9x^2 = (2x^2 + 3)^2 - (3x)^2$
 $= (2x^2 + 3x + 3)(2x^2 - 3x + 3)$
- 13 $4a^4 - 37a^2 + 9 = 4a^4 - 12a^2 + 9 - 25a^2$
 $= (2a^2 - 3)^2 - (5a)^2 = (2a^2 + 5a - 3)(2a^2 - 5a - 3)$
- 14 $4a^4 + 625 = 4a^4 + 100a^2 + 625 - 100a^2$
 $= (2a^2 + 25)^2 - (10a)^2$
 $= (2a^2 + 10a + 25)(2a^2 - 10a + 25)$
- 15 $9x^4 + 23x^2 + 16 = 9x^4 + 24x^2 + 16 - x^2 = (3x^2 + 4)^2 - x^2$
 $= (3x^2 + x + 4)(3x^2 - x + 4)$
- 16 $9a^4 - 25a^2 + 16 = 9a^4 - 24a^2 + 16 - a^2$
 $= (3a^2 - 4)^2 - a^2 = (3a^2 + a - 4)(3a^2 - a - 4)$
- 17 $9x^4 - 33x^2 + 16 = 9x^4 - 24x^2 + 16 - 9x^2$
 $= (3x^2 - 4)^2 - (3x)^2$
 $= (3x^2 + 3x - 4)(3x^2 - 3x - 4)$
- 18 $9a^4 - a^2 + 16 = 9a^4 + 24a^2 + 16 - 25a^2$
 $= (3a^2 + 4)^2 - (5a)^2 = (3a^2 + 5a + 4)(3a^2 - 5a + 4)$
- 19 $16x^4 + 4x^2a^2 + 25a^4 = 16x^4 + 40x^2a^2 + 25a^4 - 36a^2x^2$
 $= (4x^2 + 5a^2)^2 - (6ax)^2$
 $= (4x^2 + 6ax + 5a^2)(4x^2 - 6ax + 5a^2)$
- 20 $9a^4 - 19a^2x^2 + 25x^4 = 9a^4 + 30a^2x^2 + 25x^4 - 49a^2x^2$
 $= (3a^2 + 5x^2)^2 - (7ax)^2$
 $= (3a^2 + 7ax + 5x^2)(3a^2 - 7ax + 5x^2)$
- 21 $x^4 + 8x^2 + 144 = x^4 + 24x^2 + 144 - 16x^2$
 $= (x^2 + 12)^2 - (4x)^2$
 $= (x^2 + 4x + 12)(x^2 - 4x + 12)$

- 22 $a^4 - 35a^2b^2 + 25b^4 = a^4 - 10a^2b^2 + 25b^4 - 25a^2b^2$
 $= (a^2 - 5b^2)^2 - (5ab)^2$
 $= (a^2 + 5ab - 5b^2)(a^2 - 5ab - 5b^2)$
- 23 $36a^4 - 16a^2b^2 + b^4 = 36a^4 - 12a^2b^2 + b^4 - 4a^2b^2$
 $= (6a^2 - b^2)^2 - (2ab)^2$
 $= (6a^2 + 2ab - b^2)(6a^2 - 2ab - b^2)$
- 24 $49m^4 + 16n^4 - 60m^2n^2$
 $= 49m^4 - 56m^2n^2 + 16n^4 - 4m^2n^2$
 $= (7m^2 - 4n^2)^2 - (2mn)^2$
 $= (7m^2 + 2mn - 4n^2)(7m^2 - 2mn - 4n^2)$
- 25 $64a^4 + 81x^4 = 64a^4 + 144a^2x^2 + 81x^4 - 144a^2x^2$
 $= (8a^2 + 9x^2)^2 - 144a^2x^2$
 $= (8a^2 + 12ax + 9x^2)(8a^2 - 12ax + 9x^2)$
- 26 $4x^4 + (7a)^4 = 4x^4 + 4x^2(7a)^2 + (7a)^4 - 4x^2(7a)^2$
 $= (2x^2 + (7a)^2)^2 - (14ax)^2$
 $= (2x^2 + 14ax + 49a^2)(2x^2 - 14ax + 49a^2)$
- 27 $x^2 - y^2 + 2yz - z^2 = x^2 - (y - z)^2 = (x + y - z)(x - y + z)$
- 28 $4a^2 - b^2 - 9c^2 + 6bc = (2a)^2 - (b - 3c)^2 = (2a + b - 3c)(2a - b + 3c)$
- 29 $9x^2 - 4y^2 + 12yz - 9z^2 = (3x)^2 - (2y - 3z)^2$
 $= (3x + 2y - 3z)(3x - 2y + 3z)$
- 30 $a^2 - 4b^2 - 25c^2 + 20bc = a^2 - (2b - 5c)^2$
 $= (a + 2b - 5c)(a - 2b + 5c)$
- 31 $30xz + 16y^2 - 9x^2 - 25z^2$
 $= 16y^2 - (9x^2 - 30xz + 25z^2)$
 $= (4y)^2 - (3x - 5z)^2$
 $= (4y + 3x - 5z)(4y - 3x + 5z)$
- 32 $a^2 + 4b^2 - 9c^2 - 4d^2 - 4ab + 12cd$
 $= (a^2 - 4ab + 4b^2) - (9c^2 - 12cd + 4d^2)$
 $= (a - 2b)^2 - (3c - 2d)^2$
 $= (a - 2b + 3c - 2d)(a - 2b - 3c + 2d)$
- 33 $(x^2 - 2xy) - (z^2 - 2yz)$
 $= (x^2 - 2xy + y^2) - (z^2 - 2yz + y^2)$
 $= (x - y)^2 - (z - y)^2 = \{(x - y) + (z - y)\}\{(x - y) - (z - y)\}$
 $= (x - 2y + z)(x - z)$

- 34 $4a^2 - 1 + 9a^2 - 25b^2 + 12a - 10b$
 $= (4a^2 + 12a + 9a^2) - (25b^2 + 10b + 1)$
 $= (2a + 3a)^2 - (5b + 1)^2$
 $= (2a + 3a + 5b + 1)(2a + 3a - 5b - 1)$
- 35 $9x^2 - 4y^2 - 49z^2 - 3ax + 28yz + 25$
 $= (9x^2 - 30x + 25) - (4y^2 - 28yz + 49z^2)$
 $= (3x - 5)^2 - (2y - 7z)^2$
 $= (3x + 2y - 7z - 5)(3x - 2y + 7z - 5)$
- 36 $16a^2 - 16c^2 - 9b^2 - 24a + 24bc + 9$
 $= (16a^2 - 24a + 9) - (16c^2 - 24bc + 9b^2)$
 $= (4a - 3)^2 - (4c - 3b)^2$
 $= (4a - 3b + 4c - 3)(4a + 3b - 4c - 3)$
37. $49y^2 + 20z + 1^2 - 14xy - 25z^2 - 4$
 $= (1^2 - 14xy + 49y^2) - (25z^2 - 20z + 4)$
 $= (1 - 7y)^2 - (5z - 2)^2$
 $= (1 - 7y + 5z - 2)(1 - 7y - 5z + 2)$
- 38 $16x^2 + 42by - 9y^2 + 40x - 49b^2 + 25a^2$
 $= (16x^2 + 40x + 25a^2) - (9y^2 - 42by + 49b^2)$
 $= (4x + 5a)^2 - (3y - 7b)^2$
 $= (4x + 5a + 3y - 7b)(4x + 5a - 3y + 7b)$
- 39 $49x^2 - 1 + 16y^2 - 64z^2 + 16x - 56xy$
 $= (49x^2 - 56xy + 16y^2) - (64z^2 - 16x + 1)$
 $= (7x - 4y)^2 - (8z - 1)^2$
 $= (7x - 4y + 8z - 1)(7x - 4y - 8z + 1)$
- 40 $a^2 - b^2 - c^2 + d^2 - 2(ad - bc)$
 $= (a^2 - 2ad + d^2) - (b^2 - 2bc + c^2)$
 $= (a - d)^2 - (b - c)^2$
 $= (a + b - c - d)(a - b + c - d)$

Exercise 43

- $a^3 - 8b^3 = a^3 - (2b)^3 = (a - 2b)(a^2 + 2ab + 4b^2)$
- $a^4 - 27a^2 = a\{a^3 - (3^2)^2\} = a(a - 3)(a^2 + 3a + 9)$

- 3 $512x^9 + 1 = (8x^3)^3 + 1$
 $= (8x^3 + 1)((8x^3)^2 - 8x^3 + 1)$
 $= (2x^3 + 1)(64x^6 - 8x^3 + 1)$
 $= (2x + 1)(4x^2 - 2x + 1)(64x^6 - 8x^3 + 1)$
- 4 $a^6 - 512b^6 = (a^2)^3 - (8b^2)^3$
 $= (a^2 - 8b^2)(a^4 + 8a^2b^2 + 64b^4)$
 $= (a^2 - (2b)^2)(a^4 + 8a^2b^2 + 64b^4)$
 $= (a - 2b)(a^2 + 2ab + 4b^2)(a^4 + 8a^2b^2 + 64b^4)$
- 5 $27a^6 + 125x^6 = (3a^2)^3 + (5x^2)^3$
 $= (3a^2 + 5x^2)((3a^2)^2 - (3a^2)(5x^2) + (5x^2)^2)$
 $= (3a^2 + 5x^2)(9a^4 - 15a^2x^2 + 25x^4)$
- 6 $m^6 - n^6 = (m^2)^3 - (n^2)^3$
 $= (m^2 - n^2)(m^4 + mn^2 + n^4)$
 $= (m + n)(m - n)(m^2 + mn + n^2)(m^2 - mn + n^2)$
- 7 $343x^3 + 512y^3 = (7x)^3 + (8y)^3$
 $= (7x + 8y)(49x^2 - 56xy + 64y^2)$
- 8 $64x^{12} - 1 = (4x^4)^3 - 1$
 $= (4x^4 - 1)(16x^8 + 4x^4 + 1)$
 $= (2x^2 + 1)(2x^2 - 1)(16x^8 + 8x^4 + 1 - 4x^4)$
 $= (2x^2 + 1)(2x^2 - 1)((4x^4 + 1)^2 - (2x^2)^2)$
 $= (2x^2 + 1)(2x^2 - 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$
- 9 $a^6 - 64x^{12} = (a^2)^3 - (8x^4)^3$
 $= (a^2 + 8x^4)(a^4 - 8x^4)$
 $= (a^2 + (2x^3)^2)(a^4 - (2x^3)^2)$
 $= (a + 2x^3)(a^2 - 2ax^3 + 4x^6)(a - 2x^3)(a^2 + 2ax^3 + 4x^6)$
- 10 $125x^3 - 216a^3 = (5x)^3 - (6a)^3$
 $= (5x^3 - 6a^3)(25x^6 + 30ax^3 + 36a^6)$
- 11 $64a^{12}b + 343ab^{12} = ab(64a^{11} + 343b^{11})$
 $= ab((4a^3)^3 + (7b^3)^3)$
 $= ab(4a^3 + 7b^3)(16a^6 - 28a^3b^3 + 49b^6)$
- 12 $729x^{20}y^2 - 64x^2y^{20} = x^2y^2(729x^{18} - 64y^{18})$
 $= x^2y^2((9x^6)^3 - (4y^6)^3)$
 $= x^2y^2(9x^6 - 4y^6)(81x^{12} + 36x^6y^6 + 16y^{12})$

$$\begin{aligned}
&= x^2 y^2 (3x^3 + 2y^3)(3x^3 - 2y^3) \{ (81x^{12} + 72x^6 y^6 \\
&\quad + 16y^{12}) - 36x^6 y^6 \} \\
&= x^2 y^2 (3x^3 + 2y^3)(3x^3 - 2y^3) \{ (9x^6 + 4y^6)^2 \\
&\quad - (6x^3 y^3)^2 \} \\
&= x^2 y^2 (3x^3 + 2y^3)(3x^3 - 2y^3) (9x^6 + 6x^3 y^3 + 4y^6) \\
&\quad (9x^6 - 6x^3 y^3 + 4y^6)
\end{aligned}$$

- 13 $(a^3 + b^3)^3 + 8a^3 b^3 = (a^2 + b^2)^3 + (2ab)^3$
 $= (a^2 + b^2 + 2ab) \{ (a^2 + b^2)^2 - 2ab(a^2 + b^2) + (2ab)^2 \}$
 $= (a^2 + b^2 + 2ab) (a^4 + 2a^2 b^2 + b^4 - 2a^3 b - 2ab^3$
 $\quad + 4a^2 b^2)$
 $= (a + b)^2 (a^4 - 2a^3 b + 6a^2 b^2 - 2ab^3 + b^4)$
- 14 $(2x^2 - 3y^2)^3 + y^6 = \{ (2x^2 - 3y^2) + y^2 \} \{ (2x^2 - 3y^2)^2 - y^2(2x^2 - 3y^2) + y^4 \}$
 $= 2(x^2 - y^2) \{ 4x^4 - 12x^2 y^2 + 9y^4 - 2x^2 y^2 + 3y^4 + y^4 \}$
 $= 2(x + y)(x - y)(4x^4 - 14x^2 y^2 + 13y^4)$
- 15 $(2a^3 - b^3)^3 - b^9 = \{ (2a^3 - b^3) - b^3 \} \{ (2a^3 - b^3)^2 + b^3(2a^3 - b^3) + b^6 \}$
 $= 2(a^3 - b^3)(4a^6 - 4a^3 b^3 + b^6 + 2a^3 b^3 - b^6 + b^6)$
 $= 2(a - b)(a^2 + ab + b^2)(4a^6 - 2a^3 b^3 + b^6)$

Exercise 44

- 1 $3 + 1 = 4$ and $3 \times 1 = 3$, $x^2 + 4x + 3 = (x + 1)(x + 3)$
- 2 $3 + 2 = 5$ and $3 \times 2 = 6$, $x^2 + 5x + 6 = (x + 2)(x + 3)$
- 3 $3 + 4 = 7$ and $3 \times 4 = 12$, $x^2 + 7x + 12 = (x + 3)(x + 4)$
- 4 $4 + 5 = 9$ and $4 \times 5 = 20$, $x^2 + 9x + 20 = (x + 4)(x + 5)$
- 5 $6 + 3 = 9$ and $6 \times 3 = 18$, $x^2 + 9x + 18 = (x + 6)(x + 3)$
- 6 $7 + 4 = 11$ and $7 \times 4 = 28$, $x^2 + 11x + 28 = (x + 7)(x + 4)$
- 7 $-6 - 4 = -10$ and $(-6) \times (-4) = 24$,
 $x^2 - 10x + 24 = (x - 6)(x - 4)$
- 8 $-5 - 3 = -8$ and $(-5) \times (-3) = 15$,
 $x^2 - 8x + 15 = (x - 5)(x - 3)$
- 9 $-5 - 6 = -11$ and $(-5) \times (-6) = 30$,
 $x^2 - 11x + 30 = (x - 5)(x - 6)$

- 10 $-4-8=-12$ and $(-4)\times(-8)=32$,

$$r^2-12r+32=(r-4)(r-8)$$
- 11 $-12-2=-14$ and $(-12)\times(-2)=24$,

$$r^2-14r+24=(r-12)(r-2)$$
- 12 $-20-2=-22$ and $(-20)\times(-2)=40$,

$$r^2-22r+40=(r-20)(r-2)$$
- 13 $10-3=7$ and $10\times(-3)=-30$,

$$r^2+7r-30=(r+10)(r-3)$$
- 14 $8-6=2$ and $8\times(-6)=-48$,

$$r^2+2r-48=(r+8)(r-6)$$
- 15 $18-2=16$ and $18\times(-2)=-36$,

$$r^2+16r-36=(r+18)(r-2)$$
- 16 $12-3=9$ and $12\times(-3)=-36$,

$$r^2+9r-36=(r+12)(r-3)$$
- 17 $14-3=11$ and $14\times(-3)=-42$,

$$r^2+11r-42=(r+14)(r-3)$$
- 18 $18-4=14$ and $18\times(-4)=-72$,

$$r^2+14r-72=(r+18)(r-4)$$
- 19 $5-8=-3$ and $5\times(-8)=-40$,

$$r^2-3r-40=(r+5)(r-8)$$
- 20 $-16+5=-11$ and $(-16)\times5=-80$

$$r^2-11r-80=(r-16)(r+5)$$
- 21 $-32+3=-29$ and $(-32)\times3=-96$,

$$r^2-29r-96=(r-32)(r+3)$$
- 22 $-14+4=-10$ and $(-14)\times4=-56$,

$$r^2-10r-56=(r-14)(r+4)$$
- 23 $-7+6=-1$ and $(-7)\times6=-42$,

$$r^2-r-42=(r-7)(r+6)$$
- 24 $-9+8=-1$ and $(-9)\times8=-72$,

$$r^2-r-72=(r-9)(r+8)$$

- | | | |
|----|--|---|
| 25 | $\begin{array}{l} 12+10=22 \\ 12 \times 10=120 \end{array}$ | $\begin{array}{l} x^2+22x+120 \\ =(x+12)(x+10) \end{array}$ |
| 26 | $\begin{array}{l} 20-4=16 \\ 20 \times (-4)=-80 \end{array}$ | $\begin{array}{l} x^2+16x-80 \\ =(x+20)(x-4) \end{array}$ |
| 27 | $\begin{array}{l} -24+3=-21 \\ (-24) \times 3=-72 \end{array}$ | $\begin{array}{l} x^2-21x-72 \\ =(x-24)(x+3) \end{array}$ |
| 28 | $\begin{array}{l} 12-7=5 \\ 12 \times (-7)=-84 \end{array}$ | $\begin{array}{l} x^2+5x-84 \\ =(x+12)(x-7) \end{array}$ |
| 29 | $\begin{array}{l} -12-8=-20 \\ (-12) \times (-8)=96 \end{array}$ | $\begin{array}{l} x^2-20x+96 \\ =(x-12)(x-8) \end{array}$ |
| 30 | $\begin{array}{l} 26-3=23 \\ 26 \times (-3)=-78 \end{array}$ | $\begin{array}{l} x^2+23x-78 \\ =(x+26)(x-3) \end{array}$ |
| 31 | $\begin{array}{l} -12+6=-6 \\ (-12) \times 6=-72 \end{array}$ | $\begin{array}{l} x^2-6x-72 \\ =(x-12)(x+6) \end{array}$ |
| 32 | $\begin{array}{l} -21-4=-25 \\ (-21) \times (-4)=84 \end{array}$ | $\begin{array}{l} x^2-25x+84 \\ =(x-21)(x-4) \end{array}$ |
| 33 | $\begin{array}{l} -22-4=-26 \\ (-22) \times (-4)=88 \end{array}$ | $\begin{array}{l} x^2-26x+88 \\ =(x-22)(x-4) \end{array}$ |
| 34 | $\begin{array}{l} 15-8=7 \\ 15 \times (-8)=-120 \end{array}$ | $\begin{array}{l} x^2+7x-120 \\ =(x+15)(x-8) \end{array}$ |
| 35 | $\begin{array}{l} -10+8=-2 \\ (-10) \times 8=-80 \end{array}$ | $\begin{array}{l} x^2-2x-80 \\ =(x-10)(x+8) \end{array}$ |
| 36 | $\begin{array}{l} 14-6=8 \\ (14) \times (-6)=-84 \end{array}$ | $\begin{array}{l} x^2+8x-84 \\ =(x+14)(x-6) \end{array}$ |
| 37 | $\begin{array}{l} -8+7=-1 \\ (-8) \times 7=-56 \end{array}$ | $\begin{array}{l} a^2-a-56 \\ =(a-8)(a+7) \end{array}$ |
| 38 | $\begin{array}{l} -15+6=-9 \\ (-15) \times 6=-90 \end{array}$ | $\begin{array}{l} m^2-9m-90 \\ =(m-15)(m+6) \end{array}$ |
| 39 | $\begin{array}{l} 20-3=17 \\ 20 \times (-3)=-60 \end{array}$ | $\begin{array}{l} a^2+17a-60 \\ =(a+20)(a-3) \end{array}$ |
| 40 | $\begin{array}{l} -6-9=-15 \\ (-6) \times (-9)=54 \end{array}$ | $\begin{array}{l} a^2-15a+54 \\ =(a-6)(a-9) \end{array}$ |
| 41 | $\begin{array}{l} -24+2=-22 \\ (-24) \times 2=-48 \end{array}$ | $\begin{array}{l} p^2-22p-48 \\ =(p-24)(p+2) \end{array}$ |
| 42 | $\begin{array}{l} 9-8=1 \\ 9 \times (-8)=-72 \end{array}$ | $\begin{array}{l} m^2+m-72 \\ =(m+9)(m-8) \end{array}$ |

- 43 $\left. \begin{array}{l} 30-3=27 \\ \text{and } 30 \times (-3) = -90 \end{array} \right\} \quad \begin{array}{l} m^2+27m-90 \\ = (m+30)(m-3) \end{array}$
- 44 $\left. \begin{array}{l} -24-5=-29 \\ (-24) \times (-5)=120 \end{array} \right\} \quad \begin{array}{l} a^2-29a+120 \\ = (a-24)(a-5) \end{array}$
- 45 $\left. \begin{array}{l} 13-6=7 \\ 13 \times (-6) = -78 \end{array} \right\} \quad \begin{array}{l} 1^2+71-78 \\ = (1+13)(1-6) \end{array}$
- 46 $\left. \begin{array}{l} -51+2=-49 \\ (-51) \times 2 = -102 \end{array} \right\} \quad \begin{array}{l} a^2-49a-102 \\ = (a-51)(a+2) \end{array}$
- 47 $\left. \begin{array}{l} -15-4=-19 \\ (-15) \times (-4)=60 \end{array} \right\} \quad \begin{array}{l} a^2-19a+60 \\ = (a-15)(a-4) \end{array}$
- 48 $\left. \begin{array}{l} 16-4=12 \\ 16 \times (-4) = -64 \end{array} \right\} \quad \begin{array}{l} 1^2+121-64 \\ = (1+16)(1-4) \end{array}$
- 49 $\left. \begin{array}{l} -30+4=-26 \\ (-30) \times 4 = -120 \end{array} \right\} \quad \begin{array}{l} a^2-26a-120 \\ = (a-30)(a+4) \end{array}$
- 50 $\left. \begin{array}{l} 15-7=8 \\ 15 \times (-7) = -105 \end{array} \right\} \quad \begin{array}{l} 1^2+81-105 \\ = (1+15)(1-7) \end{array}$
- 51 $\left. \begin{array}{l} 6-7=-1 \\ 6 \times (-7) = -42 \end{array} \right\} \quad \begin{array}{l} 1^2-1y-42y^2 \\ = (1-7y)(1+6y) \end{array}$
- 52 $\left. \begin{array}{l} -8-4=-12 \\ (-8) \times (-4)=32 \end{array} \right\} \quad \begin{array}{l} a^2-12ab+32b^2 \\ = (a-8b)(a-4b) \end{array}$
- 53 $\left. \begin{array}{l} 6-5=1 \\ 6 \times (-5) = -30 \end{array} \right\} \quad \begin{array}{l} m^2+mn-30n^2 \\ = (m+6n)(m-5n) \end{array}$
- 54 $\left. \begin{array}{l} 4-3=1 \\ 4 \times (-3) = -12 \end{array} \right\} \quad \begin{array}{l} a^3+ab-12b^2 \\ = (a+4b)(a-3b) \end{array}$
- 55 $\left. \begin{array}{l} 3-5=-2 \\ 3 \times (-5) = -15 \end{array} \right\} \quad \begin{array}{l} a^3-2ab-15b^2 \\ = (a+3b)(a-5b) \end{array}$
- 56 $\left. \begin{array}{l} -8+1=-7 \\ (-8) \times 1 = -8 \end{array} \right\} \quad \begin{array}{l} 1^2-7xy-8y^2 \\ = (1-8y)(1+y) \end{array}$
- 57 $\left. \begin{array}{l} 8-5=3 \\ 8 \times (-5) = -40 \end{array} \right\} \quad \begin{array}{l} 1^2+3xy-40y^2 \\ = (1+8y)(1-5y) \end{array}$
- 58 $\left. \begin{array}{l} -8-6=-14 \\ (-8) \times (-6)=48 \end{array} \right\} \quad \begin{array}{l} p^2-14pq+48q^2 \\ = (p-8q)(p-6q) \end{array}$
- 59 $\left. \begin{array}{l} 10-8=2 \\ 10 \times (-8) = -80 \end{array} \right\} \quad \begin{array}{l} p^2+2pq-80q^2 \\ = (p+10q)(p-8q) \end{array}$
- 60 $\left. \begin{array}{l} 24-4=20 \\ 24 \times (-4) = -96 \end{array} \right\} \quad \begin{array}{l} 1^2+24xy-96y^2 \\ = (1+24y)(1-4y) \end{array}$

- 61 $\begin{array}{l} 5-1=4 \\ 5 \times (-1)=-5 \end{array} \quad \left. \vphantom{\begin{array}{l} 5-1=4 \\ 5 \times (-1)=-5 \end{array}} \right\} \quad \begin{array}{l} a^4+4a^2-5 \\ =(a^2+5)(a^2-1) \\ =(a^2+5)(a+1)(a-1) \end{array}$
- 62 $\begin{array}{l} 5-3=2 \\ 5 \times (-3)=-15 \end{array} \quad \left. \vphantom{\begin{array}{l} 5-3=2 \\ 5 \times (-3)=-15 \end{array}} \right\} \quad \begin{array}{l} x^4+2x^2-15 \\ =(x^2+5)(x^2-3) \end{array}$
- 63 $\begin{array}{l} 7-4=3 \\ 7 \times (-4)=-28 \end{array} \quad \left. \vphantom{\begin{array}{l} 7-4=3 \\ 7 \times (-4)=-28 \end{array}} \right\} \quad \begin{array}{l} x^4+3x^2-28 \\ =(x^2+7)(x^2-4) \\ =(x^2+7)(x+2)(x-2) \end{array}$
- 64 $\begin{array}{l} 3-1=2 \\ 3 \times (-1)=-3 \end{array} \quad \left. \vphantom{\begin{array}{l} 3-1=2 \\ 3 \times (-1)=-3 \end{array}} \right\} \quad \begin{array}{l} x^4+2x^2-3 \\ =(x^2+3)(x^2-1) \\ =(x^2+3)(x-1)(x+1) \end{array}$
- 65 $\begin{array}{l} -8-2=-10 \\ (-8) \times (-2)=16 \end{array} \quad \left. \vphantom{\begin{array}{l} -8-2=-10 \\ (-8) \times (-2)=16 \end{array}} \right\} \quad \begin{array}{l} a^4-10a^2+16 \\ =(a^2-8)(a^2-2) \\ =(a-2)(a^2+2a+4)(a^2-2) \end{array}$
- 66 $\begin{array}{l} 27-1=26 \\ 27 \times (-1)=-27 \end{array} \quad \left. \vphantom{\begin{array}{l} 27-1=26 \\ 27 \times (-1)=-27 \end{array}} \right\} \quad \begin{array}{l} x^4+26x^2-27 \\ =(x^2+27)(x^2-1) \\ =(x+3)(x-3)(x+1)(x-1) \\ \quad (x^2+1+1) \end{array}$
- 67 $\begin{array}{l} 8-1=7 \\ 8 \times (-1)=-8 \end{array} \quad \left. \vphantom{\begin{array}{l} 8-1=7 \\ 8 \times (-1)=-8 \end{array}} \right\} \quad \begin{array}{l} a^4+7a^2-8 \\ =(a^2+8)(a^2-1) \\ =(a+2)(a^2-2a+4)(a-1) \\ \quad (a^2-a+1) \end{array}$
- 68 $\begin{array}{l} -16-4=-20 \\ (-16) \times (-4)=64 \end{array} \quad \left. \vphantom{\begin{array}{l} -16-4=-20 \\ (-16) \times (-4)=64 \end{array}} \right\} \quad \begin{array}{l} x^5-20x^3+64 \\ =(x^3-16)(x^2-4) \\ =(x^3+4)(x^2-4)(x^2+2)(x^2-2) \\ =(x^3+4)(x+2)(x-2)(x^2+2) \\ \quad (x^2-2) \end{array}$
- 69 $\begin{array}{l} -16+5=-11 \\ (-16) \times 5=-80 \end{array} \quad \left. \vphantom{\begin{array}{l} -16+5=-11 \\ (-16) \times 5=-80 \end{array}} \right\} \quad \begin{array}{l} a^5-11a^3-80 \\ =(a^3-16)(a^2+5) \\ =(a^3-4)(a^2-4)(a^2+5) \\ =(a^3+4)(a+2)(a-2)(a^2+5) \end{array}$
- 70 $\begin{array}{l} -8+1=-7 \\ (-8) \times 1=-8 \end{array} \quad \left. \vphantom{\begin{array}{l} -8+1=-7 \\ (-8) \times 1=-8 \end{array}} \right\} \quad \begin{array}{l} x^{12}-7x^6-8 \\ =(x^6-8)(x^1+1) \\ =(x^2-2)(x^4+2x^2+4)(x^1+1) \\ \quad (x^4-x^2+1) \\ =(x^2+1)(x^2-2)(x^4-x^2+1) \\ \quad (x^4+2x^2+4) \end{array}$

- 71 Putting x for (a^2+2a) , the given expression

$$=x^2-1-2=(1+1)(x-2)=(a^2+2a+1)(a^2+2a-2)$$

$$=(a^2+2a-2)(a+1)^2$$
- 72 Putting a for (x^2+3x+1) , the given expression

$$=a^2+3a+2=(a+1)(a+2)=(x^2+3x+1)(x^2+3x+2)$$

$$=(x^2+3x+1)^2+1(x+2)$$
- 73 Putting a for (x^2-2x) , the given expression

$$=a^2-2a-3=(a-3)(a+1)=(x^2-2x-3)(x^2-2x+1)$$

$$=(x-3)^2+1(x-1)^2$$
- 74 Putting x for (a^2-3a) , the given expression

$$=x^2-3x-4=(x-4)(x+1)=(a^2-3a-4)(a^2-3a+1)$$

$$=(a+1)(a-4)(a^2-3a+1)$$
- 75 Putting a for (x^2-4x) , the given expression

$$=a^2-4a-5=(a-5)(a+1)=(x^2-4x-5)(x^2-4x+1)$$

$$=(x+1)(x-5)^2(x^2-4x+1)$$
- 76 Putting a for (x^2-x) , the given expression

$$=a^2-6a+12=(a-6)(a-2)$$

$$=(x^2-x-6)(x^2-x-2)$$

$$=(x-3)(x+2)(x+1)(x-2)$$
- 77 Putting a for (x^2-5x) , the given expression

$$=a^2-10a+24=(a+6)(a-4)$$

$$=(x^2-5x+6)(x^2-5x+4)$$

$$=(x-2)(x-3)^2(x-1)(x-4)$$
- 78 Putting x for (a^2+7a) , the given expression

$$=x^2-8x-180=(x-18)(x+10)$$

$$=(a^2+7a-18)(a^2+7a+10)$$

$$=(a-2)(a+9)(a+2)(a+5)$$
- 79 Putting x for (a^2+6a) , the given expression

$$=x^2-32x-320=(x-40)(x+8)$$

$$=(a^2+6a-40)(a^2+6a+8)$$

$$=(a-4)(a+10)(a+2)(a+4)$$

- 80 Putting
- a
- for
- $(x^2 - 8x)$
- , the given expression

$$\begin{aligned}
 &= a^2 - 29a + 180 = (a - 9)(a - 20) \\
 &= (x^2 - 8x - 9)(x^2 - 8x - 20) \\
 &= (x - 9)(x + 1)(x - 10)(x + 2)
 \end{aligned}$$

- 81 Putting
- a
- for
- $(3x + 4y)$
- and
- b
- for
- $(x + 2y)$
- , the given expression
- $= a^2 + ab - 2b^2$

$$\begin{aligned}
 &= (a - b)(a + 2b) \\
 &= \{(3x + 4y) - (x + 2y)\}\{(3x + 4y) + 2(x + 2y)\} \\
 &= (2x + 2y) - (5x + 8y) = 2(x + y)(5x + 8y)
 \end{aligned}$$

- 82 Putting
- x
- for
- $(2a - 5b)$
- and
- y
- for
- $(a + 2b)$
- , the given expression
- $= x^2 + 2xy - 3y^2 = (x + 3y)(x - y)$

$$\begin{aligned}
 &= \{(2a - 5b) + 3(a + 2b)\}\{(2a - 5b) - (a + 2b)\} \\
 &= (5a + b)(a - 7b)
 \end{aligned}$$

- 83 Putting
- a
- for
- $(4m - 3n)$
- and
- b
- for
- $(m + 5n)$
- , the given expression
- $= a^2 + 3ab - 4b^2$

$$\begin{aligned}
 &= (a + 4b)(a - b) \\
 &= \{(4m - 3n) + 4(m + 5n)\}\{(4m - 3n) - (m + 5n)\} \\
 &= (8m + 17n)(3m - 8n)
 \end{aligned}$$

- 84 Putting
- x
- for
- $(5a + 3b)$
- and
- y
- for
- $(a + b)$
- , the given expression

$$\begin{aligned}
 &= x^2 + 4xy - 5y^2 = (x + 5y)(x - y) \\
 &= \{(5a + 3b) + 5(a + b)\}\{(5a + 3b) - (a + b)\} \\
 &= (10a + 8b)(4a + 2b) \\
 &= 4(5a + 4b)(2a + b)
 \end{aligned}$$

- 85 Putting
- a
- for
- $(5x + 7y)$
- and
- b
- for
- $(x - y)$
- , the given expression

$$\begin{aligned}
 &= a^2 - 2ab - 8b^2 = (a + 2b)(a - 4b) \\
 &= \{(5x + 7y) + 2(x - y)\}\{(5x + 7y) - 4(x - y)\} \\
 &= (7x + 5y)(x + 11y)
 \end{aligned}$$

- 86 Putting
- x
- for
- $(7a + 3b)$
- and
- y
- for
- $(a - 4b)$
- , the given expression

$$\begin{aligned}
 &= x^2 - 3xy - 10y^2 \\
 &= (x - 5y)(x + 2y) \\
 &= \{(7a + 3b) - 5(a - 4b)\}\{(7a + 3b) + 2(a - 4b)\} \\
 &= (2a + 23b)(9a - 5b)
 \end{aligned}$$

- 87 Putting x for $(9a-4b)$ and y for $(a-3b)$, the given expression

$$\begin{aligned} &= x^2 - 5xy - 6y^2 \\ &= (x - 6y)(x + y) \\ &= \{(9a - 4b) - 6(a - 3b)\}\{(9a - 4b) + (a - 3b)\} \\ &= (3a + 14b)(10a - 7b) \end{aligned}$$

- 88 Putting x for $(2m^2 + 3n^2)$ and y for $(m^2 - 2n^2)$, the given expression

$$\begin{aligned} &= x^2 + 5xy - 14y^2 = (x - 2y)(x + 7y) \\ &= \{(2m^2 + 3n^2) - 2(m^2 - 2n^2)\}\{(2m^2 + 3n^2) + 7(m^2 - 2n^2)\} \\ &= 7n^2(9m^2 - 11n^2) \end{aligned}$$

- 89 Putting a for $(x^2 + 6xy)$ and b for $(xy - 6y^2)$, the given

$$\begin{aligned} &= a^2 - 3ab + 2b^2 \\ &= (a - b)(a - 2b) \\ &= \{(x^2 + 6xy) - (xy - 6y^2)\}\{(x^2 + 6xy) - 2(xy - 6y^2)\} \\ &= (x^2 + 4xy + 12y^2)(x^2 + 5xy + 6y^2) \\ &= (x^2 + 4xy + 12y^2)(x + 3y)(x + 2y) \end{aligned}$$

- 90 Putting x for $(a^2 - 5ab)$ and y for $(ab - 4b^2)$, the given

$$\begin{aligned} &= x^2 - 9xy + 18y^2 \\ &= (x - 3y)(x - 6y) \\ &= \{(a^2 - 5ab) - 3(ab - 4b^2)\}\{(a^2 - 5ab) - 6(ab - 4b^2)\} \\ &= (a^2 - 8ab + 12b^2)(a^2 - 11ab + 24b^2) \\ &= (a - 6b)(a - 2b)(a - 3b)(a - 8b) \\ &= (a - 2b)(a - 3b)(a - 6b)(a - 8b) \end{aligned}$$

- 91 $2x^2 + x - 15 = \frac{1}{2}(2 \cdot 2x^2 + 2x - 15 \times 2)$
 $= \frac{1}{2}(x^2 + x - 30)$ [putting a for $2x$]
 $= \frac{1}{2}(x + 6)(x - 5) = \frac{1}{2}(2x + 6)(2x - 5)$
 $= (x + 3)(2x - 5)$

- 92 $6a^2 - a - 15 = \frac{1}{6}(6 \cdot 6a^2 - 6a - 15 \times 6)$
 $= \frac{1}{6}(x^2 - x - 90)$ [putting x for $6a$]
 $= \frac{1}{6}(x + 9)(x - 10)$
 $= \frac{1}{6}(6a + 9)(6a - 10)$
 $= \frac{1}{2}(6a + 9) \cdot \frac{1}{2}(6a - 10) = (2a + 3)(3a - 5)$

$$\begin{aligned}
 93 \quad 8m^2 - 6m - 9 &= \frac{1}{8}(8 \cdot 8m^2 - 6 \cdot 8m - 72) \\
 &= \frac{1}{8}(a^2 - 6a - 72) \quad [\text{putting } a \text{ for } 8m] \\
 &= \frac{1}{8}(a - 12)(a + 6) \\
 &= \frac{1}{8}(8m - 12)(8m + 6) = \frac{1}{2}(8m - 12) \frac{1}{2}(8m + 6) \\
 &= (2m - 3)(4m + 3)
 \end{aligned}$$

$$\begin{aligned}
 94 \quad 6x^2 + 7xy - 24y^2 &= \frac{1}{6}(6 \cdot 6x^2 + 7 \cdot 6xy - 144y^2) \\
 &= \frac{1}{6}(a^2 + 7ay - 144y^2) \quad [\text{putting } a \text{ for } 6x] \\
 &= \frac{1}{6}(a + 16y)(a - 9y) \\
 &= \frac{1}{6}(6x + 16y)(6x - 9y) = \frac{1}{2}(6x + 16y) \frac{1}{3}(6x - 9y) \\
 &= (3x + 8y)(2x - 3y)
 \end{aligned}$$

$$\begin{aligned}
 95 \quad 10a^2 - 41ab + 21b^2 &= \frac{1}{10}(10 \cdot 10a^2 - 41 \cdot 10ab + 210b^2) \\
 &= \frac{1}{10}(x^2 - 41x + 210b^2) \quad [\text{putting } x \text{ for } 10a] \\
 &= \frac{1}{10}(x - 6b)(x - 35b) \\
 &= \frac{1}{10}(10a - 6b)(10a - 35b) \\
 &= \frac{1}{2}(10a - 6b) \frac{1}{5}(10a - 35b) = (5a - 3b)(2a - 7b)
 \end{aligned}$$

$$\begin{aligned}
 96 \quad 12m^2 - mn - 20n^2 &= \frac{1}{12}(12 \cdot 12m^2 - 12mn - 240n^2) \\
 &= \frac{1}{12}(x^2 - xn - 240n^2) \quad [\text{putting } x \text{ for } 12m] \\
 &= \frac{1}{12}(x - 16n)(x + 15n) \\
 &= \frac{1}{12}(12m - 16n)(12m + 15n) \\
 &= \frac{1}{3}(12m - 16n) \frac{1}{4}(12m + 15n) \\
 &= (3m - 4n)(4m + 5n)
 \end{aligned}$$

$$\begin{aligned}
 97 \quad 12x^2 + 28xy - 5y^2 &= \frac{1}{12}(12 \cdot 12x^2 + 28 \cdot 12xy - 60y^2) \\
 &= \frac{1}{12}(a^2 + 28ay - 60y^2) \quad [\text{putting } a \text{ for } 12x] \\
 &= \frac{1}{12}(a + 30y)(a - 2y) \\
 &= \frac{1}{12}(12x + 30y)(12x - 2y) \\
 &= \frac{1}{3}(12x + 30y) \frac{1}{4}(12x - 2y) = (3x + 5y)(6x - y)
 \end{aligned}$$

$$\begin{aligned}
 98 \quad 20a^2 + ab - 30b^2 &= \frac{1}{20}(20 \cdot 20a^2 + 20ab - 600b^2) \\
 &= \frac{1}{20}(x^2 + x - 600b^2) \quad [\text{putting } x \text{ for } 20a] \\
 &= \frac{1}{20}(x + 25b)(x - 24b) \\
 &= \frac{1}{20}(20a + 25b)(20a - 24b) \\
 &= \frac{1}{5}(20a + 25b) \frac{1}{4}(20a - 24b) \\
 &= (4a + 5b)(5a - 6b)
 \end{aligned}$$

$$\begin{aligned}
 99 \quad 18x^2 - 51xy + 35y^2 &= \frac{1}{18}(18 \cdot 18x^2 - 51 \cdot 18xy + 630y^2) \\
 &= \frac{1}{18}(a^2 - 51ay + 630y^2) \quad [\text{putting } a \text{ for } 18x] \\
 &= \frac{1}{18}(a - 21y)(a - 30y) \\
 &= \frac{1}{18}(18x - 21y)(18x - 30y) \\
 &= \frac{1}{6}(18x - 21y) \frac{1}{3}(18x - 30y) \\
 &= (6x - 7y)(3x - 5y)
 \end{aligned}$$

$$\begin{aligned}
 100 \quad 12x^2 + 23xy - 21y^2 &= \frac{1}{12}(12 \cdot 12x^2 + 23 \cdot 12xy - 288y^2) \\
 &= \frac{1}{12}(a^2 + 23ay - 288y^2) \quad [\text{putting } a \text{ for } 12x] \\
 &= \frac{1}{12}(a + 32y)(a - 9y) \\
 &= \frac{1}{12}(12x + 32y)(12x - 9y) \\
 &= \frac{1}{4}(12x + 32y) \frac{1}{3}(12x - 9y) \\
 &= (3x + 8y)(4x - 3y)
 \end{aligned}$$

Exercise 45.

$$\begin{aligned}
 1 \quad x^2 + 4x + 3 &= x^2 + 4x + 4 - 1 \\
 &= (x + 2)^2 - 1 = (x + 2 + 1)(x + 2 - 1) = (x + 3)(x + 1)
 \end{aligned}$$

$$\begin{aligned}
 2 \quad x^2 + 6x + 5 &= x^2 + 6x + 9 - 4 = (x + 3)^2 - (2)^2 \\
 &= \{(x + 3) + 2\}\{(x + 3) - 2\} \\
 &= (x + 5)(x + 1)
 \end{aligned}$$

$$\begin{aligned}
 3 \quad x^2 + 8x + 15 &= x^2 + 8x + 16 - 1 = (x + 4)^2 - 1 \\
 &= \{(x + 4) + 1\}\{(x + 4) - 1\} \\
 &= (x + 5)(x + 3)
 \end{aligned}$$

$$\begin{aligned}
 4 \quad x^2 - 10x + 21 &= x^2 - 10x + 25 - 4 = (x - 5)^2 - (2)^2 \\
 &= \{(x - 5) + 2\}\{(x - 5) - 2\} \\
 &= (x - 3)(x - 7)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad x^2 - 2x - 48 &= x^2 - 2x + 1 - 49 = (x - 1)^2 - (7)^2 \\
 &= \{(x - 1) + 7\}\{(x - 1) - 7\} \\
 &= (x + 6)(x - 8)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad x^2 - 4x - 45 &= x^2 - 4x + 4 - 49 = (x - 2)^2 - (7)^2 \\
 &= \{(x - 2) + 7\}\{(x - 2) - 7\} \\
 &= (x + 5)(x - 9)
 \end{aligned}$$

- 7 $r^2 - 12r + 32 = r^2 - 12r + 36 - 4 = (r-6)^2 - (2)^2$
 $= \{(r-6)+2\}\{(r-6)-2\}$
 $= (r-4)(r-8)$
- 8 $r^2 - 6r - 55 = r^2 - 6r + 9 - 64 = (r-3)^2 - (8)^2$
 $= \{(r-3)+8\}\{(r-3)-8\}$
 $= (r+5)(r-11)$
- 9 $a^2 + 2ab - c^2 + 2bc = (a^2 + 2ab + b^2) - (b^2 + c^2 - 2bc)$
 $= (a+b)^2 - (b-c)^2$
 $= \{(a+b)+(b-c)\}\{(a+b)-(b-c)\}$
 $= (a+2b-c)(a+c)$
- 10 $x^2 + 2x - y^2 + 2y = (x^2 + 2x + 1) - (y^2 - 2y)$
 $= (x+1)^2 - (y-1)^2$
 $= \{(x+1)+(y-1)\}\{(x+1)-(y-1)\}$
 $= (x+y)(x-y+2)$
- 11 $x^2 + 6x - y^2 + 4y + 5 = (x^2 + 6x + 9) - (y^2 - 4y)$
 $= (x+3)^2 - (y-2)^2$
 $= \{(x+3)+(y-2)\}\{(x+3)-(y-2)\}$
 $= (x+y+1)(x-y+5)$
- 12 $a^2 + 4ab - 5b^2 - c^2 + 6bc = (a^2 + 4ab + 4b^2) - (9b^2 + c^2 - 6bc)$
 $= (a+2b)^2 - (3b-c)^2$
 $= \{(a+2b)+(3b-c)\}\{(a+2b)-(3b-c)\}$
 $= (a+5b-c)(a-b+c)$
- 13 $r^2 - 6ry + 5y^2 - z^2 + 4yz = (r^2 - 6ry + 9y^2) - (4y^2 + z^2 - 4yz)$
 $= (r-3y)^2 - (2y-z)^2$
 $= \{(r-3y)+(2y-z)\}\{(r-3y)-(2y-z)\}$
 $= (r-y-z)(r-5y+z)$
- 14 $r^2 - 10ry + 16y^2 - 4z^2 + 12yz$
 $= (r^2 - 10ry + 25y^2) - (9y^2 + 4z^2 - 12yz)$
 $= (r-5y)^2 - (3y-2z)^2$
 $= \{(r-5y)+(3y-2z)\}\{(r-5y)-(3y-2z)\}$
 $= (r-2y-2z)(r-8y+2z)$

$$\begin{aligned}
 15 \quad a^2 - 12ab + 13b^2 - 9c^2 + 42bc \\
 &= (a^2 - 12ab + 36b^2) - (9c^2 - 42bc) \\
 &= (a - 6b)^2 - (7b - 3c)^2 \\
 &= \{(a - 6b) + (7b - 3c)\} \{(a - 6b) - (7b - 3c)\} \\
 &= (a + b - 3c)(a - 13b + 3c)
 \end{aligned}$$

$$\begin{aligned}
 16 \quad x^2 + 12xy + 36y^2 - 9z^2 + 36yz \\
 &= (x^2 + 12xy + 36y^2) - (9z^2 - 36yz) \\
 &= (x + 6y)^2 - (6y - 3z)^2 \\
 &= \{(x + 6y) + (6y - 3z)\} \{(x + 6y) - (6y - 3z)\} \\
 &= (x + 12y - 3z)(x + 3z)
 \end{aligned}$$

$$\begin{aligned}
 17 \quad x^2 - 14xy + 49y^2 - 25z^2 + 80yz \\
 &= (x^2 - 14xy + 49y^2) - (25z^2 - 80yz) \\
 &= (x - 7y)^2 - (8y - 5z)^2 \\
 &= \{(x - 7y) + (8y - 5z)\} \{(x - 7y) - (8y - 5z)\} \\
 &= (x + y - 5z)(x - 15y + 5z)
 \end{aligned}$$

$$\begin{aligned}
 18 \quad 21x^2 - 5x - 3 &= 2 \left(x^2 - \frac{5}{2}x - \frac{3}{2} \right) \\
 &= 2 \left\{ x^2 - \frac{5}{2}x + \frac{25}{16} - \left(\frac{25}{16} + \frac{3}{2} \right) \right\} \\
 &= 2 \left\{ \left(x - \frac{5}{4} \right)^2 - \left(\frac{49}{16} \right) \right\} \\
 &= 2 \left\{ \left(x - \frac{5}{4} \right)^2 - \left(\frac{7}{4} \right)^2 \right\} \\
 &= 2 \left\{ \left(x - \frac{5}{4} + \frac{7}{4} \right) \left(x - \frac{5}{4} - \frac{7}{4} \right) \right\} \\
 &= 2(x + 1)(x - 3) = (2x + 2)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 19 \quad 3x^2 - 5x - 2 &= 3(x^2 - \frac{5}{3}x - \frac{2}{3}) \\
 &= 3 \{ (x^2 - \frac{5}{3}x + \frac{25}{36}) - (\frac{25}{36} + \frac{2}{3}) \} \\
 &= 3 \{ (x - \frac{5}{6})^2 - (\frac{49}{36}) \} \\
 &= 3 \{ (x - \frac{5}{6})^2 - (\frac{7}{6})^2 \} \\
 &= 3 \{ (x - \frac{5}{6} + \frac{7}{6}) (x - \frac{5}{6} - \frac{7}{6}) \} \\
 &= 3(x + \frac{1}{3})(x - 2) = (3x + 1)(x - 2)
 \end{aligned}$$

$$\begin{aligned}
 20 \quad 3x^2 + 14x + 8 &= 3(x^2 + \frac{14}{3}x + \frac{8}{3}) \\
 &= 3 \{ x^2 + \frac{14}{3}x + \frac{49}{9} - (\frac{49}{9} - \frac{8}{3}) \}
 \end{aligned}$$

$$\begin{aligned}
&= 3\{(1 + \frac{1}{3})' - (\frac{1}{3})^2\} \\
&= 3\{(1 + \frac{1}{3})^2 - (\frac{1}{3})^2\} \\
&= 3\{(1 + \frac{1}{3}) + \frac{1}{3}\}\{(1 + \frac{1}{3}) - \frac{1}{3}\} \\
&= 3(1 + 4)(1 + \frac{1}{3}) = (1 + 4)(3 + 1)
\end{aligned}$$

$$\begin{aligned}
21 \quad 4x^2 + 7x - 2 &= 4(x^2 + \frac{7}{4}x - \frac{1}{2}) \\
&= 4\{x^2 + \frac{7}{4}x + \frac{49}{16} - (\frac{49}{16} + \frac{1}{2})\} \\
&= 4\{(x + \frac{7}{8})^2 - (\frac{9}{8})^2\} \\
&= 4\{(x + \frac{7}{8})^2 - (\frac{9}{8})^2\} \\
&= 4\{(x + \frac{7}{8}) + \frac{9}{8}\}\{(x + \frac{7}{8}) - \frac{9}{8}\} \\
&= 4(1 + 2)(1 - \frac{1}{4}) = (1 + 2)(4 - 1)
\end{aligned}$$

$$\begin{aligned}
22 \quad 6x^2 + x - 2 &= 6(x^2 + \frac{1}{6}x - \frac{2}{3}) \\
&= 6\{x^2 + \frac{1}{6}x + \frac{1}{144} - (\frac{1}{144} + \frac{2}{3})\} \\
&= 6\{(x + \frac{1}{12})^2 - (\frac{49}{144})\} \\
&= 6\{(x + \frac{1}{12})^2 - (\frac{7}{12})^2\} \\
&= 6\{(x + \frac{1}{12}) + \frac{7}{12}\}\{(x + \frac{1}{12}) - \frac{7}{12}\} \\
&= 6(1 + \frac{2}{3})(1 - \frac{1}{2}) = 3(1 + \frac{2}{3})2(1 - \frac{1}{2}) \\
&= (3 + 2)(2 - 1)
\end{aligned}$$

$$\begin{aligned}
23 \quad 6x^2 - 5x - 4 &= 6(x^2 - \frac{5}{6}x - \frac{2}{3}) \\
&= 6\{x^2 - \frac{5}{6}x + \frac{25}{144} - (\frac{25}{144} + \frac{2}{3})\} \\
&= 6\{(x - \frac{5}{12})^2 - (\frac{121}{144})\} \\
&= 6\{(x - \frac{5}{12})^2 - (\frac{11}{12})^2\} \\
&= 6\{(x - \frac{5}{12}) + \frac{11}{12}\}\{(x - \frac{5}{12}) - \frac{11}{12}\} \\
&= 6(1 + \frac{1}{2})(x - \frac{4}{3}) = 2(x + \frac{1}{2})3(x - \frac{4}{3}) \\
&= (2 + 1)(3 - 4)
\end{aligned}$$

$$\begin{aligned}
24 \quad 6x^2 + 7x - 3 &= 6\left(x^2 + \frac{7}{6}x - \frac{1}{2}\right) \\
&= 6\left\{x^2 + \frac{7}{6}x + \frac{49}{144} - \left(\frac{49}{144} + \frac{1}{2}\right)\right\} \\
&= 6\left\{\left(x + \frac{7}{12}\right)^2 - \frac{121}{144}\right\} \\
&= 6\left\{\left(x + \frac{7}{12}\right)^2 - \left(\frac{11}{12}\right)^2\right\} \\
&= 6\left\{\left(x + \frac{7}{12}\right) + \frac{11}{12}\right\}\left\{\left(x + \frac{7}{12}\right) - \frac{11}{12}\right\} \\
&= 6\left(x + \frac{3}{2}\right)\left(x - \frac{1}{3}\right) = 2\left(x + \frac{3}{2}\right)3\left(x - \frac{1}{3}\right) \\
&= (2x + 3)(3x - 1)
\end{aligned}$$

$$\begin{aligned}
25 \quad 8x^2 + 2x - 15 &= 8\left(x^2 + \frac{1}{4}x - \frac{15}{8}\right) \\
&= 8\left\{x^2 + \frac{x}{4} + \frac{1}{64} - \left(\frac{1}{64} + \frac{15}{8}\right)\right\} \\
&= 8\left\{\left(x + \frac{1}{8}\right)^2 - \frac{121}{64}\right\} \\
&= 8\left\{\left(x + \frac{1}{8}\right)^2 - \left(\frac{11}{8}\right)^2\right\} \\
&= 8\left\{\left(x + \frac{1}{8}\right) + \frac{11}{8}\right\}\left\{\left(x + \frac{1}{8}\right) - \frac{11}{8}\right\} \\
&= 8\left(x + \frac{3}{2}\right)\left(x - \frac{5}{4}\right) = (2x + 3)(4x - 5)
\end{aligned}$$

$$\begin{aligned}
26 \quad 4x^2 + 4x - 35 &= 4\left(x^2 + x - \frac{35}{4}\right) \\
&= 4\left\{x^2 + x + \frac{1}{4} - \left(\frac{1}{4} + \frac{35}{4}\right)\right\} \\
&= 4\left\{\left(x + \frac{1}{2}\right)^2 - (9)\right\} \\
&= 4\left\{\left(x + \frac{1}{2}\right) + 3\right\}\left\{\left(x + \frac{1}{2}\right) - 3\right\} \\
&= 4\left(x + \frac{7}{2}\right)\left(x - \frac{5}{2}\right) = (2x + 7)(2x - 5)
\end{aligned}$$

$$\begin{aligned}
 27 \quad 6x^2 - x - 12 &= 6 \left(x^2 - \frac{1}{6}x - 2 \right) \\
 &= 6 \left\{ x^2 - \frac{x}{6} + \frac{1}{144} - \left(\frac{1}{144} + 2 \right) \right\} \\
 &= 6 \left\{ \left(x - \frac{1}{12} \right)^2 - \frac{289}{144} \right\} \\
 &= 6 \left\{ \left(x - \frac{1}{12} \right)^2 - \left(\frac{17}{12} \right)^2 \right\} \\
 &= 6 \left\{ \left(x - \frac{1}{12} \right) + \frac{17}{12} \right\} \left\{ \left(x - \frac{1}{12} \right) - \frac{17}{12} \right\} \\
 &= 6 \left(x + \frac{4}{3} \right) \left(x - \frac{3}{2} \right) = (3x + 4)(2x - 3)
 \end{aligned}$$

$$\begin{aligned}
 28 \quad 3x^2 - 16x - 12 &= 3 \left(x^2 - \frac{16x}{3} - 4 \right) \\
 &= 3 \left\{ x^2 - \frac{16x}{3} + \frac{64}{9} - \left(\frac{64}{9} + 4 \right) \right\} \\
 &= 3 \left\{ \left(x - \frac{8}{3} \right)^2 - \frac{100}{9} \right\} \\
 &= 3 \left\{ \left(x - \frac{8}{3} \right)^2 - \left(\frac{10}{3} \right)^2 \right\} \\
 &= 3 \left\{ \left(x - \frac{8}{3} \right) + \frac{10}{3} \right\} \left\{ \left(x - \frac{8}{3} \right) - \frac{10}{3} \right\} \\
 &= 3 \left(x + \frac{2}{3} \right) (x - 6) = (3x + 2)(x - 6)
 \end{aligned}$$

$$\begin{aligned}
 29 \quad 2x^2 - 9x - 35 &= 2 \left(x^2 - \frac{9}{2}x - \frac{35}{2} \right) \\
 &= 2 \left\{ x^2 - \frac{9}{2}x + \frac{81}{16} - \left(\frac{81}{16} + \frac{35}{2} \right) \right\} \\
 &= 2 \left\{ \left(x - \frac{9}{4} \right)^2 - \left(\frac{19}{4} \right)^2 \right\} \\
 &= 2 \left\{ \left(x - \frac{9}{4} \right) + \frac{19}{4} \right\} \left\{ \left(x - \frac{9}{4} \right) - \frac{19}{4} \right\} \\
 &= 2 \left(x + \frac{5}{2} \right) (x - 7) = (2x + 5)(x - 7)
 \end{aligned}$$

$$\begin{aligned}
30 \quad 21^2 + 51 - 42 &= 2 \left(x^2 + \frac{5}{2}x - 21 \right) \\
&= 2 \left\{ x^2 + \frac{5}{2}x + \frac{25}{16} - \left(\frac{25}{16} + 21 \right) \right\} \\
&= 2 \left\{ \left(x + \frac{5}{4} \right)^2 - \frac{361}{16} \right\} \\
&= 2 \left\{ x + \frac{5}{4} \right\}^2 - \left(\frac{19}{4} \right)^2 \\
&= 2 \left\{ \left(x + \frac{5}{4} \right) + \frac{19}{4} \right\} \left\{ \left(x + \frac{5}{4} \right) - \frac{19}{4} \right\} \\
&= 2(x+6) \left(x - \frac{7}{2} \right) = (x+6)(2x-7)
\end{aligned}$$

$$\begin{aligned}
31 \quad 3x^2 + 13x - 30 &= 3 \left(x^2 + \frac{13}{3}x - 10 \right) \\
&= 3 \left\{ x^2 + \frac{13}{3}x + \frac{169}{36} - \left(\frac{169}{36} + 10 \right) \right\} \\
&= 3 \left\{ \left(x + \frac{13}{6} \right)^2 - \frac{529}{36} \right\} \\
&= 3 \left\{ \left(x + \frac{13}{6} \right)^2 - \left(\frac{23}{6} \right)^2 \right\} \\
&= 3 \left\{ \left(x + \frac{13}{6} \right) + \frac{23}{6} \right\} \left\{ \left(x + \frac{13}{6} \right) - \frac{23}{6} \right\} \\
&= 3(x+6) \left(x - \frac{5}{3} \right) = (x+6)(3x-5)
\end{aligned}$$

$$\begin{aligned}
32 \quad 12x^2 + x - 6 &= 12 \left(x^2 + \frac{x}{12} - \frac{1}{2} \right) \\
&= 12 \left\{ x^2 + \frac{x}{12} + \frac{1}{576} - \left(\frac{1}{576} + \frac{1}{2} \right) \right\} \\
&= 12 \left\{ \left(x + \frac{1}{24} \right)^2 - \frac{289}{576} \right\} \\
&= 12 \left\{ \left(x + \frac{1}{24} \right)^2 - \left(\frac{17}{24} \right)^2 \right\} \\
&= 12 \left\{ \left(x + \frac{1}{24} \right) + \frac{17}{24} \right\} \left\{ \left(x + \frac{1}{24} \right) - \frac{17}{24} \right\} \\
&= 12 \left(x + \frac{3}{4} \right) \left(x - \frac{2}{3} \right) = (4x+3)(3x-2)
\end{aligned}$$

$$\begin{aligned}
 33 \quad 2a^3 + 7ab - 15b^3 &= 2 \left(a^3 + \frac{7}{2}b - \frac{15}{2}b^3 \right) \\
 &= 2 \left\{ a^3 + \frac{7}{2}ab + \frac{49}{16}b^3 - \left(\frac{49}{16} + \frac{15}{2} \right) b^3 \right\} \\
 &= 2 \left\{ \left(a + \frac{7}{4}b \right)^3 - \frac{169b^3}{16} \right\} \\
 &= 2 \left\{ \left(a + \frac{7}{4}b \right)^3 - \left(\frac{13b}{4} \right)^3 \right\} \\
 &= 2 \left\{ \left(a + \frac{7}{4}b \right) + \frac{13b}{4} \right\} \left\{ \left(a + \frac{7}{4}b \right) - \frac{13b}{4} \right\} \\
 &= 2(a + 5b)(a - \frac{1}{2}b) = (a + 5b)(2a - 3b)
 \end{aligned}$$

$$\begin{aligned}
 34 \quad 6x^3 - 13xy^2 + 6y^3 &= 6 \left(x^3 - \frac{13}{6}xy^2 + y^3 \right) \\
 &= 6 \left\{ x^3 - \frac{13}{6}xy^2 + \frac{169}{144}y^3 - \left(\frac{169}{144} - 1 \right) y^3 \right\} \\
 &= 6 \left\{ \left(x - \frac{13}{12}y \right)^3 - \frac{25}{144}y^3 \right\} \\
 &= 6 \left\{ \left(x - \frac{13}{12}y \right)^3 - \left(\frac{5}{12}y \right)^3 \right\} \\
 &= 6 \left\{ \left(x - \frac{13}{12}y + \frac{5}{12}y \right) \left\{ \left(x - \frac{13}{12}y \right) - \frac{5}{12}y \right\} \right\} \\
 &= 6 \left(x - \frac{2}{3}y \right) \left(x - \frac{3}{2}y \right) = (3x - 2y)(2x - 3y)
 \end{aligned}$$

$$\begin{aligned}
 35 \quad 6m^3 - 11mn^2 - 10n^3 &= 6 \left(m^3 - \frac{11}{6}mn^2 - \frac{5}{3}n^3 \right) \\
 &= 6 \left\{ m^3 - \frac{11}{6}mn^2 + \frac{121}{144}n^3 - \left(\frac{121}{144} + \frac{5}{3} \right) n^3 \right\} \\
 &= 6 \left\{ \left(m - \frac{11}{12}n \right)^3 - \frac{10}{144}n^3 \right\} \\
 &= 6 \left\{ \left(m - \frac{11}{12}n \right)^3 - \left(\frac{5}{12}n \right)^3 \right\} \\
 &= 6 \left\{ \left(m - \frac{11}{12}n \right) + \frac{5}{12}n \right\} \left\{ \left(m - \frac{11}{12}n \right) - \frac{5}{12}n \right\} \\
 &= 6 \left(m + \frac{2}{3}n \right) \left(m - \frac{5}{2}n \right) = (3m + 2n)(2m - 5n)
 \end{aligned}$$

$$\begin{aligned}
 36 \quad 3p^3 + 5pq - 12q^3 &= 3 \left(p^3 + \frac{5}{3}pq - 4q^3 \right) \\
 &= 3 \left\{ p^3 + \frac{5}{3}pq + \frac{25}{9}q^3 - \left(\frac{25}{9} + 4 \right) q^3 \right\} \\
 &= 3 \left\{ \left(p + \frac{5}{3}q \right)^3 - \frac{109}{9}q^3 \right\} \\
 &= 3 \left\{ \left(p + \frac{5}{3}q \right)^3 - \left(\frac{13}{3}q \right)^3 \right\} \\
 &= 3 \left\{ \left(p + \frac{5}{3}q \right) + \frac{13}{3}q \right\} \left\{ \left(p + \frac{5}{3}q \right) - \frac{13}{3}q \right\} \\
 &= 3(p + 3q)(p - 4q) = (p + 3q)(3p - 4q)
 \end{aligned}$$

$$\begin{aligned}
 37 \quad 8a^3 - 14ab - 15b^3 &= 8(a^3 - \frac{7}{4}ab - \frac{15}{8}b^3) \\
 &= 8(a^3 - \frac{7}{4}ab - \frac{15}{8}b^3) \\
 &= 8\{a^3 - \frac{7}{4}ab + \frac{15}{8}b^3 - (\frac{1}{8}a + \frac{1}{8}b)^3\} \\
 &= 8\{(a - \frac{7}{8}b)^3 - \frac{15}{8}b^3\} \\
 &= 8\{(a - \frac{7}{8}b) - (\frac{1}{8}b)^3\} \\
 &= 8\{(a - \frac{7}{8}b) + \frac{1}{8}b\}\{(a - \frac{7}{8}b) - \frac{1}{8}b\} \\
 &= 8(a + \frac{1}{4}b)(a - \frac{5}{4}b) = (4a + 3b)(2a - 5b)
 \end{aligned}$$

$$\begin{aligned}
 38 \quad 10m^2 + 11mn - 6n^2 &= 10(m^2 + \frac{11}{10}mn - \frac{3}{5}n^2) \\
 &= 10\{m^2 + \frac{11}{10}mn + \frac{121}{100}n^2 - (\frac{11}{10}m + \frac{1}{10}n)^2\} \\
 &= 10\{(m + \frac{11}{10}n) - \frac{1}{10}n\}^2 \\
 &= 10\{(m + \frac{11}{10}n)^2 - (\frac{1}{10}n)^2\} \\
 &= 10\{(m + \frac{11}{10}n) + \frac{1}{10}n\}\{(m + \frac{11}{10}n) - \frac{1}{10}n\} \\
 &= 10(m + \frac{1}{2}n)(m - \frac{1}{10}n) = (2m + 3n)(5m - 2n)
 \end{aligned}$$

$$\begin{aligned}
 39 \quad 12x^2 + 13xy - 4y^2 &= 12(x^2 + \frac{13}{12}xy - \frac{1}{3}y^2) \\
 &= 12\{x^2 + \frac{13}{12}xy + \frac{169}{144}y^2 - (\frac{13}{12}x + \frac{1}{12}y)^2\} \\
 &= 12\{(x + \frac{13}{12}y)^2 - \frac{1}{144}y^2\} = 12\{(x + \frac{13}{12}y) - \frac{1}{12}y\}^2 \\
 &= 12\{(x + \frac{13}{12}y) + \frac{1}{12}y\}\{(x + \frac{13}{12}y) - \frac{1}{12}y\} \\
 &= 12(x + \frac{14}{3}y)(x - \frac{1}{3}y) = (3x + 4y)(4x - y)
 \end{aligned}$$

$$\begin{aligned}
 40 \quad 15a^3 - 11ab - 12b^3 &= 15(a^3 - \frac{11}{15}ab - \frac{4}{5}b^3) \\
 &= 15\{a^3 - \frac{11}{15}ab + \frac{121}{225}b^3 - (\frac{11}{15}a + \frac{1}{15}b)^3\} \\
 &= 15\{(a - \frac{11}{15}b)^3 - \frac{121}{225}b^3\} \\
 &= 15\{(a - \frac{11}{15}b)^3 - (\frac{11}{15}b)^3\} \\
 &= 15\{(a - \frac{11}{15}b) + \frac{11}{15}b\}\{(a - \frac{11}{15}b) - \frac{11}{15}b\} \\
 &= 15(a + \frac{11}{5}b)(a - \frac{1}{5}b) = (5a + 3b)(3a - 4b)
 \end{aligned}$$

$$\begin{aligned}
 41 \quad 2a^3 - 5ab + 2b^3 &= 2(a^3 - \frac{5}{2}ab + b^3) \\
 &= 2\{a^3 - \frac{5}{2}ab + \frac{25}{4}b^3 - (\frac{1}{2}a - \frac{1}{2}b)^3\} \\
 &= 2\{(a - \frac{5}{4}b)^3 - \frac{25}{4}b^3\} \\
 &= 2\{(a - \frac{5}{4}b)^3 - (\frac{5}{4}b)^3\} \\
 &= 2\{(a - \frac{5}{4}b) + \frac{5}{4}b\}\{(a - \frac{5}{4}b) - \frac{5}{4}b\} \\
 &= 2(a - \frac{1}{4}b)(a - 2b) = (2a - b)(a - 2b)
 \end{aligned}$$

$$\begin{aligned}
 42 \quad 3a^2 - 8ab - 3b^2 &= 3(a^2 - \frac{8}{3}ab - b^2) \\
 &= 3\{a^2 - \frac{8}{3}ab + \frac{64}{9}b^2 - (\frac{4}{3}a + \frac{1}{3}b)^2\} \\
 &= 3\{(a - \frac{4}{3}b)^2 - \frac{64}{9}b^2\}
 \end{aligned}$$

$$\begin{aligned}
 &= 3\{(a - \frac{1}{3}b)^2 - (\frac{2}{3}b)^2\} \\
 &= 3\{(a - \frac{1}{3}b) + \frac{2}{3}b\}\{(a - \frac{1}{3}b) - \frac{2}{3}b\} \\
 &= 3(a - \frac{1}{3}b)(a - \frac{2}{3}b) = (3a + b)(a - 3b),
 \end{aligned}$$

$$\begin{aligned}
 43 \quad 3x^2 + 8xy - 3y^2 &= 3(x^2 - \frac{8}{3}xy - y^2) \\
 &= 3\{x^2 - \frac{8}{3}xy + \frac{16}{9}y^2 - (\frac{1}{3} - 1)y^2\} \\
 &= 3\{(x + \frac{4}{3}y)^2 - \frac{25}{9}y^2\} \\
 &= 3\{(x + \frac{4}{3}y)^2 - (\frac{5}{3}y)^2\} \\
 &= 3\{(x + \frac{4}{3}y) - \frac{5}{3}y\}\{(x + \frac{4}{3}y) + \frac{5}{3}y\} \\
 &= 3(x + \frac{4}{3}y)(x - \frac{1}{3}y) = (x + 3y)(3x - y).
 \end{aligned}$$

$$\begin{aligned}
 44 \quad 4a^2 - 15a + 4 &= 4(a^2 - \frac{15}{4}a + 1) \\
 &= 4\{a^2 - \frac{15}{4}a + \frac{225}{64} - (\frac{225}{64} + 1)\} \\
 &= 4\{(a - \frac{15}{8})^2 - \frac{67}{64}\} \\
 &= 4\{(a - \frac{15}{8})^2 - (\frac{1}{8})^2\} \\
 &= 4\{(a - \frac{15}{8}) - \frac{1}{8}\}\{(a - \frac{15}{8}) + \frac{1}{8}\} \\
 &= 4(a - \frac{1}{4})(a - \frac{1}{4}) = (a - \frac{1}{4})(4a - 1)
 \end{aligned}$$

$$\begin{aligned}
 45 \quad 1a^2 - 17ab + 4b^2 &= 1(a^2 - \frac{17}{4}ab - b^2) \\
 &= 1\{a^2 - \frac{17}{4}ab - \frac{289}{64}b^2 - (\frac{289}{64} - 1)b^2\} \\
 &= 1\{(a - \frac{17}{8}b)^2 - \frac{81}{64}b^2\} \\
 &= 1\{(a - \frac{17}{8}b)^2 - (\frac{9}{8}b)^2\} \\
 &= 1\{(a - \frac{17}{8}b) - \frac{9}{8}b\}\{(a - \frac{17}{8}b) + \frac{9}{8}b\} \\
 &= 1(a - \frac{1}{4}b)(a - \frac{1}{4}b) = (4a - b)(a - 4b)
 \end{aligned}$$

$$\begin{aligned}
 46 \quad 5x^2 - 22x + 5 &= 5(x^2 - \frac{22}{5}x + 1) \\
 &= 5\{x^2 - \frac{22}{5}x + \frac{121}{25} - (\frac{121}{25} + 1)\} \\
 &= 5\{(x - \frac{11}{5})^2 - \frac{16}{5}\} \\
 &= 5\{(x - \frac{11}{5})^2 - (\frac{4}{5})^2\} \\
 &= 5\{(x - \frac{11}{5}) - \frac{4}{5}\}\{(x - \frac{11}{5}) + \frac{4}{5}\} \\
 &= 5(x - \frac{7}{5})(x - 5) = (5x + 1)(x - 5).
 \end{aligned}$$

$$\begin{aligned}
 47 \quad 5x^2 - 26x + 5 &= 5(x^2 - \frac{26}{5}x + 1) \\
 &= 5\{x^2 - \frac{26}{5}x + \frac{169}{25} - (\frac{169}{25} - 1)b^2\} \\
 &= 5\{(x - \frac{13}{5})^2 - \frac{144}{25}\} \\
 &= 5\{(x - \frac{13}{5})^2 - (\frac{12}{5})^2\} \\
 &= 5\{(x - \frac{13}{5}) - \frac{12}{5}\}\{(x - \frac{13}{5}) + \frac{12}{5}\} \\
 &= 5(x - \frac{1}{5})(x - 5) = (5x - 1)(x - 5).
 \end{aligned}$$

- 48 $6x^2 + 37x + 6 = 6\left(x^2 + \frac{37}{6}x + 1\right)$
 $= 6\left\{x^2 + \frac{1}{6}x + \frac{1}{36}x^2 - \left(\frac{1}{36}x^2 - 1\right)\right\}$
 $= 6\left\{\left(x + \frac{1}{6}\right)^2 - \frac{1}{36}\right\}$
 $= 6\left\{\left(x + \frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^2\right\}$
 $= 6\left\{\left(x + \frac{1}{6}\right) + \frac{1}{6}\right\}\left\{\left(x + \frac{1}{6}\right) - \frac{1}{6}\right\}$
 $= 6(x + 1)(x + \frac{1}{6}) = (x + 6)(6x + 1)$
- 49 $6a^2 + 35ab - 6b^2 = 6\{a^2 + \frac{35}{6}ab - b^2\}$
 $= 6\{a^2 + \frac{5}{6}ab + \frac{1}{4}a^2b^2 - (\frac{1}{4}a^2b^2 + 1)b^2\}$
 $= 6\{(a + \frac{1}{2}b)^2 - \frac{1}{4}b^2\}$
 $= 6\{(a + \frac{1}{2}b)^2 - (\frac{1}{2}b)^2\}$
 $= 6\{(a + \frac{1}{2}b) + \frac{1}{2}b\}\{(a + \frac{1}{2}b) - \frac{1}{2}b\}$
 $= 6(a + b)(a - \frac{1}{2}b) = (a + 2b)(6a - b)$
- 50 $6a^2 - 35ab - 6b^2 = 6\{a^2 - \frac{35}{6}ab - b^2\}$
 $= 6\{a^2 - \frac{5}{6}ab + \frac{1}{4}a^2b^2 - (\frac{1}{4}a^2b^2 + 1)b^2\}$
 $= 6\{(a - \frac{1}{2}b)^2 - \frac{1}{4}b^2\}$
 $= 6\{(a - \frac{1}{2}b)^2 - (\frac{1}{2}b)^2\}$
 $= 6\{(a - \frac{1}{2}b) + \frac{1}{2}b\}\{(a - \frac{1}{2}b) - \frac{1}{2}b\}$
 $= 6(a + \frac{1}{2}b)(a - b) = (6a + b)(a - b)$
- 51 $7a^2 - 50ab + 7b^2 = 7\{a^2 - \frac{50}{7}ab + b^2\}$
 $= 7\{a^2 - \frac{5}{7}ab + \frac{25}{49}b^2 - (\frac{25}{49}b^2 - 1)b^2\}$
 $= 7\{(a - \frac{5}{7}b)^2 - \frac{25}{49}b^2\}$
 $= 7\{(a - \frac{5}{7}b)^2 - (\frac{5}{7}b)^2\}$
 $= 7\{(a - \frac{5}{7}b) + \frac{5}{7}b\}\{(a - \frac{5}{7}b) - \frac{5}{7}b\}$
 $= 7(a - b)(a - 7b) = (7a - b)(a - 7b)$
- 52 $7a^2 + 48ab - 7b^2 = 7\{a^2 + \frac{48}{7}ab - b^2\}$
 $= 7\{a^2 + \frac{4}{7}ab + \frac{16}{49}b^2 - (\frac{16}{49}b^2 + 1)b^2\}$
 $= 7\{(a + \frac{4}{7}b)^2 - \frac{16}{49}b^2\}$
 $= 7\{(a + \frac{4}{7}b)^2 - (\frac{4}{7}b)^2\}$
 $= 7\{(a + \frac{4}{7}b) + \frac{4}{7}b\}\{(a + \frac{4}{7}b) - \frac{4}{7}b\}$
 $= 7(a + b)(a - \frac{4}{7}b) = (a + 7b)(7a - b)$
- 53 $7a^2 - 48ab - 7b^2 = 7\{a^2 - \frac{48}{7}ab - b^2\}$
 $= 7\{a^2 - \frac{4}{7}ab + \frac{16}{49}b^2 - (\frac{16}{49}b^2 + 1)b^2\}$
 $= 7\{(a - \frac{4}{7}b)^2 - \frac{16}{49}b^2\}$

$$\begin{aligned}
 &= 7\{(a - \frac{2}{7}b)^2 - (\frac{2}{7}b)^2\} \\
 &= 7\{(a - \frac{2}{7}b) + \frac{2}{7}b\}\{(a - \frac{2}{7}b) - \frac{2}{7}b\} \\
 &= 7(a + \frac{1}{7}b)(a - \frac{7}{7}b) = 7(a + b)(a - b)
 \end{aligned}$$

$$\begin{aligned}
 54 \quad 8x^2 + 63xy - 8y^2 &= 8(x^2 + \frac{9}{8}xy - y^2) \\
 &= 8\{x^2 + \frac{9}{8}xy + \frac{81}{64}y^2 - (\frac{81}{64}y^2 + 1)y^2\} \\
 &= 8\{(x + \frac{9}{8}y)^2 - \frac{1}{2}xy^2\} \\
 &= 8\{(x + \frac{9}{8}y)^2 - (\frac{1}{2}xy)^2\} \\
 &= 8\{(x + \frac{9}{8}y) + \frac{1}{2}y\}\{(x + \frac{9}{8}y) - \frac{1}{2}y\} \\
 &= 8(x + 8y)(x - \frac{1}{2}y) = (1 + 8y)(8x - y)
 \end{aligned}$$

$$\begin{aligned}
 55 \quad 9x^2 - 82xy + 9y^2 &= 9(x^2 - \frac{82}{9}xy + y^2) \\
 &= 9\{x^2 - \frac{82}{9}xy + \frac{164}{81}y^2 - (\frac{164}{81}y^2 - 1)y^2\} \\
 &= 9\{(x - \frac{41}{9}y)^2 - \frac{164}{81}y^2\} \\
 &= 9\{(x - \frac{41}{9}y) - (\frac{41}{9}y)^2\} \\
 &= 9\{(x - \frac{41}{9}y) + \frac{41}{9}y\}\{(x - \frac{41}{9}y) - \frac{41}{9}y\} \\
 &= 9(x - \frac{1}{9}y)(x - 9y) = (9x - y)(x - 9y)
 \end{aligned}$$

$$\begin{aligned}
 56 \quad 10x^2 + 99xy - 10y^2 &= 10(x^2 + \frac{99}{10}xy - y^2) \\
 &= 10\{x^2 + \frac{99}{10}xy + \frac{9801}{100}y^2 - (\frac{9801}{100}y^2 + 1)y^2\} \\
 &= 10\{(x + \frac{99}{10}y) - \frac{19801}{100}y^2\} \\
 &= 10\{(x + \frac{99}{10}y)^2 - (\frac{19801}{100}y^2)\} \\
 &= 10\{(x + \frac{99}{10}y) + (\frac{19801}{100}y)\}\{(x + \frac{99}{10}y) - \frac{19801}{100}y\} \\
 &= 10(x + 10y)(x - \frac{1}{10}y) = (1 + 10y)(10x - y)
 \end{aligned}$$

$$\begin{aligned}
 57 \quad \text{Putting } x \text{ for } (a+b), \text{ the given expression} \\
 &= 2x^2 + 3x - 2 = 2(x + \frac{1}{2} - 1) \\
 &= 2\{x + \frac{1}{2} + \frac{1}{4} - (\frac{9}{4} + 1)\} \\
 &= 2\{(x + \frac{1}{2})^2 - \frac{5}{4}\} \\
 &= 2\{(x + \frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2\} \\
 &= 2\{(x + \frac{1}{2}) + \frac{\sqrt{5}}{2}\}\{(x + \frac{1}{2}) - \frac{\sqrt{5}}{2}\} \\
 &= 2(x + 2)(x - \frac{1}{2}) \\
 &= (x + 2)(2x - 1) = (a + b + 2)(2a + 2b - 1)
 \end{aligned}$$

$$\begin{aligned}
 58 \quad \text{Putting } a \text{ for } (x^2 + y^2) \text{ and } b \text{ for } xy, \text{ the given expression} \\
 &= 2a^2 - 3ab - 2b^2 = 2(a^2 - \frac{3}{2}ab - b^2) \\
 &= 2\{a^2 - \frac{3}{2}ab + \frac{9}{4}b^2 - (\frac{9}{4}b^2 + 1)b^2\} \\
 &= 2\{(a - \frac{3}{2}b)^2 - \frac{5}{4}b^2\} \\
 &= 2\{(a - \frac{3}{2}b)^2 - (\frac{\sqrt{5}}{2}b)^2\}
 \end{aligned}$$

$$\begin{aligned}
 &= 2\{(a - \frac{1}{4}b) + \frac{5}{4}b\}\{(a - \frac{1}{4}b) - \frac{1}{4}b\} \\
 &= 2(a + \frac{1}{2}b)(a - \frac{1}{2}b) \\
 &= (2a + b)(a - b) \\
 &= (2x^2 + 2y^2 + 2xy)(x^2 + y^2 - 2xy) \\
 &= (2x^2 + 2y^2 + 2xy)(x - y)^2
 \end{aligned}$$

59 Putting x for $(a^2 + b^2)$ and y for ab , the given expression

$$\begin{aligned}
 &= 2x^2 + 5xy + 2y^2 = 2(x^2 + \frac{5}{2}xy + y^2) \\
 &= 2\{x^2 + \frac{5}{2}xy + \frac{25}{4}y^2 - (\frac{5}{4}y - 1)y^2\} \\
 &= 2\{(x + \frac{5}{4}y)^2 - (\frac{1}{4}y)^2\} \\
 &= 2\{(x + \frac{5}{4}y)^2 - (\frac{1}{4}y)^2\} \\
 &= 2\{(x + \frac{5}{4}y) + \frac{1}{4}y\}\{(x + \frac{5}{4}y) - \frac{1}{4}y\} \\
 &= 2(x + 2y)(x + y) \\
 &= (x + 2y)(2x + y) \\
 &= (a^2 + b^2 + 2ab)(2a^2 + 2b^2 + ab) \\
 &= (a + b)^2(2a^2 + 2b^2 + ab)
 \end{aligned}$$

60 Putting a for $(x^2 - 4xy + y^2)$ and b for xy , the given expression

$$\begin{aligned}
 &= 4a^2 + 15ab - 4b^2 \\
 &= 4\{a^2 + \frac{15}{4}ab - b^2\} \\
 &= 4\{a^2 + \frac{15}{4}ab + \frac{225}{16}b^2 - (\frac{15}{4}b + 1)b^2\} \\
 &= 4\{(a + \frac{15}{8}b)^2 - \frac{25}{16}b^2\} \\
 &= 4\{(a + \frac{15}{8}b)^2 - (\frac{5}{4}b)^2\} \\
 &= 4\{(a + \frac{15}{8}b) + \frac{5}{4}b\}\{(a + \frac{15}{8}b) - \frac{5}{4}b\} \\
 &= 4(a + 4b)(a - \frac{1}{4}b) \\
 &= (a + 4b)(4a - b) \\
 &= (x^2 - 4xy + y^2 + 4xy)(4x^2 - 16xy + 4y^2 - 4xy) \\
 &= (x^2 + y^2)(4x^2 - 12xy + 4y^2) \\
 &= (x^2 + y^2)4(x - \frac{1}{2}y)(x - y) \\
 &= (x^2 + y^2)(4x - y)(x - y)
 \end{aligned}$$

61 $2x^4 - 5x^2 - 12 = 2(x^4 - \frac{5}{2}x^2 - 6)$

$$\begin{aligned}
 &= 2\{x^4 - \frac{5}{2}x^2 + \frac{25}{4} - (\frac{25}{4} + 6)\} \\
 &= 2\{(x^2 - \frac{5}{4})^2 - \frac{121}{16}\} = 2\{(x^2 - \frac{5}{4})^2 - (\frac{11}{4})^2\} \\
 &= 2\{(x^2 - \frac{5}{4}) + \frac{11}{4}\}\{(x^2 - \frac{5}{4}) - \frac{11}{4}\} \\
 &= 2(x^2 + \frac{1}{2})(x^2 - 4) = (2x^2 + 3)(x + 2)(x - 2)
 \end{aligned}$$

$$\begin{aligned}
 62 \quad 8a^4 - 14a^2b^2 - 9b^4 &= 8(a^4 - \frac{7}{4}a^2b^2 - \frac{9}{8}b^4) \\
 &= 8\{a^4 - \frac{7}{4}a^2b^2 + \frac{49}{16}b^4 - b^4(\frac{49}{16} + \frac{9}{8})\} \\
 &= 8\{(a^2 - \frac{7}{8}b^2)^2 - \frac{1}{8}b^4\} \\
 &= 8\{(a^2 - \frac{7}{8}b^2)^2 - (\frac{1}{8}b^2)^2\} \\
 &= 8\{(a^2 - \frac{7}{8}b^2) + \frac{1}{8}b^2\}\{(a^2 - \frac{7}{8}b^2) - \frac{1}{8}b^2\} \\
 &= 8(a^2 + \frac{1}{2}b^2)(a^2 - \frac{9}{4}b^2) \\
 &= (2a^2 + b^2)(4a^2 - 9b^2) \\
 &= (2a^2 + b^2)(2a + 3b)(2a - 3b)
 \end{aligned}$$

$$\begin{aligned}
 63 \quad 9a^4 + 2a^2b^2 - 32b^4 &= 9(a^4 + \frac{2}{9}a^2b^2 - \frac{32}{9}b^4) \\
 &= 9\{a^4 + \frac{2}{9}a^2b^2 + \frac{4}{81}b^4 - b^4(\frac{4}{81} + \frac{32}{9})\} \\
 &= 9\{(a^2 + \frac{1}{9}b^2)^2 - \frac{32}{9}b^4\} \\
 &= 9\{(a^2 + \frac{1}{9}b^2)^2 - (\frac{4}{3}b^2)^2\} \\
 &= 9\{(a^2 + \frac{1}{9}b^2) + \frac{4}{3}b^2\}\{(a^2 + \frac{1}{9}b^2) - \frac{4}{3}b^2\} \\
 &= 9(a^2 + 2b^2)(a^2 - \frac{11}{9}b^2) \\
 &= (a^2 + 2b^2)(9a^2 - 11b^2) \\
 &= (a^2 + 2b^2)(3a + 4b)(3a - 4b)
 \end{aligned}$$

$$\begin{aligned}
 64 \quad 8x^6 - 65x^3 + 8 &= 8(x^6 - \frac{65}{8}x^3 + 1) \\
 &= 8\{x^6 - \frac{65}{8}x^3 + \frac{49}{64}x^3 - (\frac{49}{64}x^3 - 1)\} \\
 &= 8\{(x^3 - \frac{1}{8}x)^2 - \frac{3}{2}\frac{49}{64}\} \\
 &= 8\{(x^3 - \frac{1}{8}x)^2 - (\frac{3}{4}\frac{7}{8})^2\} \\
 &= 8\{x^3 - \frac{1}{8}x + \frac{7}{8}\}\{x^3 - \frac{1}{8}x - \frac{7}{8}\} \\
 &= 8(x^3 - \frac{1}{4})(x^3 - 8) \\
 &= (8x^3 - 1)(x^3 - 8) \\
 &= (2x - 1)(4x^2 + 2x + 1)(x - 2)(x^2 + 2x + 4)
 \end{aligned}$$

$$\begin{aligned}
 65 \quad 4a^8 - 17a^4b^4 + 4b^8 &= 4(a^8 - \frac{17}{4}a^4b^4 + b^8) \\
 &= 4\{a^8 - \frac{17}{4}a^4b^4 + \frac{289}{64}b^8 - (\frac{289}{64}b^8 - 1)b^8\} \\
 &= 4\{(a^4 - \frac{17}{8}b^4)^2 - \frac{289}{64}b^8\} \\
 &= 4\{(a^4 - \frac{17}{8}b^4)^2 - (\frac{17}{8}b^4)^2\} \\
 &= 4\{(a^4 - \frac{17}{8}b^4 + \frac{17}{8}b^4)(a^4 - \frac{17}{8}b^4 - \frac{17}{8}b^4)\} \\
 &= 4(a^4 - \frac{1}{4}b^4)(a^4 - 4b^4) \\
 &= (4a^4 - b^4)(a^4 - 4b^4) \\
 &= (2a^2 + b^2)(2a^2 - b^2)(a^2 + 2b^2)(a^2 - 2b^2)
 \end{aligned}$$

Exercise 46.

- 1 $r^2 + r + 1 + 1 = r^2(r+1) + (r+1) = (r^2+1)(r+1)$
- 2 $r^4 + r^2 - r - 1 = r^2(r+1) - (r+1)$
 $= (r+1)(r^2-1) = (r+1)(r+1)(r-1)$
 $= (r+1)^2(r-1)$
- 3 $x^3 - x^2 - r + 1 = r^2(x-1) - (r-1)$
 $= (r-1)(r^2-1) = (r-1)(r+1)(r-1)$
 $= (r-1)^2(r+1)$
- 4 $bc(a^2+1) + a(b^2+c^2) = bca^2 + ab^2 + ac^2 + b$
 $= ab(ac+b) + c(ac+b) = (ab+c)(a+b)$
- 5 $x^4 - at^3 + xbt - x^2a = r^4 + 2bt^2 - ab^2 - x^2a$
 $= r(r^2+b) - a(r^2+b^2)$
 $= (r-a)(r^2+b) = (r-ayr+b)(r^2-1b+bt^2)$
- 6 $ab(r^2+j^2) + rj(a^2+b^2) = abr^2 + rja^2 + xjb^2 + abj^2$
 $= ar(b+aj) + bj(b+aj)$
 $= (ar+bj)(br+aj)$
- 7 $x^2 + xy - yz - z^2 = r^2 - z^2 + xy - yz$
 $= (r+z)(r-z) + j(r-z) = (r+j+z)(r-z)$
- 8 $xb - ac - rc + ab = rb - rc + ab - a$
 $= r(b-c) + a(b-c) = (r+a)(b-c)$
- 9 $(2x^2+3b^2)a - (2a^2+3r^2)b$
 $= 2ar^2 - 3x^2b - 2a^2b + 3ab^2$
 $= r^2(2a-3b) - ab(2a-3b)$
 $= (x^2-ab)(2a-3b)$
- 10 $a(a+c) - b(b+c) = a^2 - b^2 + ac - bc$
 $= (a+b)(a-b) + c(a-b)$
 $= (a-b)(a+b+c)$
- 11 $4a^3 + 8ac - 12b - 9b^2$
 $= 4a^2 - 9b^2 + 4c(2a-3b)$
 $= (2a+3b)(2a-3b) + 4c(2a-3b)$
 $= (2a-3b)(2a+3b+4c)$

- 12 $a^2x^2 + acxz - b^2y^2 - bcyz$
 $= (ax + by)(ax - by) + cz(ax - by)$
 $= (a_1 - by)(a_1 + by + cz)$
- 13 $x^4 - y^2z + y^2x^2 - y^2z^2$
 $= x^4 - y^2z^2 + y^2x^2 - y^2z$
 $= (x^2 - y^2)(x^2 + y^2) + y^2(x^2 - y^2)$
 $= (x^2 - y^2)(x^2 + y^2 + y^2)$
- 14 $16x^2 - 15ab + 12bx - 25a^2$
 $= 16x^2 - 25a^2 - 12bx - 15ab$
 $= (4x + 5a)(4x - 5a) + 3b(4x - 5a)$
 $= (4x - 5a)(4x + 5a + 3b)$
- 15 $a^2(a + 2b) + b^2(2a + b)$
 $= a^3 + 2a^2b + 2ab^2 + b^3$
 $= a^3 + b^3 + 2a^2b + 2ab^2$
 $= (a + b)(a^2 - ab + b^2) + 2ab(a + b)$
 $= (a + b)(a^2 + ab + b^2)$
- 16 $m^3 - 2m^2n + 2mn^2 - n^3$
 $= m^3 - n^3 - 2m^2n + 2mn^2$
 $= (m - n)(m^2 + mn + n^2) - 2mn(m - n)$
 $= (m - n)(m^2 - mn + n^2)$
- 17 $a^4 + 2a^3b - 2ab^3 - b^4 = a^4 - b^4 + 2a^3b - 2ab^3$
 $= (a^2 - b^2)(a^2 + b^2) + 2ab(a^2 - b^2)$
 $= (a^2 - b^2)(a^2 + b^2 + 2ab)$
 $= (a - b)(a + b)(a + b)^2 = (a - b)(a + b)^3$
- 18 $x^3(1 - y) + y^3(2x - y) = x^4 - y^4 - 2x^3y + 2xy^3$
 $= (x^2 - y^2)(x^2 + y^2) - 2xy(x^2 - y^2)$
 $= (x^2 - y^2)(x^2 + y^2 - 2xy)$
 $= (x - y)(x + y)(x - y)^2 = (x - y)^3(x + y)$
- 19 $x^2 + 5a^2 + 10a + 8 = a^2 + 8 + 5a^2 + 10a$
 $= (a + 2)(a^2 - 2a + 4) + 5a(a + 2)$
 $= (a + 2)(a^2 + 3a + 4)$
- 20 $x^3 - 17x^2 + 85x - 125 = x^3 - 125 - 17x^2 + 85x$
 $= (x - 5)(x^2 + 5x + 25) - 17x(x - 5)$
 $= (x - 5)(x^2 - 12x + 25)$

- 21 $8a^3 + 18a^2b - 27ab^2 - 27b^3$
 $= 8a^3 - 27b^3 + 18a^2b - 27ab^2$
 $= (2a - 3b)(4a^2 + 6ab + 9b^2) + 9ab(2a - 3b)$
 $= (2a - 3b)(4a^2 + 15ab + 9b^2)$
 $= (2a - 3b)(4a^2 + 12ab + 3ab + 9b^2)$
 $= (2a - 3b)\{4a(a + 3b) + 3b(a + 3b)\}$
 $= (2a - 3b)(a + 3b)(4a + 3b)$
- 22 $1^3 - 2xy + y^3 - 1 + y = (1 - y)^3 - (x - y)$
 $= (1 - y)(1 - y - 1)$
- 23 $4a^2 - 4ab + b^2 - 6a + 3b = (2a - b)^2 - 3(2a - b)$
 $= (2a - b)(2a - b - 3)$
- 24 $x^4 - 2ax^3 + 2a^2x^2 - 2a^3x + a^4$
 $= x^4 + 2a^2x^2 + a^4 - 2ax^3 - 2a^3x$
 $= (x^2 + a^2)^2 - 2ax(x^2 + a^2)$
 $= (x^2 + a^2)(x^2 + a^2 - 2ax) = (x^2 + a^2)(x - a)^2$
- 25 $a^4 - 3a^3b + 4a^2b^2 - 6ab^3 + 4b^4$
 $= a^4 + 4a^2b^2 + 4b^4 - 3a^3b - 6ab^3$
 $= (a^2 + 2b^2)^2 - 3ab(a^2 + 2b^2)$
 $= (a^2 + 2b^2)(a^2 + 2b^2 - 3ab)$
 $= (a^2 + 2b^2)(a - 2b)(a - b)$
- 26 $a^3 + 3ab^2 + 2b^3 + ac + 2bc$
 $= (a + b)(a + 2b) + c(a + 2b)$
 $= (a + 2b)(a + b + c)$
- 27 $x^2 - 4xy + 3y^2 + xz - 3yz$
 $= (x - y)(x - 3y) + z(x - 3y)$
 $= (x - 3y)(x - y + z)$
- 28 $m^3 + 2pm - 5mn - 4pn + 6n^2$
 $= m^3 + 2pm - 3mn - 2mn - 4pn + 6n^2$
 $= m(m + 2p - 3n) - 2n(m + 2p - 3n)$
 $= (m + 2p - 3n)(m - 2n)$
- 29 $a^2 - 10ab - 15bc + 21b^2 + 5ac$
 $= a^2 - 7ab + 5ac - 3ab + 21b^2 - 15bc$
 $= a(a - 7b + 5c) - 3b(a - 7b + 5c)$
 $= (a - 7b + 5c)(a - 3b)$

- 30 $2x^2 + 4a(4b - 3a) + x(4b + 5a)$
 $= 2x^2 - 3ax + 4bx + 8a^2 + 4a(4b - 3a)$
 $= 2(2x - 3a + 4b) + 4a(2x + 4b - 3a)$
 $= (2x - 3a + 4b)(x + 4a)$
- 31 $a^2 - 3a(2b - 1) + 4b(2b - 3)$
 $= a^2 - 2ab + 3a - 4ab + 4b(2b - 3)$
 $= a(a - 2b + 3) - 4b(a - 2b + 3)$
 $= (a - 2b + 3)(a - 4b)$
- 32 $3x(1 + 2) - 2y(4x - 1) - 3y^2$
 $= 3x^2 - 9xy + 6x + 1y - 3y^2 + 2y$
 $= 3x(1 - 3y + 2) + y(1 - 3y + 2)$
 $= (1 - 3y + 2)(3x + y)$
- 33 $a^2 - b^2 - c^2 - 2bc + a - b - c$
 $= a^2 - (b + c)^2 + (a - b - c)$
 $= (a + b + c)(a - b - c) + (a - b - c)$
 $= (a - b - c)(a + b + c + 1)$
- 34 $x^2 - 4y^2 - 9z^2 + 12yz + 4x - 8y + 12z$
 $= x^2 - (2y - 3z)^2 + 4(1 - 2y + 3z)$
 $= (x + 2y - 3z)(x - 2y + 3z) + 4(x - 2y + 3z)$
 $= (x - 2y + 3z)(x + 2y - 3z + 4)$
- 35 $9x^2 - 4z^2 - 24xy + 16y^2 + 20y - 15x + 10z$
 $= 9x^2 - 24xy + 16y^2 - 4z^2 - 15x + 20y + 10z$
 $= (3x - 4y)^2 - (2z)^2 - 5(3x - 4y - 2z)$
 $= (3x - 4y + 2z)(3x - 4y - 2z) - 5(3x - 4y - 2z)$
 $= (3x - 4y - 2z)(3x - 4y + 2z - 5)$
- 36 $2a^2x^4 - 5a^4x^2 + 3a^6 - 2b^2x^4 + 5a^2b^2x^2 - 3a^4b^2$
 $= 2a^2x^4 - 2b^2x^4 - 5a^4x^2 + 5a^2b^2x^2 + 3a^6 - 3a^4b^2$
 $= 2x^4(a^2 - b^2) - 5a^2x^2(a^2 - b^2) + 3a^4(a^2 - b^2)$
 $= (a^2 - b^2)(2x^4 - 5a^2x^2 + 3a^4)$
 $= (a^2 - b^2)(x^2 - a^2)(2x^2 - 3a^2)$
 $= (a + b)(a - b)(1 + a)(x - a)(2x^2 - 3a^2)$
- 37 $2x^2 + (2a - 3b)x^2 - (2b + 3ab)x + 3b^2$
 $= 2x^3 + 2ax^2 - 2bx - 3bx^2 - 3abx + 3b^2$

$$= 21(1^2 + a1 - b) - 3b(1^2 + a1 - b) \\ = (1^2 + a1 - b)(21 - 3b)$$

$$\begin{aligned} 38 \quad (a^2 + b^2)x^2 - a^2b(2a + b) + a(2bx^2 - a^2) \\ = (a^2 + b^2)x^2 + 2abx^2 - a^4 - a^2b(2a + b) \\ = x^2(a^2 + b^2 + 2ab) - a^2(a^2 + 2ab + b^2) \\ = (x^2 - a^2)(a^2 + b^2 + 2ab) - (1 + a)(x - a)(a + b)^2 \end{aligned}$$

$$\begin{aligned} 39 \quad a^2(b-c) + b^2(c-a) + c^2(a-b) \\ = a^2(b-c) - a(b^2 - c^2) + b^2c - bc^2 \\ = a^2(b-c) - a(b-c)(b+c) + bc(b-c) \\ = (b-c)\{a^2 - a(b+c) + bc\} \\ = (b-c)\{a(a-b) - c(a-b)\} \\ = (b-c)(a-b)(a-c) \end{aligned}$$

$$\begin{aligned} 40 \quad a^2(b-c) + b^2(c-a) + c^2(a+b) + 2abc \\ = a^2(b+c) + a(b^2 + c^2 + 2bc) + b^2c + bc^2 \\ = a^2(b+c) + a(b+c)^2 + bc(b+c) \\ = (b+c)\{a^2 + a(b+c) + bc\} \\ = (b+c)\{a(a+b) + c(a+b)\} \\ = (b+c)(a+b)(a+c) \end{aligned}$$

$$\begin{aligned} 41 \quad a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2) \\ = a^2(b^2 - c^2) - a^2(b^2 - c^2) + b^2c^2 - b^2c^2 \\ = a^2(b^2 - c^2) - a^2(b-c)(b^2 + bc + c^2) + b^2c^2(b-c) \\ = (b-c)\{a^2(b+c) - a^2(b^2 + bc + c^2) + b^2c^2\} \\ = (b-c)(a^2b - a^2b^2 + a^2c - a^2bc - a^2c^2 + b^2c^2) \\ = (b-c)\{a^2b(a-b) + a^2c(a-b) - c^2(a^2 + b^2)\} \\ = (b-c)(a-b)(a^2b + a^2c - ac^2 - bc^2) \\ = (b-c)(a-b)\{b(a^2 - c^2) + ac(a-c)\} \\ = (b-c)(a-b)(a-c)(ab + bc + ac) \end{aligned}$$

$$\begin{aligned} 42 \quad a^4(b-c) + b^4(c-a) + c^4(a-b) \\ = a^4(b-c) - a(b^4 - c^4) + bc(b^3 - c^3) \\ = (b-c)\{a^4 - a(b^4 + b^2c + bc^2 + c^3) + bc(b^2 + bc + c^2)\} \\ = (b-c)\{-b^4(a-c) - b^2c(a-c) - bc^2(a-c) + a(a^3 - c^3)\} \\ = (b-c)(a-c)\{-b^4 - b^2c - bc^2 + a(a^2 + ac + c^2)\} \\ = (b-c)(a-c)\{c^4(a-b) + c(a^2 - b^2) + (a^3 - b^3)\} \end{aligned}$$

$$\begin{aligned}
 &= (b-c)(a-c)(a-b)(c^3+ac+bc+a^2+ab+b^2) \\
 &= (b-c)(a-c)(a-b)(a^2+b^2+c^2+ab+ac+bc)
 \end{aligned}$$

$$\begin{aligned}
 43 \quad & a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2) \\
 &= a^4(b^2-c^2)-a^2(b^4-c^4)+b^2c^2(b^2-c^2) \\
 &= (b^2-c^2)\{a^4-a^2(b^2+c^2)+b^2c^2\} \\
 &= (b^2-c^2)(a^2-b^2)(a^2-c^2) \\
 &= (a-b)(a-c)(b-c)(a+b)(b+c)(c+a)
 \end{aligned}$$

$$\begin{aligned}
 44 \quad & a^5(b-c)+b^5(c-a)+c^5(a-b) \\
 &= a^5(b-c)-a(b^5-c^5)+bc(b^4-c^4) \\
 &= (b-c)\{a^5-a(b^4+b^3c+b^2c^2+bc^3+c^4) \\
 &\quad +bc(b^3+b^2c+bc^2+c^3)\} \\
 &= (b-c)\{-b^4(a-c)-b^3c(a-c)-b^2c^2(a-c) \\
 &\quad -bc^3(a-c)+a(a^4-c^4)\} \\
 &= (b-c)(a-c)\{-b^4-b^3c-b^2c^2-bc^3+ \\
 &\quad a(a^3+a^2c+ac^2+c^3)\} \\
 &= (b-c)(a-c)\{c^2(a-b)+c^2(a^2+b^2)+c(a^3-b^3) \\
 &\quad +(a^4-b^4)\} \\
 &= (b-c)(a-c)(a-b)\{c^3+c^2(a+b)+c(a^2+ab+b^2) \\
 &\quad +(a^3+a^2b+ab^2+b^3)\} \\
 &= (a-b)(a-c)(b-c)(a^4+b^3+c^3+a^2b+a^2c \\
 &\quad +b^2a+b^2c+c^2a+c^2b+abc)
 \end{aligned}$$

Exercise 47

$$\begin{aligned}
 1 \quad & a^3+b^3-c^3+3abc = \{(a+b)^3-c^3\}-3ab(a+b)+3abc \\
 &= (a+b-c)\{(a+b)^2+(a+b)c+c^2\}-3ab \\
 &\quad (a+b-c) \\
 &= (a+b-c)(a^2+b^2+c^2-ab+ac+bc),
 \end{aligned}$$

the required quotient $= a^2+b^2+c^2-ab+ac+bc$

$$\begin{aligned}
 2 \quad & x^3-y^3-1-3xy = (x-y)^3-1+3xy(x-y)-3xy \\
 &= \{(x-y)-1\}\{(x-y)^2+(x-y)+1\}+3xy \\
 &\quad (x-y-1) \\
 &= (x-y-1)(x^2+y^2-2xy+x-y+1+3xy) \\
 &= (x-y-1)(x^2+y^2+1+xy+x-y), \\
 &\text{the required quotient} = x^2+y^2+1+xy+x-y
 \end{aligned}$$

$$\begin{aligned}
 3 \quad x^3 - 8y^3 + 27z^3 + 18xyz &= x^3 - \{(2y - 3z)^3 + 3 \cdot 2y \cdot 3z(2y - 3z)\} + 18xyz \\
 &= \{1 - (2y - 3z)\} \{x^2 + x(2y - 3z) + (2y - 3z)^2\} \\
 &\quad + 18yz(1 - 2y + 3z) \\
 &= (1 - 2y + 3z) \{x^2 + 2xy - 3xz + 4y^2 + 9z^2 - 12yz + 18yz\} \\
 &= (x - 2y + 3z) \{x^2 + 4y^2 + 9z^2 + 2xy - 3xz + 6yz\}, \\
 \text{the required quotient} &= x^2 + 4y^2 + 9z^2 + 2xy - 3xz + 6yz
 \end{aligned}$$

$$\begin{aligned}
 4 \quad 8a^3 - 27b^3 - c^3 - 18abc &= (2a)^3 + (-3b)^3 + (-c)^3 - 3 \cdot 2a \cdot (-3b) \cdot (-c) \\
 &= (2a - 3b - c) \{4a^2 + 9b^2 + c^2 + 6ab - 3bc + 2ac\}, \\
 \text{the required quotient} &= 2a - 3b - c
 \end{aligned}$$

$$\begin{aligned}
 5 \quad x^6 - 26x^3 - 27 &= x^6 - 27x^3 + x^3 - 27 \\
 &= x^3(x^3 - 27) + (x^3 - 27) \\
 &= (x^3 - 27)(x^3 + 1) \\
 &= (x - 3)(x^2 + 3x + 9)(x + 1)(x^2 - x + 1), \\
 \text{the required quotient} &= (x - 3)(x + 1) = x^2 - 2x - 3
 \end{aligned}$$

$$\begin{aligned}
 6 \quad (a^2 - bc)^3 + 27b^2c^3 &= \{(a^2 - bc) + 3bc\} \{(a^2 - bc)^2 - 3bc(a^2 - bc) + 9b^2c^3\} \\
 &= (a^2 + 2bc)(a^4 + b^2c^3 - 2a^2bc - 3a^2bc + 3b^2c^3 + 9b^2c^3) \\
 &= (a^2 + 2bc)(a^4 - 5a^2bc + 13b^2c^3), \\
 \text{the required quotient} &= a^2 + 2bc
 \end{aligned}$$

$$\begin{aligned}
 7 \quad 125 - 8a^3 + b^3 + 30ab &= (5 - 2a)^3 + b^3 + 30a(5 - 2a) + 30ab \\
 &= (5 - 2a + b) \{(5 - 2a)^2 - b(5 - 2a) + b^2\} + 30a(5 - 2a + b) \\
 &= (5 - 2a + b)(25 + 4a^2 - 20a - 5b + 2ab + b^2) + 30a \\
 &= (5 - 2a + b) \{(a^2 + 2ab + b^2) + (a^2 + 10a + 25) + 2a^2 - 5b\} \\
 &= (5 - 2a + b) \{(a + b)^2 + (a + 5)^2 + 2a^2 - 5b\}, \\
 \text{the required quotient} &= 5 - 2a + b
 \end{aligned}$$

$$\begin{aligned}
 8 \quad 2x^3 - x^2 + x + 4 &= 2x^3 + 2x^2 - 3x^2 - 3x + 4x + 4 \\
 &= 2x^2(x + 1) - 3x(x + 1) + 4(x + 1) \\
 &= (x + 1)(2x^2 - 3x + 4), \\
 \text{the required quotient} &= 2x^2 - 3x + 4
 \end{aligned}$$

$$\begin{aligned}
 9 \quad x^4 - 4x^3 + 7x^2 - 11x + 10 &= x^4 - 4x^3 + 4x^2 + 3x^2 - 6x - 5x + 10 \\
 &= x^2(x - 2)^2 + 3x(x - 2) - 5(x - 2) \\
 &= (x - 2) \{x^2(x - 2) + 3x - 5\},
 \end{aligned}$$

$$\begin{aligned}\text{the required quotient} &= x^2(x-2) + 3x - 5 \\ &= x^3 - 2x^2 + 3x - 5\end{aligned}$$

$$\begin{aligned}10 \quad r^4 - 3r^3 - 13r^2 + 13r - 6 \\ &= r^4 + 3r^3 - 6r^3 - 18r^2 + 5r^2 + 15r - 2r - 6 \\ &= r^2(r+3) - 6r^2(r+3) + 5r(2+3) - 2(r+3) \\ &= (1+3)r^3 - 6r^3 + 5r - 2, \\ \text{the required quotient} &= 1 - 6r^2 + 5r - 2\end{aligned}$$

$$\begin{aligned}11 \quad x^3 + 4x - 39 &= x^3 - 3x^2 + 3x^2 - 9x + 13x - 39 \\ &= x^2(1-3) + 3x(x-3) + 13(2-3) \\ &= (1-3)x^2 + 3x + 13, \\ \text{the required quotient} &= x^2 + 3x + 13\end{aligned}$$

$$\begin{aligned}12 \quad a^3 - 4a^2b + 24b^2 &= a^3 + 2a^2b - 6a^2b + 24b^2 \\ &= a^2(a+2b) - 6b(a^2-4b^2) \\ &= a^2(a+2b) - 6b(a+2b)(a-2b) \\ &= (a+2b)(a^2-6ab+12b^2), \\ \text{the required quotient} &= a^2 - 6ab + 12b^2\end{aligned}$$

$$\begin{aligned}13 \quad a^4 - 5a^3b + 8a^2b^2 - 7ab^3 + 3b^4 \\ &= a^4 - 3a^3b - 2a^3b + 6a^2b^2 + 2a^2b^2 - 6ab^2 - ab^3 + 3b^4 \\ &= a^2(a-3b) - 2a^2b(a-3b) + 2ab^2(a-3b) - b^3(a-3b) \\ &= (a-3b)(a^2-2a^2b+2ab^2-b^3), \\ \text{the required quotient is} \quad &a^2 - 2a^2b + 2ab^2 - b^3\end{aligned}$$

$$\begin{aligned}14 \quad x^4 + x^3 - 10x^2 + 7x - 4 \\ &= x^4 + 4x^3 - 3x^3 - 12x^2 + 2x^2 + 8x - 1 - 4 \\ &= x^2(1+4) - 3x^2(x+4) + 21(1+4) - (1+4) \\ &= (x+4)(x^2 - 3x^2 + 2x - 1), \\ \text{the required quotient is} \quad &x^3 - 3x^2 + 2x - 1\end{aligned}$$

$$\begin{aligned}15 \quad x^3 - 3xy^2 - 76y^3 \\ &= x^3 - 16xy^2 + 19xy^2 - 76y^3 \\ &= x(x^2 - 16y^2) + 19y^2(x - 4y) \\ &= (x-4y)(x^2 + 4xy + 19y^2), \\ \text{the required quotient is} \quad &x^2 + 4xy + 19y^2\end{aligned}$$

$$\begin{aligned}16 \quad r^3 + 8x^2 + 19x + 12 \\ &= r^3 + 8x^2 + 7x + 12x + 12\end{aligned}$$

$$=1(x+1)(x+7)+12(1+1) \\ = (1+1)(x^2+7x+12) = (1+1)(1+3)(1+4)$$

$$17. \quad x^3+9x^2+26x+24 \\ = x^3+9x^2+14x+12x+24 \\ = 1(x^2+9x+14)+12(1+2) \\ = 1(x+2)(x+7)+12(1+2) \\ = (1+2)(x^2+7x+12) = (1+2)(1+3)(1+4)$$

$$18. \quad x^3-6x^2+11x-6 = x^3-6x^2+9x+2x-6 \\ = 1(x^2-6x+9)+2(x-3) \\ = 1(x-3)^2+2(1-3) \\ = (1-3)(x^2-3x+2) \\ = (x-3)(x-1)(1-2) \\ = (x-1)(1-2)(1-3)$$

$$19. \quad x^3+5x^2-2x-24 = x^3+5x^2-14x+12x-24 \\ = 1(x^2+5x-14)+12(x-2) \\ = 1(1+7)(x-2)+12(1-2) \\ = (1-2)(x^2+7x+12) = (1-2)(x+3)(1+4)$$

$$20. \quad x^3-4x^2+x+2 = x^3-4x^2+3x-2x+2 \\ = 1(x^2-4x+3)-2(1-1) \\ = 1(x-3)(x-1)-2(1-1) \\ = (x-1)(x-3)-2 = (1-1)(x^2-3x-2)$$

$$21. \quad x^3+5x^2+4x-6x-6 \\ = x(x^2+5x+4)-6(1+1) \\ = x(x+4)(1+1)-6(1+1) = (1+1)(x^2+4x-6)$$

$$22. \quad x^3-6x^2+13x-10 \\ = x^3-6x^2+8x+5x-10 \\ = 1(x^2-6x+8)+5(1-2) = x(x-4)(1-2)+5(1-2) \\ = (1-2)(x^2-4x+5)$$

$$23. \quad x^4-3x^3-9x^2+12x+20 \\ = x^4-3x^3-5x^2-4x^2+12x+20 \\ = x^3(x^2-3x-5)-4(x^2-3x-5) \\ = (x^2-3x-5)(1+2)(1-2)$$

$$24. \quad x^4-3x^3-x^2+13x-10 \\ = x^4-x^3-2x^3+2x^2-3x^2+3x+10x-10 \\ = x^3(x-1)-2x^2(x-1)-3x(x-1)+10(1-1)$$

$$\begin{aligned}
 &= (r-1)(x^2 - 2x^2 - 3x + 10) \\
 &= (1-1)(x^3 + 2x^2 - 4x^2 - 8x + 5x + 10) \\
 &= (1-1)\{x^2(r+2) - 4x(r+2) + 5(r+2)\} \\
 &= (1-1)(r+2)(x^2 - 4x + 5)
 \end{aligned}$$

$$\begin{aligned}
 25 \quad x^4 - 5x^3 + x^2 + 13x + 6 &= x^4 - 5x^3 - 6x^2 + 7x^2 + 13x + 6 \\
 &= x^2(x^2 - 5x - 6) + (7x^2 + 13x + 6) \\
 &= x^2(x+1)(x-6) + \{7x(1+1) + 6(1+1)\} \\
 &= x^2(r+1)(1-6) + (1+1)(7x+6) \\
 &= (1+1)x^2 - 6x^2 + 7x + 6 \\
 &= (1+1)x^3 - 6x^2 + 9x - 2x + 6 \\
 &= (1+1)\{x(1-3)^2 - 2(x-3)\} = (1+1)(1-3)(x^2 - 3x - 2)
 \end{aligned}$$

$$\begin{aligned}
 26 \quad x^4 + 5x^3 - 8x^2 - 30x + 36 &= x^4 + 3x^3 + 2x^3 + 6x^2 - 14x^2 - 42x + 12x + 36 \\
 &= x^2(r+3) + 2x^2(r+3) - 14x(1+3) + 12(1+3) \\
 &= (1+3)(x^3 + 2x^2 - 14x + 12) \\
 &= (1+3)(x^2 - 2x^2 + 4x^2 - 8x - 6x + 12) \\
 &= (1+3)\{x^2(1-2) + 4x(1-2) - 6(1-2)\} \\
 &= (1+3)x(1-2)(x^2 + 4x - 6)
 \end{aligned}$$

$$\begin{aligned}
 27 \quad x^4 - 7x^3 + 9x^2 + 26x - 56 &= x^4 - 7x^3 + 12x^2 - 3x^2 + 26x - 56 \\
 &= x^2(x^2 - 7x + 12) - (3x^2 - 12x - 14x + 56) \\
 &= x^2(x-3)(x-4) - \{3x(x-4) - 14(x-4)\} \\
 &= x^2(r-3)(r-4) - (1-4)(3x-14) \\
 &= (r-4)(x^3 - 3x^2 - 3x + 14) \\
 &= (1-4)(x^2 + 2x^2 - 5x^2 - 10x + 7x + 14) \\
 &= (1-4)\{x^2(1+2) - 5x(1+2) + 7(1+2)\} \\
 &= (1-4)(1+2)(x^2 - 5x + 7)
 \end{aligned}$$

$$\begin{aligned}
 28 \quad x^3 - 7x^2 + 13x - 15 &= x^3 - 7x^2 + 10x + 3x - 15 \\
 &= x(x^2 - 7x + 10) + 3(x-5) \\
 &= x(x-5)(x-2) + 3(1-5) \\
 &= (1-5)(x^3 - 2x + 3)
 \end{aligned}$$

$$\begin{aligned}
 29 \quad x^3 - 5x + 12 &= x^3 + 27 - 5x - 15 \\
 &= (1+3)(x^2 - 3x + 9) - 5(1+3) \\
 &= (1+3)(x^3 - 3x + 9 - 5) = (x+3)(x^2 - 3x + 4)
 \end{aligned}$$

$$\begin{aligned}
 30 \quad x^3 - 6x^2 + 3x &= x^3 + 8 - 6(x^2 - 4) \\
 &= (x+2)(x^2 - 2x + 4) - 6(x+2)(x-2) \\
 &= (x+2)(x^2 - 2x + 4 - 6x + 12) \\
 &= (x+2)(x^2 - 8x + 16) = (x+2)(x-4)^2
 \end{aligned}$$

$$\begin{aligned}
 31 \quad 2x^3 - 3x^2 - 4 &= 2x^3 - 16 - 3x^2 + 12 \\
 &= 2(x^3 - 8) - 3(x^2 - 4) \\
 &= 2(x-2)(x^2 + 2x + 4) - 3(x-2)(x+2) \\
 &= (x-2)(2x^2 + 4x + 8 - 3x - 6) \\
 &= (x-2)(2x^2 + x + 2)
 \end{aligned}$$

$$\begin{aligned}
 32 \quad x^3 - 9xy^2 - 10y^3 &= x^3 - 4xy^2 - 5xy^2 - 10y^3 \\
 &= x(x^2 - 4y^2) - 5y^2(x + 2y) \\
 &= x(x - 2y)(x + 2y) - 5y^2(x + 2y) \\
 &= (x + 2y)(x^2 - 2xy - 5y^2)
 \end{aligned}$$

$$\begin{aligned}
 33 \quad a^3 + 4a^2b - 9b^3 &= a^3 + 3a^2b + a^2b - 9b^3 \\
 &= a^2(a + 3b) + b(a^2 - 9b^2) \\
 &= a^2(a + 3b) + b(a + 3b)(a - 3b) \\
 &= (a + 3b)(a^2 + ab - 3b^2)
 \end{aligned}$$

$$\begin{aligned}
 34 \quad 5a^3 - 3a^2b - 28b^3 &= 5a^3 - 10a^2b + 7a^2b - 28b^3 \\
 &= 5a^2(a - 2b) + 7b(a^2 - 4b^2) \\
 &= 5a^2(a - 2b) + 7b(a - 2b)(a + 2b) \\
 &= (a - 2b)(5a^2 + 7ab + 14b^2)
 \end{aligned}$$

$$\begin{aligned}
 35 \quad 8x^3 + 4x - 3 &= 8x^3 - 1 + 4x - 2 \\
 &= (2x - 1)(4x^2 + 2x + 1) + 2(2x - 1) \\
 &= (2x - 1)(4x^2 + 2x + 3)
 \end{aligned}$$

$$\begin{aligned}
 36 \quad 2x^3 + 5x^2 - 4x - 3 &= 2x^3 - 2x^2 + 7x^2 - 7x + 3x - 3 \\
 &= 2x^2(x - 1) + 7x(x - 1) + 3(x - 1) \\
 &= (x - 1)(2x^2 + 7x + 3) \\
 &= (x - 1)(2x^2 + 6x + x + 3) \\
 &= (x - 1)(2x(x + 3) + (x + 3)) = (x - 1)(x + 3)(2x + 1)
 \end{aligned}$$

$$\begin{aligned}
 37 \quad x^3 - 3x - 2 &= x^3 - 8 - 3x + 6 \\
 &= (x - 2)(x^2 + 2x + 4) - 3(x - 2) \\
 &= (x - 2)(x^2 + 2x + 4 - 3) \\
 &= (x - 2)(x^2 + 2x + 1) = (x - 2)(x + 1)^2
 \end{aligned}$$

$$\begin{aligned}
 38 \quad 2a^3 - a^2b - b^3 &= a^3 - b^3 + a^3 - a^2b \\
 &= (a-b)(a^2 + ab + b^2) + a^2(a-b) \\
 &= (a-b)(a + ab + b^2 + a^2) = (a-b)(2a^2 + ab + b^2)
 \end{aligned}$$

$$\begin{aligned}
 39 \quad 3x^3 + 8x^2 - 8x - 3 &= 3x^3 - 3 + 8x^2 - 8x \\
 &= 3(x^3 - 1) + 8x(x - 1) \\
 &= 3(x-1)(x^2 + x + 1) + 8x(x-1) \\
 &= (x-1)(3x^2 + 3x + 3 + 8x) \\
 &= (x-1)(3x^2 + 11x + 3)
 \end{aligned}$$

$$\begin{aligned}
 40 \quad x^3 - 6xy^2 + 9y^3 &= x^3 - 9xy^2 + 3xy^2 + 9y^3 \\
 &= x(x^2 - 9y^2) + 3y^2(x + 3y) \\
 &= x(x-3y)(x+3y) + 3y^2(x+3y) \\
 &= (x+3y)(x^2 - 3xy + 3y^2)
 \end{aligned}$$

$$\begin{aligned}
 41 \quad x^2 + bx - (a^2 - 3ab + 2b^2) &= x^2 + bx - (a-b)(a-2b) \\
 &= x^2 + \{(a-b) - (a-2b)\}x - (a-b)(a-2b) \\
 &= \{x + (a-b)\}\{x - (a-2b)\} \\
 &= (x+a-b)(x-a+2b)
 \end{aligned}$$

$$\begin{aligned}
 42 \quad x^4 + 4abx^2y^2 - (a^2 - b^2)^2y^4 &= x^4 + 4abx^2y^2 + 4a^2b^2y^4 - y^4\{(a^2 - b^2)^2 + 4a^2b^2\} \\
 &= (x^2 + 2ab)^2y^2 - y^4\{a^4 + b^4\} \\
 &= \{x^2 + 2ab\}y^2\{x^2 + 2ab\} - y^4\{a^4 + b^4\} \\
 &= \{x^2 + y^2(a+b)^2\}\{x^2 - y^2(a-b)^2\} \\
 &= \{x^2 + y^2(a+b)^2\}\{x+y(a-b)\}\{x-y(a-b)\}
 \end{aligned}$$

$$\begin{aligned}
 43 \quad a^4 + 2(x^2 + y^2)a^2b^2 + (x^2 - y^2)^2b^4 &= a^4 + 2(x^2 + y^2)a^2b^2 + (x^2 + y^2)^2b^4 - (x^2 - y^2)^2b^4 \\
 &\quad + (x^2 - y^2)^2b^4 \\
 &= \{a^2 + (x^2 + y^2)b^2\}^2 - b^4\{(x^2 + y^2)^2 - (x^2 - y^2)^2\} \\
 &= \{a^2 + (x^2 + y^2)b^2\}^2 - b^4(4x^2y^2) \\
 &= \{a^2 + (x^2 + y^2)b^2 + 2b^2xy\}\{a^2 + (x^2 + y^2)b^2 - 2b^2xy\} \\
 &= \{a^2 + b^2(x^2 + y^2 + 2xy)\}\{a^2 + b^2(x^2 + y^2 - 2xy)\} \\
 &= \{a^2 + b^2(x+y)^2\}\{a^2 + b^2(x-y)^2\}
 \end{aligned}$$

$$\begin{aligned}
 44 \quad a^2 + (x+y)a - 2x^2 + 5xy - 2y^2 &= a^2 + (x+y)a - (2x^2 - 4xy - xy + 2y^2) \\
 &= a^2 + (x+y)a - \{2x(x-2y) - y(x-2y)\}
 \end{aligned}$$

$$\begin{aligned}
 &= a^2 + \{(2x - y) - (1 - 2y)\}a - (21 - y)(1 - 2y) \\
 &= \{a + (21 - y)\}\{a - (1 - 2y)\} \\
 &= (a + 2x - y)(a - 1 + 2y)
 \end{aligned}$$

$$\begin{aligned}
 45 \quad &1(\tau + a) - 2a^2 + 3b(a + \tau) + 2b^2 \\
 &= x^2 + x(a + 3b) - 2a^2 + 3ab + 2b^2 \\
 &= x^2 + x(a + 3b) - 2a^2 + 4ab - ab + 2b^2 \\
 &= x^2 + 1(a + 3b) - 2a(a - 2b) - b(a - 2b) \\
 &= x^2 + 1(a + 3b) - (a - 2b)(2a + b) \\
 &= x^2 + 1\{(2a + b) - (a - 2b)\} - (2a + b)(a - 2b) \\
 &= \{1 + (2a + b)\}\{1 - (a - 2b)\} \\
 &= (\tau + 2a + b)(1 - a + 2b)
 \end{aligned}$$

$$\begin{aligned}
 46 \quad &\tau^2 + 4xy + 3y^2 + 2yz - z^2 \\
 &= x^2 + 4xy + 3y^2 + 3yz - yz - z^2 \\
 &= x^2 + 4xy + 3y(y + z) - z(y + z) \\
 &= x^2 + 1\{(y + z) + (3y - z)\} + (y + z)(3y - z) \\
 &= (1 + y + z)(x + 3y - z)
 \end{aligned}$$

$$\begin{aligned}
 47 \quad &4a^2 - 4ab - 3b^2 + 12bc - 9c^2 \\
 &= 4a^2 - 4ab - 3(b^2 - 4bc + 3c^2) \\
 &= 4a^2 - 4ab - 3\{b(b - 3c) - c(b - 3c)\} \\
 &= 4a^2 - 4ab - 3(b - 3c)(b - c) \\
 &= (2a^2)^2 + 2a\{(b - 3c) - 3(b - c)\} - 3(b - c)(b - 3c) \\
 &= \{2a + (b - 3c)\}\{2a - 3(b - c)\} \\
 &= (2a + b - 3c)(2a - 3b + 3c)
 \end{aligned}$$

$$\begin{aligned}
 48 \quad &\tau^4 + 6\tau^2 + 8\tau^2 + 6x - 9 \\
 &= x^4 + 6x^3 + 8x^2 + 12x - 6x - 9 \\
 &= x^4 + 6x^3 + 4x(2x + 3) - 3(2x + 3) \\
 &= (x^2)^2 + (x^2)\{(2x + 3) + (4x - 3)\} + (2x + 3)(4x - 3) \\
 &= (x^2 + 2x + 3)(x^2 + 4x - 3)
 \end{aligned}$$

$$\begin{aligned}
 49 \quad &a^4 - 4a^3b - 5a^2b^2 + 6ab^3 - b^4 \\
 &= a^4 - 4a^3b - b^2(5a^2 - 6ab + b^2) \\
 &= a^4 - 4a^3b - b^2(5a^2 - 5ab - ab + b^2) \\
 &= a^4 - 4a^3b - b^2\{5a(a - b) - b(a - b)\} \\
 &= a^4 - a^2 \cdot 4ab - b^2(a - b)(5a - b) \\
 &= (a^2)^2 + a^2\{(ab - a^2) - (5ab - b^2)\} - (ab - b^2)(5ab - b^2)
 \end{aligned}$$

$$= \{a^2 + (ab - b^2)\} \{a^2 - (5ab - 2^2)\}$$

$$= (a^2 + ab - b^2)(a^2 - 5ab + b^2)$$

$$50 \quad 4x^4 - 20x^3 + 24x^2 + 6x - 9$$

$$= 4x^4 - 20x^3 + 3(8x^2 + 2x - 3)$$

$$= 4x^4 - 20x^3 + 3(8x^2 + 6x - 4x - 3)$$

$$= 4x^4 - 20x^3 + 3\{2x(4x + 3) - (4x + 3)\}$$

$$= 4x^4 - 20x^3 + 3(4x + 3)(2x - 1)$$

$$= (2x^2)^2 + 2x^2\{(3 - 6x) - (3 + 4x)\} - (3 - 6x)(3 + 4x)$$

$$= (2x^2 + 3 - 6x)(2x^2 - 3 - 4x)$$

$$= (2x^2 - 6x + 3)(2x^2 - 4x - 3)$$

$$51 \quad x^4 - 2x^3 + 2x^2 - 2x + 1$$

$$= x^4 + 2x^2 + 1 - 2x(x^2 + 1)$$

$$= (x^2 + 1)^2 - 2x(x^2 + 1)$$

$$= (x^2 + 1)(x^2 + 1 - 2x) = (x^2 + 1)(x - 1)^2$$

$$52 \quad a^4 - 9a^2 + 30a - 25 = a^4 - (9a^2 - 30a + 25)$$

$$= a^4 - (3a - 5)^2 = (a^2 + 3a - 5)(a^2 - 3a + 5)$$

$$53 \quad a^3 - 2abx - (ac - b^2)x^2 + bcx^3$$

$$= a^3 - 2abx + b^2x^2 - acx^2 + bctx^3$$

$$= (a - bx)^2 - cx^2(a - bx) = (a - bx)(a - bx - cx^2)$$

$$54 \quad x^4y^4 + x^2y^2 - z^2 + 2xyx + 1$$

$$= x^4y^4 + 2x^2y^2 + 1 - x^2y^2 - z^2 + 2xyx$$

$$= (x^2y^2 + 1)^2 - (xy - z)^2$$

$$= (x^2y^2 + 1 + xy - z)(x^2y^2 + 1 - xy + z)$$

$$= (x^2y^2 + xy - z + 1)(x^2y^2 - xy + z + 1)$$

$$55 \quad x^2(y^2 - z^2) + 4xyz - y^2 + z^2$$

$$= x^2y^2 + 2xyz + z^2 - (x^2z^2 - 2xyz + y^2)$$

$$= (xy + z)^2 - (xz - y)^2$$

$$= (xy + z + xz - y)(xy + z - xz + y)$$

$$= \{x(y + z) - y + x\}\{x(y - z) + y + x\}$$

$$56 \quad (a^2 - b^2)(x^2 + y^2) + 2(a^2 + b^2)xy$$

$$= a^2(x^2 + y^2 + 2xy) - b^2(x^2 + y^2 - 2xy)$$

$$= a^2(x + y)^2 - b^2(x - y)^2$$

$$= \{a(x + y) + b(x - y)\}\{a(x + y) - b(x - y)\}$$

$$= \{x(a + b) + y(a - b)\}\{x(a - b) + y(a + b)\}$$

$$\begin{aligned}
 57 \quad & x^4 - 4x^3 - x^2 + 10x + 4 \\
 &= x^4 - 4x^3 + 4x^2 - 5x^2 + 10x + 4 \\
 &= (x^2 - 2x)^2 - 5(x^2 - 2x) + 4 \\
 &= (x^2 - 2x)^2 - (-4 - 1)(x^2 - 2x) + (-4)(-1) \\
 &= (x^2 - 2x - 4)(x^2 - 2x - 1)
 \end{aligned}$$

$$\begin{aligned}
 58 \quad & a^4 - 6a^3 + 15a^2 - 18a + 5 \\
 &= a^4 - 6a^3 + 9a^2 + 6a^2 - 18a + 5 \\
 &= (a^2 - 3a)^2 + 6(a^2 - 3a) + 5 \\
 &= (a^2 - 3a)^2 + (5 + 1)(a^2 - 3a) + 5 \cdot 1 \\
 &= (a^2 - 3a + 5)(a^2 - 3a + 1)
 \end{aligned}$$

$$\begin{aligned}
 59 \quad & 4x^4 + 12x^3 - 5x^2 - 21x + 12 \\
 &= 4x^4 + 12x^3 + 9x^2 - 14x^2 - 21x + 12 \\
 &= (2x^2 + 3x)^2 - 7(2x^2 + 3x) + 12 \\
 &= (2x^2 + 3x)^2 - (-3 - 4)(2x^2 + 3x) + (-3)(-4) \\
 &= (2x^2 + 3x - 3)(2x^2 + 3x - 4)
 \end{aligned}$$

$$\begin{aligned}
 60 \quad & x^4 - 5x^2y + 6x^2y^2 - 5xy^3 + y^4 \\
 &= x^4 + x^2y^2 + y^4 - 5xy^2(x^2 - y + y^2) \\
 &= (x^2 + xy + y^2)(x^2 - xy + y^2) - 5xy^2(x^2 - xy + y^2) \\
 &= (x^2 - xy + y^2)(x^2 + xy + y^2 - 5xy) \\
 &= (x^2 - xy + y^2)(x^2 - 4xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 61 \quad & x^4 - 5x^3 + 14x^2 - 20x + 16 \\
 &= x^4 + 8x^3 + 16 - 5x^3 - 20x + 6x^2 \\
 &= (x^2 + 4)^2 - 5x(x^2 + 4) + 6x^2 \\
 &= (x^2 + 4)^2 + (x^2 + 4)(-3x - 21) + (-3x)(-21) \\
 &= \{(x^2 + 4) - 3x\}\{(x^2 + 4) - 21\} \\
 &= (x^2 - 3x + 4)(x^2 - 21 + 4)
 \end{aligned}$$

$$\begin{aligned}
 62 \quad & a^4 - 7a^3b + 14a^2b^2 - 14ab^3 + 4b^4 \\
 &= a^4 + 4a^2b^2 + 4b^4 - 7a^3b - 14ab^3 + 10a^2b^2 \\
 &= (a^2 + 2b^2)^2 - 7ab(a^2 + 2b^2) + 10a^2b^2 \\
 &= (a^2 + 2b^2)^2 + (a^2 + 2b^2)(-5ab - 2ab) + (-5ab)(-2ab) \\
 &= \{(a^2 + 2b^2) - 5ab\}\{(a^2 + 2b^2) - 2ab\} \\
 &= (a^2 - 5ab + 2b^2)(a^2 - 2ab + 2b^2)
 \end{aligned}$$

$$\begin{aligned}
 63 \quad & x^4 + 4x^3 - 11x^2 + 20x + 25 \\
 &= x^4 + 10x^3 + 25 + 4x^3 + 20x - 21x^2
 \end{aligned}$$

$$\begin{aligned}
&= (r^2 + 5)^2 + 4r(r^2 + 5) - 21r^2 \\
&= (x^2 + 5)^2 + (x^2 + 5)(7r - 3r) + 7r(-3r) \\
&= \{(x^2 + 5) + 7r\}\{(x^2 + 5) - 3r\} \\
&= (x^2 + 7r + 5)(x^2 - 3r + 5)
\end{aligned}$$

$$\begin{aligned}
64 \quad a^4 + 4a^2b - 10a^2b^2 + 4ab^3 + b^4 \\
&= a^4 + 2a^2b^2 + b^4 + 4ab(a^2 + b^2) - 12a^2b^2 \\
&= (a^2 + b^2)^2 + (a^2 + b^2)(6ab - 2ab) + 6ab(-2ab) \\
&= \{(a^2 + b^2) + 6ab\}\{(a^2 + b^2) - 2ab\} \\
&= (a^2 + b^2 + 6ab)(a - b)^2
\end{aligned}$$

$$\begin{aligned}
65 \quad r^4 + 8x^3 + 24r^2 + 32r - 20 \\
&= x^4 + 8x^3 + 16x^2 + 8x^2 + 32r - 20 \\
&= (x^2 + 4x)^2 + 8(x^2 + 4x) - 20 \\
&= (x^2 + 4x)^2 + (x^2 + 4x)(10 - 2) + 10 \times (-2) \\
&= (x^2 + 4x + 10)(x^2 + 4x - 2)
\end{aligned}$$

$$\begin{aligned}
66 \quad (r+1)(r+3)(r-4)(r-6) + 13 \\
&= (r+1)(r-4)(r+3)(r-6) + 13 \\
&= (x^2 - 3r - 4)(x^2 - 3r - 18) + 13 \\
&= (x^2 - 3r)^2 - 22(x^2 - 3r) + 72 + 13 \\
&= (x^2 - 3r)^2 - 22(x^2 - 3r) + 85 \\
&= (x^2 - 3r)^2 + (x^2 - 3r)(-17 - 5) + (-17)(-5) \\
&= (x^2 - 3r - 17)(x^2 - 3r - 5)
\end{aligned}$$

$$\begin{aligned}
67 \quad (r+2)(r+3)(r+4)(x+5) - 360 \\
&= (r+2)(x+5)(x+3)(r+4) - 360 \\
&= (x^2 + 7r + 10)(x^2 + 7r + 12) - 360 \\
&= (x^2 + 7r)^2 + 22(x^2 + 7r) + 120 - 360 \\
&= (x^2 + 7r)^2 + 22(x^2 + 7r) - 240 \\
&= (x^2 + 7r)^2 + (x^2 + 7r)(30 - 8) + (30)(-8) \\
&= (x^2 + 7r + 30)(x^2 + 7r - 8) \\
&= (x^2 + 7r + 30)(r + 8)(r - 1)
\end{aligned}$$

$$\begin{aligned}
68 \quad 1(2r+1)(x-2)(2r-3) - 63 \\
&= x(2x-3)(2r+1)(r-2) - 63 \\
&= (2x^2 - 3x)(2x^2 - 3r - 2) - 63 \\
&= (2x^2 - 3r)^2 - 2(2x^2 - 3r) - 63 \\
&= (2x^2 - 3r)^2 + (2x^2 - 3r)(7 - 9) + 7(-9)
\end{aligned}$$

$$\begin{aligned}
 &= (2x^2 - 3x + 7)(2x^2 - 3x - 9) \\
 &= (2x^2 - 3x + 7)(2x^2 - 6x + 3x - 9) \\
 &= (2x^2 - 3x + 7)\{2x(x - 3) + 3(x - 3)\} \\
 &= (2x^2 - 3x + 7)(x - 3)(2x + 3)
 \end{aligned}$$

$$\begin{aligned}
 69 \quad & rj(1+j) + j^2s(y+z) + rs(z+x) + 3xy^2z \\
 &= rj(r+j) + 1yz + j^2s(y+z) + xj^2s + xs(z+x) + rjz \\
 &= rj(x+j+z) + j^2s(y+z+x) + xs(z+r+j) \\
 &= (r+j+z)(xy+j^2s+1z) \\
 &= (a(b-c) + b(c-a) + c(a-b))(1j+j^2s+z1) \\
 &= 0 \times (rj+j^2s+zx) = 0
 \end{aligned}$$

$$\begin{aligned}
 70 \quad & (j-z)(y^2-z^2) - x\{(y-z)^2 + x(y+z) + x^2\} \\
 &= (j-z)^2(y+z) - x(j-z)^2 - x^2(y+z) + 1^2 \\
 &= (j-z)^2(y+z-x) - x^2(y+z-x) \\
 &= (j+z-1)\{(y-z)^2 - x^2\} \\
 &= (j+z-x)(y-z+x)(y-z-x) \\
 &= (j+z-1)(j-z-x)\{(a^2-b^2) + (b^2-c^2) - (a^2-c^2)\} \\
 &= (j+z-x)(j-z-x) \times 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 71 \quad & x^2 - 2(j-2)x - 3y^2 + 20y - 3z \\
 &= x^2 + xj - 4x - 3xj - 3y^2 + 12y + 8x + 8y - 3z \\
 &= x(x+j-4) - 3y(x+j-4) + 8(x+j-4) \\
 &= (1+j-4)(x-3y+8) = 0 \times (x-3y+8) = 0
 \end{aligned}$$

$$\begin{aligned}
 72 \quad & r^2 - y^2 + 41 + 14j - 45 \\
 &= (r+j)(r-j) + 9x + 9y - 51 + 5j - 45 \\
 &= 256 + 9(x+y) - 5(x-y) - 45 \\
 &= 256 + 9 \cdot 25 - 5 \cdot 6 - 45 \\
 &= 150 + 225 - 30 - 45 = 375 - 75 = 300
 \end{aligned}$$

$$\begin{aligned}
 73 \quad & r^6 + j^6 - z^6 + 3x^2y^2z^2 \\
 &= (1^2)^3 + (j^2)^3 + (-z^2)^3 - 3x^2y^2(-z^2) \\
 &= (x^2 + y^2 - z^2)(x^4 + y^4 + z^4 - x^2y^2 + x^2z^2 + y^2z^2) \\
 &= \{(a^2 - b^2)^2 + 4a^2b^2 - (a^2 + b^2)^2\}(x^4 + y^4 + z^4 \text{ etc}) \\
 &= \{(a^2 - b^2)^2 - (a^2 + b^2)^2 + 4a^2b^2\} \times (\text{etc}) \\
 &= (-4a^2b^2 + 4a^2b^2) \times \text{etc} = 0
 \end{aligned}$$

- 74 $8xy(x^2+y^2)=4x^3y(2x^2+2y^2)$
 $=\{(x+y)^2-(x-y)^2\}\{(x+y)^2+(x-y)^2\}$
 $=(3-2)^2(3+2)=5$
- 75 $x^6+y^6=(x+y)(x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5)$
 $=a(x^4+2x^2y^2+y^4-x^4y-2x^2y^3-x^2y^3+x^2y^2)$
 $=a\{(x^2+y^2)^2-xy(x^2+2xy+y^2)+x^2y^2\}$
 $=a\{(x+y)^2-2xy\}^2-xy(x+y)^2+b^4]$
 $=a[(a^2-2b^4)^2-b^2a^2+b^4]$
 $=a(a^4-4a^2b^4+4b^4-b^2a^2+b^4)=a(a^4-5a^2b^2+5b^4)$
- 76 $x^3+y^3+z^3-3xyz$
 $=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$
 $=(x+y+z)\frac{1}{2}(2x^2+2y^2+2z^2-2xy-2yz-2xz)$
 $=\frac{1}{2}(x+y+z)\{(x^2+y^2-2xy)+(x^2+z^2-2xz)$
 $\quad\quad\quad+(y^2+z^2-2yz)\}$
 $=\frac{1}{2}(658+668+674)\{(y-x)^2+(z-x)^2+(z-y)^2\}$
 $=\frac{1}{2}(2000)\{(10)^2+(6)^2+(16)^2=1000(100+36+256)$
 $\quad\quad\quad=1000 \times 392 = 392000$

Exercise 48.

- 1 $a^4b^3=a^2 \times b^2 \times b$, $a^3b^2=a^2 \times a \times b^2$,
the H C F is a^2b^2
- 2 $12a^3b=3 \times 2^2 \times a^3 \times b$, $20a^2c^3=5 \times 2^2 \times a^2 \times c^3$,
the H C F is $2^2 \times a^2=4a^2$
- 3 $91y^2z^3=3^2 \times 7 \times y^2z^3$, $24x^3y^4=3 \times 2^3 \times x^3y^4$,
the H C F is $3 \times xy^2=3xy^2$
- 4 $20a^3x^4y^6=2^2 \times 5 \times a^3x^4y^6$, $75a^2y^3=5^2 \times 3 \times a^2y^3$,
the H C F is $5a^2y^3$
- 5 $18m^2n^4=3^2 \times 2 \times m^2n^4$, $45m^5n^3=3^2 \times 5 \times m^5n^3$,
the H C F is $3^2 \times m^2n^3=9m^2n^3$
- 6 $16a^3x^4y=2^4 \times a^3x^4y$, $40a^2y^3x=5 \times 2^3 \times a^2y^3x$,
 $28x^3a=7 \times 2^2 \times x^3a$,
the H C F is $2^2 \times ax=4ax$

$$\begin{aligned} 7 \quad 24m^2np^5 &= 3 \times 2^3 \times m^2np^5, \\ 60mn^3p &= 5 \times 2^2 \times 3 \times mn^3p, \\ 84m^2p^3 &= 7 \times 2^2 \times 3 \times m^2p^3, \end{aligned}$$

$$\text{the H C F is } 2^2 \times 3 \times mp = 12mp$$

$$\begin{aligned} 8 \quad 45r^3y^2z^4 &= 3^2 \times 5 \times r^3y^2z^4, \\ 75r^4y^4z^3 &= 3 \times 5^2 \times r^4y^4z^3, \\ 90r^4y^3z^2 &= 2 \times 3^2 \times 5 \times r^4y^3z^2, \\ \text{the H C F} &= 3 \times 5 \times r^3y^2z^2 = 15r^3y^2z^2 \end{aligned}$$

$$\begin{aligned} 9 \quad 36a^2b^2c^4i^5 &= 2^2 \times 3^2 \times a^2b^2c^4i^5, \\ 54a^5c^2x^4 &= 2 \times 3^3 \times a^5c^2x^4, \\ 90a^4b^3c^6 &= 2 \times 3^2 \times 5 \times a^4b^3c^6, \\ \text{the H C F is } &2 \times 3^2 a^2c^2 = 18a^2c^2 \end{aligned}$$

$$\begin{aligned} 10 \quad 72a^3b^4c^6 &= 2^3 \times 3^2 \times a^3b^4c^6, \\ 96b^3c^4d^5 &= 2^5 \times 3 \times b^3c^4d^5, \\ 120c^3d^4a^6 &= 2^3 \times 3 \times 5 \times a^6c^3d^4, \\ \text{the H C F is } &2^3 \times 3 \times c^3 = 24c^3 \end{aligned}$$

$$\begin{aligned} 11 \quad 48a^6i^4y^3z^2 &= 2^4 \times 3 \times a^6i^4y^3z^2, \\ 60x^5y^4z^3b^4 &= 2^2 \times 3 \times 5 \times x^5y^4z^3b^4, \\ 72j^5x^4b^3a^2 &= 2^3 \times 3^2 \times a^2b^3j^5x^4, \\ 84z^5b^4a^3i^3 &= 2^2 \times 3 \times 7 \times a^3b^4z^5i^3, \\ \text{the H C F is } &2^2 \times 3 \times z^2 = 12z^2 \end{aligned}$$

$$\begin{aligned} 12 \quad 75m^4n^3p^5q^6 &= 3 \times 5^2 \times m^4n^3p^5q^6, \\ 90m^3n^6p^6q^4 &= 2 \times 3^2 \times 5 \times m^3n^6p^6q^4, \\ 105m^6n^4p^3q^5 &= 3 \times 5 \times 7 \times m^6n^4p^3q^5, \\ 135m^6n^6p^4q^2 &= 3^3 \times 5 \times m^6n^6p^4q^2, \\ \text{the H C F is } &3 \times 5 \times m^3n^3p^3q^2 = 15m^3n^3p^3q^2 \end{aligned}$$

$$\begin{aligned} 13 \quad 54a^2b^5c^3d^4 &= 2 \times 3^3 \times a^2b^5c^3d^4, \\ 72a^5b^3c^4d^2 &= 2^3 \times 3^2 \times a^5b^3c^4d^2, \\ 108a^3b^4c^5d^2 &= 2^2 \times 3^3 \times a^3b^4c^5d^2, \\ 126a^4b^3c^2d^6 &= 2 \times 3^2 \times 7 \times a^4b^3c^2d^6, \\ \text{the H C F is } &2 \times 3^2 \times a^2b^3c^2d^2 = 18a^2b^3c^2d^2 \end{aligned}$$

$$\begin{aligned}
 14 \quad & 18a^3x^4y^5 = 2 \times 3^2 \times a^3x^4y^5, \\
 & 42a^4y^3z^4 = 2 \times 3 \times 7 \times a^4y^3z^4, \\
 & 60a^3y^4z^5 = 2^2 \times 3 \times 5 \times a^3y^4z^5, \\
 & 78a^2x^4z^3 = 2 \times 3 \times 13 \times a^2x^4z^3,
 \end{aligned}$$

the H C F is $2 \times 3 = 6$

$$\begin{aligned}
 15 \quad & 32a^2b^3x^4y^5z^6 = 2^5 \times a^2b^3x^4y^5z^6, \\
 & 40a^3x^5y^4z^8 = 2^3 \times 5 \times a^3x^5y^4z^8, \\
 & 56b^3x^2y^7z^4 = 2^3 \times 7 \times b^3x^2y^7z^4, \\
 & 72x^6a^5y^2z^3 = 2^3 \times 3^2 \times a^5x^6y^2z^3, \\
 & 96b^4a^8x^3y^8 = 2^5 \times 3 \times a^8b^4x^3y^8,
 \end{aligned}$$

the H C F is $2^3 \times x^2y^2 = 8x^2y^2$

Exercise 49

$$\begin{aligned}
 1 \quad & a^3 - ab^2 = a(a^2 - b^2) = a(a+b)(a-b), \text{ and} \\
 & a^4 + 2a^3b + a^2b^2 = a^2(a^2 + 2ab + b^2) = a^2(a+b)^2,
 \end{aligned}$$

the H C F is $a(a+b)$

$$\begin{aligned}
 2 \quad & x^5y^3 - x^3y^5 = x^3y^3(x^2 - y^2) = x^3y^3(x+y)(x-y), \text{ and} \\
 & x^6y^4 + x^4y^6 = x^4y^4(x^2 + y^2),
 \end{aligned}$$

the H C F is $x^3y^3(x+y)$

$$\begin{aligned}
 3 \quad & 6(x^2 - 9) = 3 \times 2(x+3)(x-3), \text{ and} \\
 & 15(x^3 + 27) = 3 \times 5(x+3)(x^2 - 3x + 9),
 \end{aligned}$$

the H C F is $3(x+3)$

$$\begin{aligned}
 4 \quad & 12(a^4 - a^2b^2c^2) = 2^2 \times 3 \times a^2(a^4 - b^2c^2) \\
 & \quad = 2^2 \times 3 \times a^2(a^2 + bc)(a^2 - bc), \\
 & \text{and } 20(a^4b^2c^3 + a^2b^4c^3) = 2^2 \times 5 \times a^2b^2c^3(a^2 + bc), \\
 & \text{the H C F is } 2^2 \times a^2(a^2 + bc) = 4a^2(a^2 + bc)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & m^4n^2 - 2m^5n^3 + m^4n^4 = m^4n^2(m^2 - 2mn + n^2) = m^4n^2(m-n)^2, \\
 & \text{and } (m^3n - mn^2)^3 = m^3n^3(m-n)^3, \\
 & \text{the H C F is } m^3n^3(m-n)
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & 4a^4x - 9a^3x^3 = a^2x(4a^2 - 9x^2) = a^2x(2a+3x)(2a-3x), \\
 & \text{and } 4a^3x^2 + 6ax^3 = 2ax^2(2a+3x),
 \end{aligned}$$

the H C F is $ax(2a+3x)$

$$\begin{aligned}
 7 \quad 18a^4b^3 - 32a^2b^5 &= 2a^2b^3(9a^2 - 16b^2) \\
 &= 2a^2b^3(3a + 4b)(3a - 4b), \\
 \text{and} \quad 18a^4b^3 + 24a^7b^7 &= 2 \times 3 \times a^3b^3(3a + 4b),
 \end{aligned}$$

the H C F is $2a^3b^3(3a + 4b)$

$$\begin{aligned}
 8 \quad 9x^4y^4 - 36x^2y^6 &= 9x^2y^4(x^2 - 4y^2) \\
 &= 3^2 \times x^2y^4(x + 2y)(x - 2y), \\
 \text{and} \quad 24x^4y^2 - 48x^2y^7 &= 2^3 \times 3 \times x^2y^2(x^2 - 2y^5),
 \end{aligned}$$

the H C F is $3x^2y^2(x - 2y)$

$$\begin{aligned}
 9 \quad 6a^3b^2 - 24ab^4 &= 6ab^2(a^2 - 4b^2) \\
 &= 2 \times 3 \times ab^2(a + 2b)(a - 2b), \\
 \text{and} \quad 4a^5b + 32a^2b^4 &= 4a^2b(a^3 + 8b^3) \\
 &= 2^2 \times a^2b(a + 2b)(a^2 - 2ab + 4b^2),
 \end{aligned}$$

the H C F is $2ab(a + 2b)$

$$\begin{aligned}
 10 \quad 48x^2a^3(x + a)^3(x^2a^2 - xa^3) \\
 &= 2^4 \times 3 \times x^2a^3(x + a)^3 \times xa^2(x - a) \\
 &= 2^4 \times 3 \times x^3a^4(x + a)^2(x - a), \text{ and} \\
 64(x^5a^6 - x^2a^3)(x^3a + x^2a^2) &= 2^6 \times x^3a^2(x^2 - a^3) \times x^3a(x + a) \\
 &= 2^6 x^4a^7(x - a)(x^2 + ax + a^2)(x + a),
 \end{aligned}$$

the H C F is $2^4 \times x^3a^4(x + a)(x - a) = 16x^3a^4(x^2 - a^2)$

$$\begin{aligned}
 11 \quad 24(x^2 - a^2) &= 2^3 \times 3 \times (x - a)(x^2 + ax + a^2), \text{ and} \\
 40(x^4 + x^3a^2 + a^4) &= 2^3 \times 5 \times (x^2 + ax + a^2)(x^2 - ax + a^2), \\
 \text{the H C F is } 2^3 \times (x^2 + ax + a^2) &= 8(x^2 + ax + a^2)
 \end{aligned}$$

$$\begin{aligned}
 12 \quad 56(x^6a^2 - x^2a^6) &= 2^3 \times 7 \times x^2a^2(x^4 - a^4) \\
 &= 2^3 \times 7 \times x^2a^2(x^2 + a^2)(x + a)(x - a), \\
 \text{and } 72(x^5a^7 + 3a^5x^3 + 2a^7x) \\
 &= 2^3 \times 3^2 \times a^5x(x^4 + 3a^2x^2 + 2a^4) \\
 &= 2^3 \times 3^2 \times a^5x(x^2 + 2a^2)(x^2 + a^2), \\
 \text{the H C F is } 2^3 \times a^5x(x^2 + a^2) &= 8a^5x(x^2 + a^2)
 \end{aligned}$$

$$\begin{aligned}
 13 \quad 30(a^3 + 4ab + 3b^3) &= 2 \times 3 \times 5 \times (a + b)(a + 3b), \\
 \text{and } 42(a^3 + ab - 6b^3) &= 2 \times 3 \times 7 \times (a - 2b)(a + 3b), \\
 \text{the H C F is } 2 \times 3(a + 3b) &= 6(a + 3b)
 \end{aligned}$$

$$\begin{aligned}
 14 \quad 28(x^3 - 3x^2 - 10x) &= 2^2 \times 7 \times x(x^2 - 3x - 10) \\
 &= 2^2 \times 7 \times x(x+2)(x-5), \\
 \text{and } 52(x^4 - 8x^3 + 15x^2) &= 2^2 \times 13 \times x^2(x^2 - 8x + 15) \\
 &= 2^2 \times 13 \times x^2(x-3)(x-5), \\
 \text{the H C F is } 2^2 \times x(x-5) &= 4x(x-5)
 \end{aligned}$$

$$\begin{aligned}
 15 \quad x^4y + 3x^3y^2 - 18x^2y^3 &= x^2y(x^2 + 3xy - 18y^2) \\
 &= x^2y(x-3y)(x+6y), \\
 \text{and } x^3y^2 + 10x^2y^3 + 24xy^4 &= xy^2(x^2 + 10xy + 24y^2) \\
 &= xy^2(x+4y)(x+6y), \\
 \text{the H C F is } xy(x+6y)
 \end{aligned}$$

$$\begin{aligned}
 16 \quad a^4x^3 - 4a^3x^2 - 12a^2x &= a^2x^2(a^2 - 4ax - 12x^2) \\
 &= a^2x^2(a+2x)(a-6x), \\
 \text{and } a^6x^3 + 8a^4x^2 + 12a^3x &= a^3x^2(a^2 + 8ax + 12x^2) \\
 &= a^3x^2(a^2 + 8ax + 12x^2) \\
 &= a^3x^2(a+2x)(a+6x), \\
 \text{the H C F is } a^2x^2(a+2x)
 \end{aligned}$$

$$\begin{aligned}
 17 \quad 4x^3 + 12x^2 + 9x &= x(4x^2 + 12x + 9) = x(2x+3)^2, \\
 \text{and } 4x^3 - 2x - 12 &= 2(2x^2 - x - 6) = 2(2x^2 - 4x + 3x - 6) \\
 &= 2\{2x(x-2) + 3(x-2)\} \\
 &= 2(x-2)(2x+3), \\
 \text{the H C F is } 2x+3
 \end{aligned}$$

$$\begin{aligned}
 18 \quad a^2 - ab - 2b^2 &= a^2 - 2ab + ab - 2b^2 \\
 &= a(a-2b) + b(a-2b) \\
 &= (a-2b)(a+b), \\
 \text{and } a^3 - a^2b - 4ab^2 + 4b^3 &= a^2(a-b) - 4b^2(a-b) \\
 &= (a-b)(a^2 - 4b^2) \\
 &= (a-b)(a+2b)(a-2b), \\
 \text{the H C F is } (a-2b)
 \end{aligned}$$

$$\begin{aligned}
 19 \quad x^2 + 3x - 10 &= (x+5)(x-2), \\
 \text{and } x^3 - x^2 - 14x + 24 &= x^2 - 2x^2 + x^2 - 2x - 12x + 24 \\
 &= x^2(x-2) + x(x-2) - 12(x-2) \\
 &= (x-2)(x^2 + x - 12) = (x-2)(x+4)(x-3), \\
 \text{the H C F is } x-2
 \end{aligned}$$

$$\begin{aligned}
 20 \quad 54(r^3 + 8a^3) &= 2 \times 3^3 \times (1 + 2a)(r^3 - 2ar + 4a^3), \\
 &\quad \text{and } 90(r^3 + 7ar^2 + 16a^2r + 12a^3) \\
 &= 2 \times 3^2 \times 5 \times (r^3 + 2ar^2 + 5a^2r + 10a^3r + 6a^3r + 12a^3) \\
 &= 2 \times 3^2 \times 5 \times \{r^3(r + 2a) + 5ar(r + 2a) + 6a^2(r + 2a)\} \\
 &= 2 \times 3^2 \times 5 \times (1 + 2a)(r^2 + 5ar + 6a^2) \\
 &= 2 \times 3^4 \times 5 \times (r + 2a)(1 + 2a)(1 + 3a), \\
 \text{the H C F is } 2 \times 3^2(1 + 2a) &= 18(1 + 2a)
 \end{aligned}$$

$$\begin{aligned}
 21 \quad (a^3 - b^3)(a + b)^2 &= (a - b)(a^2 + ab + b^2)(a + b)^2, \\
 a^4 - b^4 &= (a^2 + b^2)(a + b)(a - b), \\
 \text{and } 3a^4 + 2a^3b - 5a^2b^2 &= a^2(3a^2 + 2ab - 5b^2) \\
 &= a^2(3a^2 + 5ab - 3ab - 5b^2) \\
 &= a^2\{a(3a + 5b) - b(3a + 5b)\} \\
 &= a^2(3a + 5b)(a - b), \\
 \text{the H C F is } (a - b)
 \end{aligned}$$

$$\begin{aligned}
 22 \quad (2r - 3)^2(r^2 + r - 2) &= (2r - 3)^2(r + 2)(r - 1), \\
 4r^3 - r - 18 &= 4r^2 + 8r - 9r - 18 \\
 &= 4r(r + 2) - 9(r + 2) \\
 &= (r + 2)(4r - 9), \\
 \text{and } 21r^2 - 23r - 54 &= 21r^2 + 4r - 27r - 54 \\
 &= 2r(r + 2) - 27(r + 2) \\
 &= (r + 2)(2r - 27), \\
 \text{the H C F is } (r + 2)
 \end{aligned}$$

$$\begin{aligned}
 23 \quad 8(27a^5b + a^3b^4) &= 2^3a^3b(27a^2 + b^3) \\
 &= 2^3a^3b(3a + b)(9a^2 - 3ab + b), \\
 12(6a^4b^2 - 7a^3b^3 - 3a^2b^4) &= 2^2 \times 3 \times a^2b^2(6a^2 - 7ab - 3b^2) \\
 &= 2^2 \times 3 \times a^2b^2(6a^2 + 2ab - 9ab - 3b^2) \\
 &= 2^2 \times 3 \times a^2b^2\{2a(3a + b) - 3b(3a + b)\} \\
 &= 2^2 \times 3 \times a^2b^2(3a + b)(2a - 3b), \\
 \text{and } 40(3a^3b^2 + 13a^2b^3 + 4ab^4) &= 2^3 \times 5 \times ab^2(3a^2 + 13ab + 4b^2) \\
 &= 2^3 \times 5 \times ab^2(3a^2 + 12ab + ab + 4b^2) \\
 &= 2^3 \times 5 \times ab^2\{3a(a + 4b) + b(a + 4b)\} \\
 &= 2^3 \times 5 \times ab^2(a + 4b)(3a + b), \\
 \text{the H C F is } 2^2 \times ab(3a + b) &= 4ab(3a + b)
 \end{aligned}$$

$$\begin{aligned}
 24 \quad x^4 - 13x^2 + 36 &= (x^2 - 4)(x^2 - 9) \\
 &= (x + 2)(x - 2)(x + 3)(x - 3), \\
 3x^3 + 13x^2 + 8x - 12 &= 3x^3 + 9x^2 + 4x^2 + 12x - 4x - 12 \\
 &= 3x^2(x + 3) + 4x(x + 3) - 4(x + 3) \\
 &= (x + 3)(3x^2 + 4x - 4) \\
 &= (x + 3)(3x^2 + 6x - 2x - 4) \\
 &= (x + 3)\{3x(x + 2) - 2(x + 2)\} \\
 &= (x + 3)(x + 2)(3x - 2),
 \end{aligned}$$

$$\begin{aligned}
 \text{and } 4x^3 + 17x^2 + 9x - 18 &= 4x^3 + 8x^2 + 9x^2 + 18x - 9x - 18 \\
 &= 4x^2(x + 2) + 9x(x + 2) - 9(x + 2) \\
 &= (x + 2)(4x^2 + 9x - 9) \\
 &= (x + 2)(4x^2 + 12x - 3x - 9) \\
 &= (x + 2)\{4x(x + 3) - 3(x + 3)\} \\
 &= (x + 2)(x + 3)(4x - 3),
 \end{aligned}$$

the H C F is $(x + 2)(x + 3) = x^2 + 5x + 6$

Exercise 50

$$\begin{array}{r}
 1 \quad 2x^3 + 5x - 3 \overline{) 2x^3 + 3x^2 - 32x + 15} \quad (x - 1 \\
 \underline{2x^3 + 5x^2 - 3x} \\
 -2x^3 - 29x + 15 \\
 \underline{-2x^3 - 5x + 3} \\
 -12 \mid -24x + 12 \\
 2x - 1 \overline{) 2x^2 + 5x - 3} \quad (x + 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

the required H C F = $2x - 1$

[N B The above process may, for convenience, be put in the manner shown below —

$$\begin{array}{r}
 2x^3 + 3x^2 - 32x + 15 \\
 \underline{-(2x^3 + 5x - 3)x} \\
 -2x^2 - 29x + 15 \\
 \underline{-(2x^2 + 5x - 3) \times (-1)} \\
 -12 \mid -24x + 12 \\
 2x - 1
 \end{array}$$

$$\begin{array}{r} 21^2 + 5x - 3 \\ -(21 - 1)x \\ \hline 61 - 3 \\ -(21 - 1)3 \end{array}$$

the required $H \subset \Gamma = 2n - 1$

$$\begin{array}{r} 2 \quad 3x^2 + 16x - 12 \overline{) 3x^3 + 4x^2 - 28x + 16} \left(x - 4 \right. \\ \underline{3x^3 + 16x^2 - 12x} \\ -12x^2 - 16x + 16 \\ \underline{-12x^2 - 64x + 48} \\ 16148x - 32 \end{array}$$

the required H C F = $31 - 2$

[N B The above process may, for convenience, be put in the manner shown below —

$$\begin{array}{r} 3x^2 + 4x^2 - 28x + 16 \\ -(3x^2 + 16x - 12)x \\ \hline -12x^2 - 16x + 16 \\ -(3x^2 + 16x - 12) \times (-4) \\ \hline 16148x - 32 \\ \hline 3x - 2 \end{array}$$

$$\begin{array}{r} 3x^2 + 16x - 12 \\ -(3x - 2) \cdot 1 \\ \hline 18x - 12 \\ -(3x - 2) \cdot 6 \end{array}$$

the required H C F = 31 - 2]

$$\begin{array}{r} 3 \quad 2x^3 + 3ax^2 - 45a^2x - 100a^3 \\ - (2x^3 - 3ax - 20a^3) \quad 1 \\ \hline 6ax^2 - 25a^2x - 100a^3 \\ - (2x^3 - 3ax - 20a^3) \quad 3a \\ \hline - 8a^2x - 16a^2x - 40a^3 \\ \hline 2x + 5a \end{array}$$

$$\begin{array}{r} 2x^2 - 3ax - 20a^2 \\ -(2x + 5a)x \\ \hline -8ax - 20a^2 \\ -(7x + 5a)(-4a) \end{array}$$

the required H C F = $(21 + 5n)$

- 4 1st expression = $x(3x^3 + 7x^2 - 14x - 24)$,
 2nd expression = $2x^2(3x^2 - 5x - 12)$

Now to find x , the H C F of

$$\begin{array}{r}
 3x^3 + 7x^2 - 14x - 24 \text{ and } 3x^2 - 5x - 12 \\
 \underline{3x^3 + 7x^2 - 14x - 24} \\
 - (3x^2 - 5x - 12)x \\
 \hline
 12x^2 - 2x - 24 \\
 \underline{-(3x^2 - 5x - 12)4} \\
 6|18x + 24 \\
 3x + 4 \\
 \\
 3x^2 - 5x - 12 \\
 \underline{-(3x + 4)x} \\
 -9x - 12 \\
 \underline{-(3x + 4)(-3)}
 \end{array}$$

Thus $x = 3x + 4$

the required H C F = $x(3x + 4)$

5

$$\begin{array}{r}
 6a^3 - 11a^2 - 3a + 2 \\
 \underline{-(3a^3 + 20a^2 + 23a - 10)2} \\
 -51a^2 - 49a + 22 \\
 \\
 3a^3 + 20a^2 + 23a - 10 \\
 17 \\
 \hline
 51a^3 + 340a^2 + 391a - 170 \\
 \underline{-(-51a^2 - 49a + 22)(-a)} \\
 291a^2 + 413a - 170 \\
 17 \\
 \hline
 4947a^2 + 7021a - 2890 \\
 \\
 \underline{-(-51a^2 - 49a + 22) \times (-97)} \\
 756|2268a - 756 \\
 3a - 1 \\
 \\
 -51a^2 - 49a + 22 \\
 \underline{-(3a - 1) \times (-17a)} \\
 -66a + 22 \\
 \underline{-(3a - 1) \times (-22)}
 \end{array}$$

the required H C F = $3a - 1$

$$\begin{array}{r}
 6 \quad 4a^3 + 12a^2b - 7ab^2 - 30b^3 \\
 \quad \underline{3} \\
 \quad 12a^3 + 36a^2b - 21ab^2 - 90b^3 \\
 -(6a^3 - 25a^2b + 32ab^2 - 12b^3) \times 2 \\
 \quad \underline{b | 86a^2b - 85ab^2 - 66b^3} \\
 \quad \quad 86a^2 - 85ab - 66b^2 \\
 \\
 \quad 6a^2 - 25a^2b + 32ab^2 - 12b^3 \\
 \quad \underline{43} \\
 \quad 258a^3 - 1075a^2b + 1376ab^2 - 516b^3 \\
 -(86a^2 - 85ab - 66b^2) 3a \\
 \quad \underline{-2b | -820a^2b + 1574ab^2 - 516b^3} \\
 \quad \quad 410a^2 - 767ab + 258b^2 \\
 \quad \quad \underline{43} \\
 \quad \quad 17630a^2 - 33841ab + 11094b^2 \\
 -(86a^2 - 85ab - 66b^2) 205 \\
 \quad \underline{-8208b | -16416ab + 24624b^2} \\
 \quad \quad \quad 2a \quad -3b \\
 \\
 \quad 86a^2 - 85ab - 66b^2 \\
 -(2a - 3b) 43a \\
 \quad \underline{44ab - 66b^2} \\
 \quad \quad -(2a - 3b) 22b
 \end{array}$$

the required H C F = $2a - 3b$

$$\begin{array}{r}
 7 \quad 3x^3 + 5x^2 + 5x + 2 \\
 \quad \underline{3} \\
 \quad 6x^3 + 10x^2 + 10x + 4 \\
 -(2x^3 + 5x^2 + 5x + 3) 3 \\
 \quad \underline{-5 | -5x^2 - 5x - 5} \\
 \quad \quad x^2 + x + 1 \\
 \\
 \quad 2x^3 + 5x^2 + 5x + 3 \\
 -(x^2 + x + 1) 2x \\
 \quad \underline{3x^2 + 3x + 3} \\
 \quad \quad -(x^2 + x + 1) 3
 \end{array}$$

the required H C F = $x^2 + x + 1$

$$\begin{array}{r}
 8 \quad 4x^3 - 7x^2y + 7xy^2 - 3y^3 \\
 \quad \underline{3} \\
 \quad 12x^3 - 21x^2y + 21xy^2 - 9y^3 \\
 -(3x^3 - 7x^2y + 7xy^2 - 4y^3) 4 \\
 \quad \underline{7y | 7x^2y - 7xy^2 + 7y^3} \\
 \quad \quad x^2 - xy + y^2
 \end{array}$$

$$\begin{array}{r} 3x^3 - 7xy^2 + 7xy^2 - 4y^3 \\ -(x^3 - xy + y^3) 3x \\ \hline -4x^2y + 4xy^2 - 4y^3 \\ -(x^3 - xy + y^3) \times (-4y) \end{array}$$

the required H C F = $x^2 - xy + y^2$

- 9 1st expression = $x(6x^2 + 7x^2 + 5x + 2)$,
2nd expression = $2x^2(2x^2 - 9x^2 - 4x - 5)$

Now to find x , the H C F of

$$\begin{array}{r} 6x^3 + 7x^2 + 5x + 2 \text{ and } 2x^3 - 9x^2 - 4x - 5 \\ \hline 6x^3 + 7x^2 + 5x + 2 \\ -(2x^3 - 9x^2 - 4x - 5) 3 \\ \hline 17|34x^2 + 17x + 17 \\ \quad 2x^2 + x + 1 \\ \hline \quad 2x^3 - 9x^2 - 4x - 5 \\ \quad -(2x^2 + x + 1)x \\ \hline \quad \quad -10x^2 - 5x - 5 \\ \quad \quad -(2x^2 + x + 1)(-5) \end{array}$$

Thus $x = 2x^2 + x + 1$

the required H C F = $x(2x^2 + x + 1)$

10

$$\begin{array}{r} 3x^4 + 10x^3 + 7x^2 + 4x + 1 \\ \quad 2 \\ \hline 6x^4 + 20x^3 + 14x^2 + 8x + 2 \\ -(2x^3 + 3x^2 - 7x - 3) 3x \\ \hline 11x^3 + 35x^2 + 17x + 2 \\ \quad 2 \\ \hline 22x^3 + 70x^2 + 34x + 4 \\ -(2x^3 + 3x^2 - 7x - 3) 11 \\ \hline 37|37x^2 - 111x + 37 \\ \quad x^2 + 3x + 1 \\ \hline \quad 2x^3 + 3x^2 - 7x - 3 \\ \quad -(x^2 + 3x + 1) 2x \\ \hline \quad \quad -3x^2 - 9x - 3 \\ \quad \quad -(x^2 + 3x + 1) \times (-3) \end{array}$$

the required H C F = $x^2 + 3x + 1$

11

$$\begin{array}{r}
 31^3 + 131^2 + 91^2 + 91 + 2 \\
 \hline
 4 \\
 12x^4 + 52x^3 + 36x^2 + 36x + 6 \\
 -(4x^3 + 13x^2 - 8x - 3) 31 \\
 \hline
 13x^3 + 60x^2 + 45x + 8 \\
 \hline
 4 \\
 52x^3 + 240x^2 + 180x + 32 \\
 -(4x^3 + 13x^2 - 8x - 3) 13 \\
 \hline
 7171x^2 + 2841x + 71 \\
 \hline
 x^2 + 4x + 1 \\
 \\
 4x^3 + 13x^2 - 8x - 3 \\
 -(x^2 + 4x + 1) 4x \\
 \hline
 -3x^3 - 12x - 3 \\
 -(x^2 + 4x + 1) \times (-3) \\
 \hline
 \end{array}$$

the required H C F = $x^2 + 4x + 1$

12

$$\begin{array}{r}
 21a^3 + 17a^2x + 9ax^2 + x^3 \\
 \hline
 4 \\
 84a^3 + 68a^2x + 36ax^2 + 4x^3 \\
 -(12a^3 + 11a^2x + 6ax^2 + x^3) 7 \\
 \hline
 -31 \mid -9a^2x - 6ax^2 - 3x^3 \\
 \hline
 3a^2 + 2ax + x^2 \\
 \\
 12a^3 + 11a^2x + 6ax^2 + x^3 \\
 -(3a^3 + 2ax + x^3) 4a \\
 \hline
 3a^2x + 2ax^2 + x^3 \\
 -(3a^2 + 2ax + x^2) x \\
 \hline
 \end{array}$$

the required H C F = $3a^2 + 2ax + x^2$

13

$$\begin{array}{r}
 65a^3 + 54a^2x + 22ax^2 + 3x^3 \\
 \hline
 7 \\
 455a^3 + 378a^2x + 154ax^2 + 21x^3 \\
 -(35a^3 + 31a^2x + 13ax^2 + 2x^3) 13 \\
 \hline
 -5x \mid -25a^2x - 15ax^2 - 5x^3 \\
 \hline
 5a^2 + 3ax + x^2 \\
 \\
 35a^3 + 31a^2x + 13ax^2 + 2x^3 \\
 -(5a^3 + 3ax + x^2) 7a \\
 \hline
 10a^2x + 6ax^2 + 2x^3 \\
 -(5a^2 + 3ax + x^2) 2x \\
 \hline
 \end{array}$$

the required H C F = $5a^2 + 3ax + x^2$

$$\begin{array}{r}
 14 \quad \quad \quad 91r^3 - 25ax^2 + 20a^2x + 4a^3 \\
 \quad \quad \quad \underline{10} \\
 \quad \quad \quad 910r^3 - 250ar^2 + 200a^2r + 40a^3 \\
 \quad \quad \quad -(70r^3 - 9ax^2 + 11a^2r + 6a^3) 13 \\
 \quad \quad \quad \underline{-19a | -133ar^2 + 57a^2r - 38a^3} \\
 \quad \quad \quad \quad \quad \quad 7x - 3a1 + 2a^2 \\
 \quad \quad \quad 70r^3 - 9ar^2 + 11a^2r + 6a^3 \\
 \quad \quad \quad \underline{-(7x^2 - 3ax + 2a^2) 101} \\
 \quad \quad \quad 21ax^2 - 9a^2r + 6a^3
 \end{array}$$

the required H C F = $7r^2 - 3ax + 2a^2$

$$\begin{array}{r}
 15 \quad \quad \quad 85r^3 - 36r^2 + 25r + 6 \\
 \quad \quad \quad \underline{15} \\
 \quad \quad \quad 1275r^3 - 540r^2 + 375r + 90 \\
 \quad \quad \quad -(75r^3 - 35x^2 + 24r + 4) 17 \\
 \quad \quad \quad \underline{11 | 55x^2 - 33x + 22} \\
 \quad \quad \quad \quad \quad \quad 5x^2 - 3x + 2 \\
 \quad \quad \quad 75x^3 - 35r^2 + 24r + 4 \\
 \quad \quad \quad \underline{-(5x^2 - 3r + 2) 15x} \\
 \quad \quad \quad \quad \quad \quad 10x^2 - 6r + 4 \\
 \quad \quad \quad \quad \quad \quad \underline{-(5r^2 - 3x + 2) 2}
 \end{array}$$

the required H C F = $5x^2 - 3x + 2$

$$\begin{array}{r}
 16 \quad \quad \quad 49x^3 - 49r^2 + 5x + 3 \\
 \quad \quad \quad \underline{5} \\
 \quad \quad \quad 245x^3 - 245x^2 + 25x + 15 \\
 \quad \quad \quad -(35x^3 - 34x^2 + 3x + 2) 7 \\
 \quad \quad \quad \underline{-7r^2 + 4x + 1} \\
 \quad \quad \quad 35r^3 - 34x^2 + 3x + 2 \\
 \quad \quad \quad \underline{-(-7r^2 + 4x + 1) (-5r)} \\
 \quad \quad \quad \quad \quad \quad -14x^2 + 8x + 2 \\
 \quad \quad \quad \quad \quad \quad \underline{-(-7r^2 + 4x + 1) 2}
 \end{array}$$

the required H C F = $-7r^2 + 4x + 1$

$$\begin{array}{l}
 17 \quad \text{1st expression} = x^3(6r^3 + 21ax^2 + 30ar^2 + 24a^3), \\
 \quad \text{2nd expression} = 2r^3(2x^4 + ax^3 + 7a^2x^2 + 5a^3x + 12a^4) \\
 \quad \quad \quad 2x^4 + ax^3 + 7a^2x^2 + 5a^3x + 12a^4 \\
 \quad \quad \quad \underline{3} \\
 \quad \quad \quad 6x^4 + 3ax^3 + 21a^2x^2 + 15a^3x + 36a^4 \\
 \quad \quad \quad -(6r^3 + 21ax^2 + 30a^2r + 24a^3)x \\
 \quad \quad \quad \underline{-18ax^3 - 9a^2x^2 - 9a^3x + 36a^4} \\
 \quad \quad \quad -(6r^3 + 21ax^2 + 30a^2r + 24a^3) \times (-3a) \\
 \quad \quad \quad \underline{27a^2 | 54a^2r^2 + 81a^3x + 108a^4} \\
 \quad \quad \quad \quad \quad \quad 2x^2 + 3ar + 4a^2
 \end{array}$$

$$\frac{6x^5 + 21ax^2 + 30a^2x + 24a^3 - (2x^3 + 3ax + 4a^2) 3x}{12ax^2 + 18a^2x + 24a^3}$$

$$\frac{-(2x^2 + 3ax + 4a^2) 6a}{12ax^2 + 18a^2x + 24a^3}$$

the required H C F = $x^2(2x^2 + 3ax + 4a^2)$

18 1st expression = $2 \times 2 \times (a^4 + 8a^2 + 18a^2 + 11a + 2)$,

2nd expression = $2 \times 3(a^4 + 9a^2 + 23a^2 + 13a + 2)$

$$\frac{a^4 + 9a^2 + 23a^2 + 13a + 2 - (a^4 + 8a^2 + 18a^2 + 11a + 2) 1}{a(a^2 + 5a + 2)}$$

$$\frac{a^4 + 8a^2 + 18a^2 + 11a + 2 - (a^2 + 5a + 2) a^2}{3a^3 + 16a^2 + 11a + 2 - (a^2 + 5a + 2) 3a}$$

the reqd H C F = $2(a^2 + 5a + 2)$

$$19 \quad \frac{6x^4 + 21x^2 + 3x - 6}{-(2x^4 - 19x^2 + 31x - 6) 3}$$

$$\frac{3(21x^3 + 57x - 6ax + 12)}{7x^3 + 19x^2 - 20x + 4}$$

$$\frac{2x^4 - 19x^2 + 21x - 6}{7}$$

$$\frac{14x^4 - 133x^2 + 147x - 42}{-(7x^3 + 19x^2 - 20x + 4) 21}$$

$$\frac{-38x^3 - 93x^2 + 139x - 42}{7}$$

$$\frac{-266x^3 - 651x^2 + 973x - 294}{-(7x^3 + 19x^2 - 20x + 4) \times (-38)}$$

$$\frac{71(71x^2 + 213x - 142)}{x^2 + 3x - 2}$$

$$\frac{7x^3 + 19x^2 - 20x + 4}{-(x^2 + 3x - 2) 7x}$$

$$\frac{-2x^2 - 6x + 4}{-(x^2 + 3x - 2) \times (-2)}$$

$$\frac{-2x^2 - 6x + 4}{-(x^2 + 3x - 2) \times (-2)}$$

the reqd H C F = $x^2 + 3x - 2$

20 1st expression = $6(2x^4 - 5x^3 + 21x^2 + 15)$,
 2nd expression = $5(3x^4 - 5x^3 + 29x^2 - 15)$
 $- 15 + 29x^2 - 5x^3 + 3x^4$
 $-(15 + 21x^2 - 5x^3 + 2x^4) \times (-1)$

$$\begin{array}{r} 5x^4 | 50x^4 - 5x^3 - 5x^3 + 5x^3 \\ \hline 10 - x - x^2 + x^3 \\ \hline 15 + 21x^2 - 5x^3 + 2x^4 \\ \hline 30 + 42x^2 - 10x^3 + 4x^4 \\ \hline -(10 - x - x^2 + x^3) 3 \\ \hline 45x - 7x^2 - 3x^3 + 4x^4 \\ \hline 90x - 14x^2 - 6x^3 + 8x^4 \\ \hline -(10 - x - x^2 + x^3) 9x \\ \hline -x^4 - 5x^3 + 3x^3 - x^4 \\ \hline 5 - 3x + x^2 \\ \hline 10 - x - x^2 + x^3 \\ \hline -(5 - 3x + x^2) 2 \\ \hline 5x - 3x^2 + x^3 \\ \hline -(5 - 3x + x^2) x \\ \hline \end{array}$$

the required H C $\Gamma = 5 - 3x + x^2$

21 1st expression = $9(2x^4 + 13x^3 + 18x^2 + 8x + 1)$,
 2nd expression = $12x^4 + 68x^3 + 72x^2 + 108x + 20$
 $12x^4 + 68x^3 + 72x^2 + 108x + 20$
 $-(2x^4 + 13x^3 + 18x^2 + 8x + 1) \times 6$
 $-2 | -10x^3 - 36x^2 + 60x + 14$
 $5x^3 + 18x^2 - 30x - 7$
 $2x^4 + 13x^3 + 18x^2 + 8x + 1$
 5
 $10x^3 + 65x^2 + 90x + 40x + 5$
 $-(5x^3 + 18x^2 - 30x - 7) 2x$
 $29x^3 + 150x^2 + 54x + 5$
 10
 $290x^3 + 1500x^2 + 540x + 50$
 $-(5x^3 + 18x^2 - 30x - 7) 58$
 $456 | 456x^3 + 2280x^2 + 456$
 $x^3 + 5x^2 + 1$

$$\begin{array}{r}
 5x^3 + 18x^2 - 30x - 7 \\
 -(x^3 + 5x + 1) 5x \\
 \hline
 -7x^2 - 35x - 7 \\
 -(x^3 + 5x + 1) \times (-7) \\
 \hline
 \end{array}$$

the required H C F = $x^2 + 5x + 1$

22

$$\begin{array}{r}
 x^5 - 5x^2 + 6x + 12 \\
 -(x^4 - 8x^3 - 24x - 32)x \\
 \hline
 8x^3 + 19x^2 + 38x + 12 \\
 x^4 - 8x^2 - 24x - 32 \\
 8 \\
 \hline
 8x^4 - 64x^2 - 192x - 256 \\
 -(8x^3 + 19x^2 + 38x + 12)x \\
 \hline
 -19x^3 - 102x^2 - 204x - 256 \\
 8 \\
 \hline
 -152x^3 - 816x^2 - 1632x - 2048 \\
 -(8x^3 + 19x^2 + 38x + 12)(-19) \\
 \hline
 -455 \mid -455x^3 - 910x^2 - 1820x - 2048 \\
 x^3 + 2x + 4 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8x^3 + 19x^2 + 38x + 12 \\
 -(x^3 + 2x + 4) 8x \\
 \hline
 3x^2 + 6x + 12 \\
 -(x^3 + 2x + 4) 3 \\
 \hline
 \end{array}$$

the reqd H C F = $x^3 + 2x + 4$

23

$$\begin{array}{r}
 x^5 - 4x^3 + 45x + 75 \\
 -(x^4 + 5x^3 + 3x^2 - 14x - 40)x \\
 \hline
 -5x^4 - 7x^3 + 14x^2 + 85x + 75 \\
 -(x^4 + 5x^3 + 3x^2 - 14x - 40) \times (-1) \\
 \hline
 18x^3 + 29x^2 + 15x - 125 \\
 x^4 + 5x^3 + 3x^2 - 14x - 40 \\
 18 \\
 \hline
 18x^4 + 90x^3 + 54x^2 - 252x - 720 \\
 -(18x^3 + 29x^2 + 15x - 125)x \\
 \hline
 61x^3 + 39x^2 - 127x - 720 \\
 18 \\
 \hline
 1098x^3 + 702x^2 - 2286x - 12960 \\
 -(18x^3 + 29x^2 + 15x - 125) 61 \\
 \hline
 -1067 \mid -1067x^3 - 3201x^2 - 5335x - 12960 \\
 x^3 + 3x + 5 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 18x^3 + 29x^2 + 15x - 125 \\
 -(x^3 + 3x + 5) 18 \\
 \hline
 -25x^2 - 75x - 125 \\
 -(x^3 + 3x + 5) \times (-25)
 \end{array}$$

the required H C F = $x^3 + 3x + 5$

- 24 1st expression = $2 \times 2 \times (x^5 - 2x^3a^2 + 7x^2a^3 - 6xa^4 + 6a^5)$,
 2nd expression = $2 \times 3 \times (x^4 + 4x^3a - 2x^2a^2 - 4xa^3 + 16a^4)$

$$\begin{array}{r}
 x^5 - 2x^3a^2 + 7x^2a^3 - 6xa^4 + 6a^5 \\
 -(x^4 + 4x^3a - 2x^2a^2 - 4xa^3 + 16a^4) x \\
 \hline
 -4x^4a + 11x^3a^2 - 22xa^4 + 6a^5 \\
 -(x^4 + 4x^3a - 2x^2a^2 - 4xa^3 + 16a^4) \times (-4a) \\
 \hline
 a^5 | 16x^4a^2 + 3x^3a^3 - 38xa^4 + 70a^5 \\
 16x^3 + 3x^2a - 38xa^2 + 70a^3 \\
 \hline
 x^4 + 4x^3a - 2x^2a^2 - 4xa^3 + 16a^4 \\
 16 \\
 \hline
 16x^4 + 64x^3a - 32x^2a^2 - 64xa^3 + 256a^4 \\
 -(16x^3 + 3x^2a - 38xa^2 + 70a^3)x \\
 \hline
 61x^3a + 6x^2a^2 - 134xa^3 + 256a^4 \\
 16 \\
 \hline
 976x^3a + 96a^2x^3 - 2144xa^3 + 4096a^4 \\
 -(16x^3 + 3x^2a - 38xa^2 + 70a^3) 61a \\
 \hline
 -87a^3 - 87x^2a^2 + 174xa^3 - 174a^4 \\
 x^2 - 2xa + 2a^2 \\
 \hline
 16x^3 + 3x^2a - 38xa^2 + 70a^3 \\
 -(x^2 - 2xa + 2a^2) 16x \\
 \hline
 35x^2a - 70xa^2 + 70a^3 \\
 -(x^2 - 2xa + 2a^2) 35a
 \end{array}$$

the reqd H C F = $2(x^2 - 2xa + 2a^2)$

- 25 $6x^5 + 41x^4y + 5x^3y^2 + 4x^2y^3 + 8y^5$

$$\begin{array}{r}
 3 \\
 18x^5 + 12x^4y + 15x^3y^2 + 12x^2y^3 + 24y^5 \\
 -(9x^4 + 18x^3y - 13x^2y^2 - 38xy^3 - 12y^4) 2x \\
 \hline
 48x^4y + 41x^3y^2 + 88x^2y^3 + 24xy^4 + 24y^5 \\
 -(9x^4 + 18x^3y - 13x^2y^2 - 38xy^3 - 12y^4) \times (-2y) \\
 \hline
 xy | 66x^4y + 5x^3y^2 + 62x^2y^3 - 52xy^4 \\
 66x^3 + 5x^2y + 62xy^2 - 52y^3
 \end{array}$$

$$\begin{array}{r}
9x^4 - 18x^3y - 13x^2y^2 - 38xy^3 - 12y^4 \\
\underline{13} \\
117x^4 - 234x^3y - 169x^2y^2 - 494xy^3 - 156y^4 \\
- (66x^3 + 5x^2y + 62xy^2 - 52y^3) 3y \\
\underline{117x^4 - 432x^3y - 184x^2y^2 - 680xy^3} \\
22 \\
2574x^4 - 9504x^3y - 4048x^2y^2 - 14960xy^3 \\
- (66x^3 + 5x^2y + 62xy^2 - 52y^3) 39x \\
\underline{- 3233xy^4 - 9699x^2y^3 - 6466x^3y^2 - 12932xy^3} \\
3x^2 + 21y + 4y^2 \\
66x^3 + 5x^2y + 62xy^2 - 52y^3 \\
- (3x^2 + 2xy + 4y^2) 22x \\
\underline{- 39x^3y - 26xy^2 - 52y^3} \\
- (3x^2 + 2xy + 4y^2) \times (-13y)
\end{array}$$

the required H C F = $3x^2 + 21y + 4y^2$

$$\begin{array}{r}
26 \quad 4x^5 + 11x^4 \qquad \qquad \qquad + 81 \\
- (2x^5 \qquad - 11x^3 \qquad - 9)^2 \\
\underline{11 \mid 11x^4 + 22x^3 \qquad \qquad + 99} \\
x^4 + 2x^2 \qquad \qquad + 9 \\
2x^5 \qquad - 11x^3 - 9 \\
- (x^4 + 2x^2 \qquad \qquad + 9) 21 \\
\underline{- 1 \mid - 4x^3 - 11x^2 - 18x - 9} \\
4x^3 + 11x^2 + 18x + 9 \\
x^4 \qquad \qquad + 2x^2 + 9 \\
4 \\
4x^4 \qquad + 8x^2 \qquad + 36 \\
- (4x^3 + 11x^2 + 18x + 9)x \\
\underline{- 11x^3 - 10x^2 - 9x + 36} \\
4 \\
- 44x^3 - 40x^2 - 36x + 144 \\
- (4x^3 + 11x^2 + 18x + 9) \times (-11) \\
\underline{81 \mid 81x^2 + 162x + 243} \\
x^2 + 2x + 3 \\
4x^3 + 11x^2 + 18x + 9 \\
- (x^2 + 2x + 3) 4x \\
\underline{3x^2 + 6x + 9} \\
- (x^2 + 2x + 3) 3
\end{array}$$

the required H C F = $x^2 + 2x + 3$

- 27 1st expression = $5a(12a^4 + 2a^3 - 9a^2 + 9a - 10)$,
 2nd expression = $2(16a^4 + 52a^3 - 10a^2 - 61a + 15)$

$$\begin{array}{r}
 16a^4 + 52a^3 - 10a^2 - 61a + 15 \\
 \underline{3} \\
 48a^4 + 156a^3 - 30a^2 - 183a + 45 \\
 -(12a^4 + 2a^3 - 9a^2 + 9a - 10)^4 \\
 \hline
 148a^3 + 6a^2 - 219a + 85 \\
 12a^4 + 2a^3 - 9a^2 + 9a - 10 \\
 \underline{37} \\
 444a^4 + 74a^3 - 333a^2 + 333a - 370 \\
 -(148a^3 + 6a^2 - 219a + 85) 3a \\
 \hline
 2156a^3 + 324a^2 + 78a - 370 \\
 28a^3 + 162a^2 + 39a - 185 \\
 \underline{37} \\
 1036a^3 + 5994a^2 + 1443a - 6845 \\
 -(148a^3 + 6a^2 - 219a + 85) 7 \\
 \hline
 148815952a^2 + 2976a - 7440 \\
 4a^2 + 2a - 5 \\
 148a^3 + 6a^2 - 219a + 85 \\
 -(4a^2 + 2a - 5) 37a \\
 \hline
 -68a^3 - 34a + 85 \\
 -(4a^2 + 2a - 5) \times (-17)
 \end{array}$$

the required H C F = $4a^2 + 2a - 5$

28

$$\begin{array}{r}
 3x^6 - 3x^4 - 18x^3 + x^2 + 2x + 3 \\
 -(x^6 + 2x^4 - 5x^3 - 7x + 3) 3x \\
 \hline
 -6x^6 - 3x^4 - 3x^3 + 22x^2 - 71 + 3 \\
 -(x^6 + 2x^4 - 5x^3 - 7x + 3) \times (-6) \\
 \hline
 9x^4 - 3x^3 - 8x^2 - 49x + 21 \\
 x^6 + 2x^4 - 5x^3 - 7x + 3 \\
 \underline{9} \\
 9x^6 + 18x^4 - 45x^3 - 63x + 27 \\
 -(9x^4 - 3x^3 - 8x^2 - 49x + 21)x \\
 \hline
 21x^4 + 8x^3 + 4x^2 - 84x + 27 \\
 \underline{3} \\
 63x^4 + 24x^3 + 12x^2 - 252x + 81 \\
 -(9x^4 - 3x^3 - 8x^2 - 49x + 21) 7 \\
 \hline
 45x^3 + 68x^2 + 91x - 66
 \end{array}$$

$$\begin{array}{r}
9x^4 - 3x^3 - 8x^2 - 49x + 21 \\
\hline
5 \\
\hline
45x^4 - 15x^3 - 40x^2 - 245x + 105 \\
-(45x^3 + 68x^2 + 91x - 66) \times 2 \\
\hline
-83x^3 - 131x^2 - 179x + 105 \\
\hline
22 \\
\hline
-1826x^3 - 2882x^2 - 3938x + 2310 \\
-(45x^3 + 68x^2 + 91x - 66) \times (-35) \\
\hline
-2512x^3 - 2511x^2 - 5021x - 7531 \\
\hline
x^2 + 2x + 3 \\
45x^3 + 68x^2 + 91x - 66 \\
-(x^2 + 2x + 3) \times 45x \\
\hline
-22x^2 - 44x - 66 \\
\hline
-(x^2 + 2x + 3) \times (-22) \\
\hline
\text{the required H C F} = x^2 + 2x + 3
\end{array}$$

Exercise 51.

- 1 Let $A = x^3 - 3x^2 - 4x + 12$,
and $B = x^3 - 7x^2 + 16x - 12$,
 $A + B = 2x^3 - 10x^2 + 12x = 2x(x^2 - 5x + 6)$
and $A - B = 4x^2 - 20x + 24 = 4(x^2 - 5x + 6)$,
the required H C F $= x^2 - 5x + 6$

- 2 Let $A = 2x^3 - 17x + 12$,
and $B = 4x^4 - 2x^3 - 34x^2 + 41x - 12$,
 $A + B = 4x^4 - 34x^2 + 24x$
 $= 2x(2x^3 - 17x + 12)$,
and $2A - B = 2x^3 - 17x + 12$,
the required H C F $= 2x^3 - 17x + 12$

- 3 Let $A = 4x^3 + 13x^2 + 19x + 4$,
and $B = 2x^3 + 5x^2 + 5x - 4$,
 $A + B = 6x^3 + 18x^2 + 24x = 6x(x^2 + 3x + 4)$,
and $A - 2B = 3x^3 + 9x + 12 = 3(x^2 + 3x + 4)$,
the required H C F $= x^2 + 3x + 4$

- 4 Let $A = 3x^3 - 5x^2 + 7$,
and $B = 6x^4 - 7x^3 - 5x^2 + 14x + 7$,

$$\begin{aligned}
 B - A &= 6x^4 - 10x^3 + 14x \\
 &= 2x(3x^3 - 5x^2 + 7), \\
 \text{the required H C F} &= 3x^3 - 5x^2 + 7
 \end{aligned}$$

5 Let $A = 6x^4 - 11x^3 + 16x^2 - 22x + 8$,
 and $B = 6x^4 - 11x^3 - 8x^2 + 22x - 8$,
 $A + B = 12x^4 - 22x^3 + 8x^2 = 2x^2(6x^2 - 11x + 4)$,
 and $A - B = 24x^2 - 44x + 16 = 4(6x^2 - 11x + 4)$,
 the required H C F $= 6x^2 - 11x + 4$

6 Let $A = 2x^4 + 19x^3 + 20x^2 - 31x + 8$,
 and $B = 2x^4 + 7x^3 - 64x^2 + 62x + 6$,
 $A - B = 12x^3 + 84x^2 - 93x + 24$
 $= 3(4x^3 + 28x^2 - 31x + 8)$

Again let $C = 4x^3 + 28x^2 - 31x + 8$,
 $A - C = 2x^4 + 15x^3 - 8x^2$
 $= x^2(2x^2 + 15x - 8)$

Let $C' = 2x^2 + 15x - 8$,
 $C + C' = 4x^3 + 30x^2 - 16x$
 $= 2x(2x^2 + 15x - 8)$,
 the required H C F $= 2x^2 + 15x - 8$

7 Let $A = 3x^4 - 7x^3 - 27x^2 - 6x + 2$,
 and $B = 3x^4 - 13x^3 - 40x^2 - 9x + 3$,
 $A - B = 6x^3 + 13x^2 + 3x - 1 = C$ (suppose),
 and $3A - 2B = 3x^4 + 5x^3 - x^2 = x^2(3x^2 + 5x - 1)$

Let $C' = 3x^2 + 5x - 1$,
 $C - C' = 6x^3 + 10x^2 - 2x = 2x(3x^2 + 5x - 1)$,
 the required H C F $= 3x^2 + 5x - 1$

8 Let $A = 5x^4 - 18x^3 - 7x^2 + 12x + 3$,
 and $B = 5x^4 - 23x^3 - 9x^2 + 16x + 4$,
 $A - B = 5x^3 + 2x^2 - 4x - 1 = C$ (suppose),
 and $4A - 3B = 5x^4 - 3x^3 - x^2 = x^2(5x^2 - 3x - 1)$

Let $C' = 5x^2 - 3x - 1$,
 $C - C' = 5x^3 - 3x^2 - x = x(5x^2 - 3x - 1)$,
 the required H C F $= 5x^2 - 3x - 1$

- 9 Let $A = 2x^3 - 5x^2 - 17x + 2$,
and $B = 6x^5 + 23x^4 + 34x^3 + 21x^2 - 2x - 2$,
 $A + B = 6x^5 + 25x^4 + 29x^3 + 4x^2 - 4x$
 $= x(6x^4 + 25x^3 + 29x^2 + 4x - 4)$
Also let $6x^4 + 25x^3 + 29x^2 + 4x - 4 = C$,
 $2A - C = 10x^4 + 15x^3 - 5x^2 - 5x^2(2x^2 + 3x - 1)$
Let $B' = 2x^2 + 3x - 1$,
 $C - 4B' = 6x^4 + 25x^3 + 21x^2 - 8x$
 $= x(6x^3 + 25x^2 + 21x - 8)$
Again let $C' = 6x^3 + 25x^2 + 21x - 8$,
 $C' - 8B' = 6x^3 + 9x^2 - 3x = 3x(2x^2 + 3x - 1)$,
the required $H \ C \ \Gamma = 2x^2 + 3x + 1$
- 10 Let $A = 6x^6 + 9x^4 - 13x^2 - 4x^2 + 9x - 3$,
and $B = 9x^6 + 12x^4 - 18x^2 - 5x^2 + 12x - 4$,
 $3A - 2B = 3x^4 - 3x^2 - 2x^2 + 3x - 1 = C$ (suppose),
 $A - 3C = 6x^6 - 4x^3 + 2x^2 = 2x^2(3x^4 - 2x + 1)$
Also let $3x^4 - 2x + 1 = C'$,
 $C + C' = 3x^4 - 2x^3 + 1 = x(3x^3 - 2x + 1)$,
the required $H \ C \ \Gamma = 3x^3 - 2x + 1$
- 11 Let $A = x^6 - x^2 + 8$, and $B = x^6 - x^2 + 4$,
 $A - B = -x^2 + x^2 + 4 = C$ (suppose),
and $2B - A = x^6 + x^2 - 2x^2 = x^2(x^4 + x - 2)$
Let $C = x^3 + x - 2$,
 $C + C' = x^2 + x + 2 = A'$ (suppose),
 $A' + C' = x^3 + x^2 + 2x = x(x^2 + x + 2)$,
the required $H \ C \ F = x^2 + x + 2$
- 12 Let $A = 3x^6 + 139x^2 - 44$,
and $B = 39x^6 + 139x^4 - 16$,
 $B - 13A = 139x^4 - 1807x^2 + 556$
 $= 139(x^4 - 13x^2 + 4)$
Let $C = x^4 - 13x^2 + 4$,
 $B + 4C = 39x^6 + 143x^4 - 52x^2$
 $= 13x^2(3x^4 + 11x^2 - 4)$

Also let $A' = 3x^3 + 11x^2 - 4$,

$$\begin{aligned} C + A' &= x^4 + 3x^3 - 2x^2 \\ &= x^2(x^2 + 3x - 2) \end{aligned}$$

Let $x^2 + 3x - 2 = B'$,

$$\begin{aligned} A' - 2B' &= 3x^3 + 9x^2 - 6x \\ &= 3x(x^2 + 3x - 2), \end{aligned}$$

the required H C F $= x^2 + 3x - 2$

Exercise 52.

$$\begin{array}{r} 1 \quad 2x^3 + 7x^2 - 5x - 4 \\ -(x^3 + 8x^2 + 11x - 20) 2 \\ \hline -9 \mid -9x^2 - 27x + 36 \\ \quad x^2 + 3x - 4 \end{array}$$

$$\begin{array}{r} \quad 2x^3 + 8x^2 + 11x - 20 \\ -(x^2 + 3x - 4) 1 \\ \hline \quad 5x^2 + 15x - 20 \\ -(x^2 + 3x - 4) 5 \\ \hline \quad 2x^3 + 19x^2 + 49x + 20 \\ -(x^2 + 3x - 4) 21 \\ \hline \quad 13x^2 + 57x + 20 \\ -(x^2 + 3x - 4) 13 \\ \hline \quad 18 \mid 18x + 72 \\ \quad \quad x + 4 \end{array}$$

$$\begin{array}{r} \quad x^2 + 3x - 4 \\ -(x + 4)x \\ \hline \quad -x - 4 \\ -(x + 4) \times (-1) \end{array}$$

the reqd H C F $= x + 4$

$$\begin{array}{l} 2 \quad \text{Let } A = 2x^4 + 3x^3 + 8x^2 + 15x - 10, \\ \text{and } B = 2x^4 - 5x^3 + 12x^2 - 25x + 10, \\ A + B = 4x^4 - 2x^3 + 20x^2 - 10x \\ = 2x(2x^3 - x^2 + 10x - 5), \end{array}$$

$$\begin{array}{l} \text{and } A - B = 8x^3 - 4x^2 + 40x - 20 \\ = 4(2x^3 - x^2 + 10x - 5), \end{array}$$

the H C F of A and $B = 2x^3 - x^2 + 10x - 5$

$$\begin{array}{r} 2x^4 - 5x^3 + 10x^2 - 20x + 8 \\ -(2x^3 - x^2 + 10x - 5)x \\ \hline \quad -4x^3 - 15x + 8 \\ -(2x^3 - x^2 + 10x - 5) \times (-2) \\ \hline \quad -2x^3 + 5x - 2 \end{array}$$

$$\begin{array}{r} 2x^3 - x^2 + 10x - 5 \\ -(-2x^2 + 5x - 2) \times (-1) \\ \hline 4x^2 + 8x - 5 \\ -(-2x^2 + 5x - 2) \times (-2) \\ \hline 9 \mid 182 - 9 \\ \quad 2x - 1 \end{array}$$

$$\begin{array}{r} -2x^2 + 5x - 2 \\ -(2x - 1) \times (-x) \\ \hline 4x - 2 \\ -(2x - 1) \cdot 2 \end{array}$$

the reqd H C F = $2x - 1$

- 3 Let $A = 2x^4 + 5x^2 - 16x^2 - 10x + 24$,
and $B = 2x^4 + 5x^2 - 10x^2 + 5x - 12$,
 $A - B = -6x^2 - 15x + 36 = -3(2x^2 + 5x - 12)$,
and $A + 2B = 6x^4 + 15x^2 - 36x^2 = 3x^2(2x^2 + 5x - 12)$,
the H C F of A and $B = 2x^2 + 5x - 12$

$$\begin{array}{r} 2x^4 + 7x^3 - 19x^2 - 14x + 30 \\ -(2x^3 + 5x - 12)x^2 \\ \hline 2x^3 - 7x^2 - 14x + 30 \\ -(2x^3 + 5x - 12)x \\ \hline -12x^2 - 21 + 30 \\ -(2x^3 + 5x - 12) \times (-6) \\ \hline 14 \mid 28x - 42 \\ \underline{2x - 3} \end{array}$$

the required H C F = $2x - 3$

- 4 Let $A = 2x^4 - 4x^3 - 69x^2 - 2x - 35$,
and $B = 2x^4 - 6x^3 - 55x^2 - 3x - 28$,
 $A - B = 2x^2 - 14x^2 + x - 7$,
and $5B - 4A = 2x^4 - 14x^3 + x^2 - 7x$
 $= x(2x^3 - 14x^2 + x - 7)$,

$$\begin{array}{r} 2x^4 + 18x^3 + 41x^2 + 9x + 20 \\ -(2x^3 - 14x^2 + x - 7)x \\ \hline 4 \mid 32x^3 + 40x^2 + 16x + 20 \\ \quad 8x^3 + 10x^2 + 4x + 5 \\ \hline \quad -(2x^3 - 14x^2 + x - 7) \cdot 4 \\ \quad \quad 33 \mid 66x^2 + 33 \\ \quad \quad \quad 2x^2 + 1 \end{array}$$

the required H C F = $2x^2 + 1$

- 5 Let $A = 3a^3 + 28a^2b + 52ab^2 - 48b^3$,
and $B = 3a^3 + 4a^2b - 28ab^2 + 16b^3$.

$$A - B = 24a^2b + 80ab^2 - 64b^3$$

$$= 80(3a^2 + 10ab - 8b^2),$$

$$\text{and } A + 3B = 12a^3 + 40a^2b - 32ab^2$$

$$= 4a(3a^2 + 10ab - 8b^2),$$

$$\text{the H C F of } A \text{ and } B = 3a^2 + 10ab - 8b^2$$

$\begin{array}{r} 3a^3 + 10a^2b - 44ab^2 + 24b^3 \\ - (3a^2 + 10ab - 8b^2)a \\ \hline -12b^2 \quad \quad -36ab^2 + 24b^3 \\ \quad \quad 3a \quad -2b \end{array}$	$\begin{array}{r} 3a^2 + 10ab - 8b^2 \\ - (3a - 2b)a \\ \hline 12ab - 8b^2 \\ - (3a - 2b)4b \\ \hline \end{array}$
--	--

$$\text{the required H C F} = 3a - 2b$$

6 Let $A = 6a^3 - 37a^2b + 57ab^2 - 20b^3$,

$$\text{and } B = 3a^3 - 8a^2b - 31ab^2 + 60b^3,$$

$$2B - A = 21a^2b - 119ab^2 + 140b^3$$

$$= 7b(3a^2 - 17ab + 20b^2),$$

$$\text{and } 3A + B = 21a^3 - 119a^2b + 140ab^2$$

$$= 7a(3a^2 - 17ab + 20b^2),$$

$$\text{the H C F of } A \text{ and } B = 3a^2 - 17ab + 20b^2$$

$\begin{array}{r} 6a^3 + 5a^2b - 34ab^2 + 15b^3 \\ - (3a^2 - 17ab + 20b^2)2a \\ \hline 39a^2b - 74ab^2 + 15b^3 \\ - (3a^2 - 17ab + 20b^2)13b \\ \hline 49b^2 \quad \quad 147ab^2 - 245b^3 \\ \quad \quad 3a - 5b \end{array}$	$\begin{array}{r} 3a^2 - 17ab + 20b^2 \\ - (3a - 5b)a \\ \hline -12ab + 20b^2 \\ - (3a - 5b) \times (-4b) \\ \hline \end{array}$
---	--

$$\text{the required H C F} = 3a - 5b$$

7 Let $A = 3x^4 + 11x^3 - 32x^2 - 44x + 80$,

$$\text{and } B = 3x^4 - x^3 - 52x^2 + 124x - 80,$$

$$A + B = 6x^4 + 10x^3 - 84x^2 + 80x$$

$$= 2x(3x^3 + 5x^2 - 42x + 40),$$

$$\text{and } A - B = 12x^3 + 20x^2 - 168x + 160$$

$$= 4(3x^3 + 5x^2 - 42x + 40),$$

$$\text{the H C F of } A \text{ and } B = 3x^3 + 5x^2 - 42x + 40$$

$$= C \text{ (suppose)}$$

$$\text{Let } C = 3x^3 + 5x^2 - 42x + 40,$$

$$C - 5C = 3x^4 - 13x^3 - 108x^2 + 160x$$

$$= x(3x^3 - 13x^2 - 108x + 160)$$

$$= xD \text{ (suppose),}$$

$$\begin{aligned} C-D &= 18x^2 + 66x - 120 \\ &= 6(3x^2 + 11x - 20), \end{aligned}$$

$$\begin{aligned} \text{and } 4C-D &= 9x^3 + 33x^2 - 60x \\ &= 3x(3x^2 + 11x + 20), \end{aligned}$$

the H C F of A , B and C

$$= 3x^2 + 11x - 20$$

$$\begin{array}{r} 3x^4 + 2x^3 - 20x^2 - 8x + 32 \\ -(3x^2 + 11x - 20) \cdot 1^2 \end{array}$$

$$-9x^3 - 8x + 32$$

$$-(3x^2 + 11x - 20) \times (-3x)$$

$$33x^3 - 66x + 32$$

$$-(3x^2 + 11x - 20) \cdot 11$$

$$-63x - 189x + 252$$

$$3x - 4$$

the required H C F = $3x - 4$

$$3x^2 + 11x - 20$$

$$-(3x - 4) \cdot x$$

$$15x - 20$$

$$-(3x - 4) \cdot 5$$

8

$$\text{Let } A = 6x^6 - 28x^4 + 17x^3 + 54x^2 - 39x - 18,$$

$$\text{and } B = 6x^5 + 8x^4 - 79x^3 - 36x^2 + 105x + 36,$$

$$B-A = 36x^4 - 96x^3 - 90x^2 + 144x + 54$$

$$= 6(6x^4 - 16x^3 - 15x^2 + 24x + 9),$$

$$\text{and } 2A+B = 18x^6 - 48x^4 - 45x^3 + 72x^2 + 27x$$

$$= 3x(6x^5 - 16x^3 - 15x^2 + 24x + 9),$$

the H C F of A & $B = 6x^4 - 16x^3 - 15x^2 + 24x + 9$

= C' (suppose)

$$\text{Let also } C = 6x^6 + 14x^4 - 53x^3 - 37x^2 + 66x + 24,$$

$$\text{and } D = 2x^6 - 2x^4 - 31x^3 + 51x^2 + 42x - 72,$$

$$C-3D = 20x^4 + 40x^3 - 190x^2 - 60x + 240$$

$$= 10(2x^4 + 4x^3 - 19x^2 - 6x + 24) = 10 E \text{ (suppose),}$$

$$C-E = 6x^6 + 12x^4 - 57x^3 - 18x^2 + 72x$$

$$= 3x(2x^5 + 4x^3 - 19x^2 - 6x + 24),$$

the H C F of C and D

$$= 2x^4 + 4x^3 - 19x^2 - 6x + 24 = D' \text{ (suppose),}$$

$$3D'-C' = 28x^3 - 42x^2 - 42x + 63$$

$$= 7(4x^3 - 6x^2 - 6x + 9)$$

$$= 7 E' \text{ (suppose),}$$

$$\begin{aligned}
 C' - E' &= 6r^4 - 20r^3 - 9r^2 + 30r \\
 &= 1(6r^3 - 20r^2 - 9r + 30) \\
 &= 1 G \text{ (suppose) ,} \\
 3E' - 2G &= 22x^2 - 33 = 11(2x^2 - 3), \\
 \text{and } 10E' - 3G &= 22x^3 - 33r = 11r(2x^2 - 3), \\
 \text{the required H C F} &= 2x^2 - 3
 \end{aligned}$$

Exercise 53

- 1 1st expression $= a^2 \times b$,
 2nd „ $= a \times b^2$,
 the required L C M $= a^2 \times b^2 = a^2 b^2$
- 2 1st expression $= a^3 \times b^3$
 2nd „ $= a^2 \times b \times c$,
 the required L C M $= a^3 \times b^2 \times c = a^3 b^2 c$
- 3 1st expression $= 2 \times 3 \times r^2 \times y^4$,
 2nd „ $= 2 \times 5 \times x \times y^2$,
 the required L C M $= 2 \times 3 \times 5 \times r^2 \times y^4 = 30r^2 y^4$
- 4 1st expression $= 2^2 \times m^3 \times n^3$,
 2nd „ $= 2^2 \times 7 \times m^4 \times n^3 \times p$,
 the required L C M $= 2^2 \times 7 \times m^4 \times n^3 \times p = 28m^4 n^3 p$
- 5 1st expression $= 2^3 \times x^2 \times y^3 \times z$,
 2nd „ $= 2^2 \times 3 \times r^3 \times y^2 \times z^2$,
 the required L C M $= 2^3 \times 3 \times r^3 \times y^3 \times z^2 = 24x^3 y^3 z^2$
- 6 1st expression $= 2^3 \times a^2 \times b \times c$,
 2nd „ $= 2 \times 5 \times a \times b^2 \times c$,
 3rd „ $= 2 \times 7 \times a \times b \times c^2$,
 the reqd L C M $= 2^3 \times 5 \times 7 \times a^2 \times b^2 \times c^2$
 $= 140a^2 b^2 c^2$
- 7 1st expression $= 2^3 \times a^3 \times b^3 \times c$,
 2nd „ $= 2^2 \times 3 \times a \times b^3 \times c^2$,
 3rd „ $= 2^2 \times 5 \times a^2 \times b \times c^3$,
 the reqd L C M $= 2^3 \times 3 \times 5 \times a^3 \times b^3 \times c^3 = 120a^3 b^3 c^3$

- 8 1st expression $= 2 \times 3 \times x^4 \times y$,
 2nd „ $= 3^2 \times 1^2 \times y^3 \times x$,
 3rd „ $= 2^2 \times 3 \times a^2 \times 1 \times y^3$,
 4th „ $= 3 \times 5 \times a \times x \times x^2$,
 the reqd L C M $= 3^2 \times 2^2 \times 5 \times 1^1 \times y^3 \times x^3 \times a^2$
 $= 180x^4y^3x^2a^2$
- 9 1st expression $= ab(a^2 - b^2) = ab(a+b)(a-b)$,
 2nd „ $= a^2b^2(a+b)$,
 the reqd L C M $= a^2b^2(a+b)(a-b) = a^2b^2(a^2 - b^2)$
- 10 1st expression $= 2^3 \times (1-y)^2$,
 2nd „ $= 2 \times 3 \times (1-y)(1+y)$,
 3rd „ $= 2^3 \times (x+y)^3$,
 the reqd L C M $= 2^3 \times 3 \times (x-y)^3(1+y)^2 = 24(x^2 - y^2)^2$
- 11 1st expression $= (1-3)(x-1)$,
 2nd „ $= (x-2)(1-3)$,
 the reqd L C M $= (1-1)(1-2)(1-3)$
- 12 1st expression $= a(a^2 + 2ax - 3x^2) = a(a+3x)(a-x)$,
 2nd „ $= a^2(a^2 + ax - 6x^2) = a^2(a+3x)(a-x)$,
 the reqd L C M $= a^2(a-x)(a-2x)(a+3x)$
- 13 1st expression $= a^2(a-2)(a+2)$,
 2nd „ $= a^2(a^2 + 2a - 8) = a^2(a+4)(a-2)$,
 the reqd L C M $= a^2(a-2)(a+2)(a+4)$
- 14 1st expression $= 2^3 \times a^2 \times x^2$,
 2nd „ $= 2 \times x \times (x-a)(x+a)$,
 3rd „ $= 2 \times 3 \times a^3 \times x(x+a)(x^2 - ax + a^2)$,
 the required L C M
 $= 2^3 \times 3 \times a^3 \times x^2 \times (1-a)(x+a)(x^2 - ax + a^2)$
 $= 12a^3x^2(x-a)(1+a)(x^2 - ax + a^2)$
- 15 1st expression $= 2^2 \times 3 \times (x+5)(1-2)$,
 2nd „ $= 2^4 \times (x+6)(x-2)$,
 the reqd L C M $= 2^4 \times 3 \times (1+6)(x+5)(x-2)$
 $= 48(1+6)(x+5)(x-2)$

- 16 1st expression $= (x+5)(x-3)$,
 2nd „ $= (x+5)(x+4)$,
 3rd „ $= (x+7)(x-3)$,
 the reqd L C M $= (x+7)(x+5)(x+4)(x-3)$
- 17 1st expression $= 3a^2(4a^2-9b^2) = 3a^2(2a+3b)(2a-3b)$,
 2nd „ $= (2a+3b)(a-b)$,
 3rd „ $= (2a-3b)(a+b)$,
 the reqd L C M $= 3a^2(2a+3b)(2a-3b)(a+b)(a-b)$
 $= 3a^2(4a^2-9b^2)(a^2-b^2)$
- 18 1st expression $= (2a+3b)(4a^2-6ab+9b^2)$,
 2nd „ $= (2a-3b)(4a^2+6ab+9b^2)$,
 3rd „ $= 16x^4+72a^2b^2+81b^4-36a^2b^2$
 $= (4a^2+9b^2)^2-6ab^2$
 $= (4a^2+6ab+9b^2)(4a^2-6ab+9b^2)$,
 the reqd L C M
 $= (2a+3b)(2a-3b)(4a^2+6ab+9b^2)$
 $= (8a^3+27b^3)(8a^2-27b^2)$
- 19 1st expression $= 2x^2(4x^2-25y^2) = 2x^2(2x+5y)(2x-5y)$,
 2nd „ $= 3x(4x^2+8xy-5y^2) = 3x(2x+5y)(2x-y)$,
 3rd „ $= 4(4x^2-12xy+5y^2) = 4(2x-5y)(2x-y)$,
 the reqd L C M $= 3 \times 4 \times x^2(2x+5y)(2x-5y)(2x-y)$
 $= 12x^2(4x^2-25y^2)(2x-y)$
- 20 1st expression $= (2x-3a)^2$,
 2nd „ $= (3x+a)(2x-3a)$,
 3rd „ $= (3x-a)(2x-3a)$,
 the reqd L C M $= (2x-3a)^2(3x+a)(3x-a)$
 $= (2x-3a)^2(9x^2-a^2)$
- 21 1st expression $= 2x^2+6x+9$,
 2nd „ $= 2x^2(2x^2-6x+9)$,
 3rd „ $= 4x^4+36x^2+81-36x^2$
 $= (2x^2+9)^2-(6x)^2$
 $= (2x^2+6x+9)(2x^2-6x+9)$,
 the reqd L C M $= 2x(2x^2+6x+9)(2x^2-6x+9)$
 $= 2x(4x^4+81)$

- 22 1st expression $= (3a-1)^2$,
 2nd „ $= 2(3a^2 + 5a - 2) = 2(3a-1)(a+2)$,
 3rd „ $= 3(3a^2 - 7a + 2) = 3(3a-1)(a-2)$,
 the reqd L C M $= 2 \times 3(3a-1)^2(a+2)(a-2)$
 $= 6(3a-1)^2(a^2-4)$
- 23 1st expression $= (2x)^2 - 3(2x)^2 + 3(2x) - 1 = (2x-1)^2$,
 2nd „ $= (2x-1)(4x^2-1) = (2x-1)^2(2x+1)$,
 3rd „ $= (2x-1)(x+3)$,
 the reqd L C M $= (2x-1)^2(2x+1)(x+3)$
 $= (2x-1)^2(4x^2-1)(x+3)$
- 24 1st expression $= (x-4)(x-2)$,
 2nd „ $= (x-3)(x-4)$,
 3rd „ $= (x-3)(x+5)$,
 the reqd L C M $= (x-2)(x-3)(x-4)(x+5)$
- 25 1st expression $= (2x-1)(3x+1)$,
 2nd „ $= (3x+1)(x+2)$,
 3rd „ $= (2x-1)(x+2)$,
 the reqd L C M $= (2x-1)(3x+1)(x+2)$
- 26 1st expression $= (1+2x)^2 - (4x^2)^2$
 $= (1+2x+4x^2)(1+2x-4x^2)$,
 2nd „ $= (1+2x)(1-8x^2)$
 $= (1+2x)(1-2x)(1+2x+4x^2)$,
 the reqd L C M $= (1-4x^2)(1+2x+4x^2)(1+2x-4x^2)$
- 27 1st expression $= (9x^2-1)(x^2-3) = (3x+1)(3x-1)(x^2-3)$,
 2nd „ $= (9x^2-1)(3x^2-1) = (3x+1)(3x-1)(3x^2-1)$,
 3rd „ $= (9x^2-1)(3x^2+1) = (3x+1)(3x-1)(3x^2+1)$,
 4th „ $= (x^2-3)^2$,
 the reqd L C M $= (x^2-3)^2(9x^2-1)(9x^2+1)$

Exercise 54

1

$$\begin{array}{r} 3x^2 + 14x^2 + 13x - 8 \\ -(3x^3 + 2x^2 - 11x + 4) \\ \hline 12 \mid 12x^2 + 24x - 12 \\ \quad x^2 + 2x - 1 \end{array}$$

$$\begin{array}{r}
 3x^2 + 2x^2 - 11x + 4 \\
 -(x^2 - 2x - 1) 3x \\
 \hline
 -4x^2 - 8x - 4 \\
 -(x^2 - 2x - 1) \times (-4)
 \end{array}$$

Thus the H C F = $x^2 + 2x - 1$

The L C M

$$\begin{aligned}
 &= \frac{3x^2 + 2x^2 - 11x + 4}{x^2 + 2x - 1} (3x^2 + 14x^2 + 13x - 8) \\
 &= (3x - 4)(5x^2 + 14x + 13x - 8) \\
 &= 9x^4 + 30x^3 - 17x^2 - 76x + 32
 \end{aligned}$$

2

$$\begin{array}{r}
 6x^2 + 17x^2 + 9x - 1 \\
 -(6x^2 - 7x^2 - 27x - 8) \\
 \hline
 12x^2 + 36x - 12 \\
 12 \mid 24x^2 - 36x - 12 \\
 \hline
 2x^2 + 3x - 1 \\
 6x^2 - 7x^2 - 27x + 8 \\
 -(2x^2 + 3x - 1) 3x \\
 \hline
 -16x^2 - 24x + 8 \\
 -(2x^2 + 3x - 1) \times (-8)
 \end{array}$$

Thus the H C F = $2x^2 + 3x - 1$

The L C M

$$\begin{aligned}
 &= \frac{6x^2 - 7x^2 - 27x + 8}{2x^2 + 3x - 1} (6x^2 - 17x^2 + 9x - 4) \\
 &= (3x - 8)(6x^2 + 17x^2 + 9x - 4) \\
 &= 18x^4 - 3x^3 - 109x^2 - 84x + 32
 \end{aligned}$$

3

$$\begin{array}{r}
 12x^2 - 4x^2 - 25x + 12 \\
 -(12x^2 - 28x^2 - 7x - 12) \\
 \hline
 8x^2 + 24x^2 - 32x \\
 8x \mid 24x^2 - 32x \\
 \hline
 3x - 4 \\
 12x^2 - 28x^2 - 7x - 12 \\
 -(3x - 4) 4x^2 \\
 \hline
 -12x^2 - 7x - 12 \\
 -(3x - 4)(-4x) \\
 \hline
 -9x + 12 \\
 -(3x - 4)(-3)
 \end{array}$$

Thus the H C F = $3x - 4$

$$\begin{aligned}
 \text{The L C M} &= \frac{12x^3 - 28x^2 + 7x + 12}{3x - 4} (12x^3 - 4x^2 - 25x + 12) \\
 &= (4x^2 - 4x - 3)(12x^3 - 4x^2 - 25x + 12) \\
 &= 48x^5 - 64x^4 - 120x^3 + 160x^2 + 27x - 36
 \end{aligned}$$

4 Let $A = 9x^3 - 12x^2 - 15x + 20$,
 and $B = 15x^3 + 12x^2 - 25x - 20$,
 $A + B = 24x^3 - 40x = 8x(3x^2 - 5) = 8x C$ (suppose),
 $A + 4C = 9x^3 - 15x = 3x(3x^2 - 5)$

Thus, the H C F of A and $B = 3x^2 - 5$

$$\begin{aligned}
 \text{The L C M} &= \frac{9x^3 - 12x^2 - 15x + 20}{3x^2 - 5} (15x^3 + 12x^2 - 25x - 20) \\
 &= (3x - 4)(15x^3 + 12x^2 - 25x - 20) \\
 &= 45x^4 - 24x^3 - 123x^2 + 40x + 80
 \end{aligned}$$

5 Let $A = 4x^3 - 10x^2 - 18x + 45$,
 and $B = 6x^3 + 8x^2 - 27x - 36$,
 $2B - 3A = 46x^2 - 207 = 23(2x^2 - 9)$,
 and $B - 4C = 6x^3 - 27x = 3x(2x^2 - 9)$

Thus, the H C F of A and $B = 2x^2 - 9$

$$\begin{aligned}
 \text{The L C M} &= \frac{4x^3 - 10x^2 - 18x + 45}{2x^2 - 9} (6x^3 + 8x^2 - 27x - 36) \\
 &= (2x - 5)(6x^3 + 8x^2 - 27x - 36) \\
 &= 12x^4 - 14x^3 - 94x^2 + 63x + 180
 \end{aligned}$$

6 Let $A = 4x^4 + 4x^3 + 7x^2 + 11x + 4$,
 and $B = 6x^4 + 7x^3 + 4x^2 + 5x + 2$,
 $2B - A = 8x^4 + 10x^3 + x^2 - x$
 $= x(8x^3 + 10x^2 + x - 1) = x C$ (suppose),
 $B + 2C = 6x^4 + 23x^3 + 24x^2 + 7x$
 $= x(6x^3 + 23x^2 + 24x + 7) = x D$ (suppose),
 $7C + D = 62x^3 + 93x^2 + 31x$
 $= 31x(2x^2 + 3x + 1) = 31x C'$ (suppose),
 $C + C' = 8x^3 + 12x^2 + 4x = 4x(2x^2 + 3x + 1)$

Thus, the H C F of A and $B = 2x^2 + 3x + 1$

$$\begin{aligned}\text{The L C M} &= \frac{4x^4 + 4x^3 + 7x^2 + 11x + 4}{2x^2 + 3x + 1} (6x^4 + 7x^3 + 4x^2 + 5x + 2) \\ &= (2x^2 - 1 + 4)(6x^4 + 7x^3 + 4x^2 + 5x + 2) \\ &= 12x^6 + 8x^5 + 25x^4 + 34x^3 + 15x^2 + 18x + 18\end{aligned}$$

$$\begin{aligned}7 \quad \text{Let } A &= 8x^4 - 6x^3 - 8x^2 + 9x - 6, \\ \text{and } B &= 16x^4 - 12x^3 + 20x^2 - 9x + 6, \\ A + B &= 24x^4 - 18x^3 + 12x^2 \\ &= 6x^2(4x^2 - 3x + 2) = 6x^2 C \text{ (suppose)}, \\ A + 3C &= 8x^4 - 6x^3 + 4x^2 = 2x^2(4x^2 - 3x + 2)\end{aligned}$$

Thus, the H C F of A and $B = 4x^2 - 3x + 2$

$$\begin{aligned}\text{The L C M} &= \frac{8x^4 - 6x^3 - 8x^2 + 9x - 6}{4x^2 - 3x + 2} (16x^4 - 12x^3 + 20x^2 - 9x + 6) \\ &= (2x^2 - 3)(16x^4 - 12x^3 + 20x^2 - 9x + 6) \\ &= 32x^6 - 24x^5 - 8x^4 + 18x^3 - 48x^2 + 27x - 18\end{aligned}$$

$$\begin{aligned}8 \quad \text{Let } A &= 4x^4 + 8x^3 + 21x^2 + 18x + 27, \\ \text{and } B &= 3x^4 + 6x^3 + 17x^2 + 16x + 24, \\ 4B - 3A &= 5x^2 + 10x + 15 \\ &= 5(x^2 + 2x + 3) = 5C \text{ (suppose)}, \\ B - 8C &= 3x^4 + 6x^3 + 9x^2 = 3x^2(x^2 + 2x + 3)\end{aligned}$$

Thus, the H C F of A and B

$$= x^2 + 2x + 3$$

The L C M

$$\begin{aligned}&= \frac{4x^4 + 8x^3 + 21x^2 + 18x + 27}{x^2 + 2x + 3} (3x^4 + 6x^3 + 17x^2 + 16x + 24) \\ &= (4x^2 + 9)(3x^4 + 6x^3 + 17x^2 + 16x + 24) \\ &= 12x^6 + 24x^5 + 95x^4 + 118x^3 + 249x^2 + 144x + 216\end{aligned}$$

$$9 \quad \text{We have } l = \frac{xy}{h} \text{ or } xy = hl,$$

$$\text{and } h + l = x + y,$$

$$\begin{aligned}h^3 + l^3 &= (h + l)^3 - 3hl(h + l) \\ &= (x + y)^3 - 3xy(x + y) \\ &= x^3 + y^3.\end{aligned}$$

Exercise 55

1

$$\begin{array}{r}
 6x^2 + x - 2 \\
 -(3x^2 - 10x - 8) \cdot 2 \\
 \hline
 7 \overline{) 21x + 14} \\
 3x + 2 \\
 \hline
 3x^2 - 10x - 8 \\
 -(3x + 2) \cdot 2 \\
 \hline
 -12x - 8 \\
 -(3x + 2)(-4)
 \end{array}$$

Thus, the L C M of the 1st and 3rd expressions

$$\begin{aligned}
 &= \frac{3x^2 - 10x - 8}{3x + 2} (6x^2 + x - 2) \\
 &= (x - 4)(6x^2 + x - 2) = 6x^3 - 23x^2 - 6x + 8
 \end{aligned}$$

$$\begin{array}{r}
 6x^3 - 23x^2 - 6x + 8 \\
 \hline
 12x^3 - 46x^2 - 12x + 16 \\
 -(4x^2 - 20x + 9) \cdot 3x \\
 \hline
 14x^2 - 39x + 16 \\
 \hline
 28x^2 - 78x + 32 \\
 -(4x^2 - 20x + 9) \cdot 7 \\
 \hline
 31 \overline{) 62x - 31} \\
 2x - 1
 \end{array}
 \qquad
 \begin{array}{r}
 4x^2 - 20x + 9 \\
 -(2x - 1) \cdot 2x \\
 \hline
 -18x + 9 \\
 -(2x - 1) \times (-9)
 \end{array}$$

$$\begin{aligned}
 \text{The reqd L C M} &= \frac{4x^2 - 20x + 9}{2x - 1} (6x^3 - 23x^2 - 6x + 8) \\
 &= (2x - 9)(6x^3 - 23x^2 - 6x + 8) \\
 &= 12x^4 - 100x^3 + 195x^2 + 70x - 72
 \end{aligned}$$

2

$$\begin{array}{r}
 6x^2 - 7x - 3 \\
 -(3x^2 - 23x - 8) \cdot 2 \\
 \hline
 13 \overline{) 39x + 13} \\
 3x + 1
 \end{array}
 \qquad
 \begin{array}{r}
 3x^2 - 23x - 8 \\
 -(3x + 1) \cdot 2 \\
 \hline
 -24x - 8 \\
 -(3x + 1) \times (-8)
 \end{array}$$

Thus, the L C M of the 1st two expressions

$$\begin{aligned}
 &= \frac{3x^2 - 23x - 8}{3x + 1} (6x^2 - 7x - 3) \\
 &= (x - 8)(6x^2 - 7x - 3) \\
 &= 6x^3 - 55x^2 + 53x + 24
 \end{aligned}$$

$$\begin{array}{r} 6x^2 - 55x^2 + 53x + 24 \\ -(2x^2 - 11x + 12) \cdot 32 \\ \hline -22x^2 + 17x + 24 \\ -(2x^2 - 11x + 12) \times (-11) \\ \hline -52 \mid -104x + 156 \\ \quad 2x - 3 \end{array}$$

$$\begin{array}{r} 2x^2 - 11x + 12 \\ -(2x - 3)x \\ \hline -8x + 12 \\ -(2x - 3) \times (-4) \end{array}$$

The reqd L C M

$$\begin{aligned} &= \frac{2x^2 - 11x + 12}{2x - 3} (6x^3 - 55x^2 + 53x + 24) \\ &= (x - 4)(6x^3 - 55x^2 + 53x + 24) \\ &= 6x^4 - 79x^3 + 273x^2 - 188x - 96 \end{aligned}$$

$$\begin{array}{r} 3 \quad 12x^2 - 11x + 2 \\ -(6x^2 - 19x + 10) 2 \\ \hline 9 \mid 27x - 18 \\ \quad 3x - 2 \end{array}$$

$$\begin{array}{r} 6x^2 - 19x + 10 \\ -(3x - 2)2x \\ \hline -15x + 10 \\ -(3x - 2) \times (-5) \end{array}$$

Thus, the L C M of the 1st two expressions

$$\begin{aligned} &= \frac{6x^2 - 19x + 10}{3x - 2} (12x^2 - 11x + 2) \\ &= (2x - 5)(12x^2 - 11x + 2) \\ &= 24x^3 - 82x^2 + 59x - 10 \end{aligned}$$

$$\begin{array}{r} 24x^3 - 82x^2 + 59x - 10 \\ -(8x^2 + 10x - 3) 3x \\ \hline -112x^3 + 68x - 10 \\ -(8x^2 + 10x - 3) \times (-14) \\ \hline 52 \mid 208x - 52 \\ \underline{1x - 1} \end{array}$$

$$\begin{array}{r} 8x^2 + 10x - 3 \\ -(4x - 1)2x \\ \hline 12x - 3 \\ -(4x - 1)3 \end{array}$$

The required L C M

$$\begin{aligned} &= \frac{8x^2 + 10x - 5}{4x - 1} (24x^3 - 82x^2 + 59x - 10) \\ &= (2x + 3)(24x^3 - 82x^2 + 59x - 10) \\ &= 48x^4 - 92x^3 - 128x^2 + 157x - 30 \end{aligned}$$

4 Let $A = 4x^4 + 5x^3 - 7x^2 - 6x + 3$.

and $B = 2x^4 + 4x^3 + x^2 + 6x - 3$.

$$A + B = 6x^4 + 12x^3 - 6x^2$$

$$= 6x^2(x^2 + 2x - 1) = 6^2 C \text{ (suppose) ,}$$

$$B-3C=2x^4+4x^3-2x^2=2x^2(x^2+2x-1)$$

Thus, the H C F of A and $B = x^2 + 2x - 1$,
and the L C M of the 1st two expressions

$$\begin{aligned}
 &= \frac{2x^4 + 4x^3 + x^2 + 6x - 3}{x^2 + 2x - 1} (4x^4 + 8x^3 - 7x^2 - 6x + 3) \\
 &= (2x^2 + 3)(4x^4 + 8x^3 - 7x^2 - 6x + 3) \\
 &= 8x^6 + 16x^5 - 2x^4 + 12x^3 - 15x^2 - 18x + 9 \\
 &\quad \begin{array}{r} 8x^6 + 16x^5 - 2x^4 + 12x^3 - 15x^2 - 18x + 9 \\ -(8x^4 + 4x^3 - 2x^2 - 3x - 3)x^2 \\ \hline 12x^6 + 15x^3 - 12x^2 - 18x + 9 \end{array} \\
 &\quad \begin{array}{r} 2 \\ \hline 24x^6 + 30x^3 - 24x^2 - 36x + 18 \\ -(8x^4 + 4x^3 - 2x^2 - 3x - 3)3x \\ \hline -12x^4 + 36x^3 - 15x^2 - 27x + 18 \end{array} \\
 &\quad \begin{array}{r} 2 \\ \hline -24x^4 + 72x^3 - 36x^2 - 54x + 36 \\ -(8x^4 + 4x^3 - 2x^2 - 3x - 3)(-3) \\ \hline 318x^3 - 36x^2 - 63x + 27 \\ 28x^3 - 12x^2 - 21x + 9 \end{array} \\
 &\quad \begin{array}{r} 8x^3 + 4x^2 - 2x^2 - 3x - 3 \\ 7 \\ \hline 56x^3 + 28x^2 - 14x^2 - 21x - 21 \\ -(28x^3 - 12x^2 - 21x + 9)2x \\ \hline 52x^2 + 28x^2 - 39x - 21 \end{array} \\
 &\quad \begin{array}{r} 7 \\ \hline 364x^2 + 196x^2 - 273x - 147 \\ -(28x^3 - 12x^2 - 21x + 9)13 \\ \hline 881352x^2 - 264 \end{array} \\
 &\quad \begin{array}{r} 4x^2 - 3 \\ 28x^3 - 12x^2 - 21x + 9 \\ -(4x^2 - 3)7x \\ \hline -12x^2 + 9 \\ -(4x^2 - 3)(-3) \end{array}
 \end{aligned}$$

The reqd L C M

$$\begin{aligned}
 &= \frac{8x^4 + 4x^3 - 2x^2 - 3x - 3}{4x^2 - 3} (8x^6 + 16x^5 - 2x^4 + 12x^3 - 15x^2 - 18x + 9) \\
 &= (2x^2 + x + 1)(8x^6 + 16x^5 - 2x^4 + 12x^3 - 15x^2 - 18x + 9) \\
 &= 16x^8 + 40x^7 + 20x^6 + 38x^5 - 20x^4 - 39x^3 - 15x^2 - 9x + 9
 \end{aligned}$$

Miscellaneous Exercise (2).

I.

- 1 See Art I, Chap V Page 45

Since $-4 = -1 - 1 - 1 - 1$,

$$\begin{aligned} (-4) \times (-8) &= -(-8) - (-8) - (-8) - (-8) \\ &= 8 + 8 + 8 + 8 = 32 \end{aligned}$$

- 2 (i)
- $y^3(x-2z) - y^3xz + y(xz^2 - x^3 - 2z^3) + (x^3z - xz^3)$

(ii) $(xy^3 - z^3y) + z(x^3 - xy^2 - 2y^3) + z^2xy - z^3(x + 2y)$

- 3
- $$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3x \frac{1}{x} \left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$
- $$= p^3 + 3p$$

- 4
- $$x^5 - y^5 = x^4(x-y) + x^3y(1-y) + x^2y^2(x-y) + xy^3(x-y) + y^4(x-y),$$

$$\frac{x^5 - y^5}{x - y} = x^4 + x^3y + x^2y^2 + xy^3 + y^4$$

- 5 The given expression

$$\begin{aligned} &= \{(a+b+c) + (a-b+c)\} \{(a+b+c) - (a-b+c)\} \\ &\quad + \{(a+b-c) + (b+c-a)\} \\ &\quad \quad \{(a+b-c) - (b+c-a)\} \\ &= (2a+2c) 2b + 2b(2a-2c) = 4b\{(a+c) + (a-c)\} \\ &= 4b \ 2a = 8ab \end{aligned}$$

Again $8ab = 8 \ a \ b = 8 \times (-4) \times (-4) = 128$

- 6
- $$(a-b+c+d)(a+b+c-d)$$
- $$= \{(a+c) - (b-d)\} \{(a+c) + (b-d)\} = (a+c)^2 - (b-d)^2$$

- 7
- $$4x^2 + 12xy + 9y^2 - 8x - 12y = (2x+3y)^2 - 4(2x+3y)$$
- $$= (2x+3y)(2x+3y-4)$$

- 8 Let
- $A = 2x^4 - 12x^3 + 19x^2 - 6x + 9$
- ,

and $B = 4x^3 - 18x^2 + 19x - 3$,

$$A + 3B = 2x^4 - 35x^3 + 51x^2$$

$$= x(2x^3 - 35x^2 + 51x) = x \ C \text{ (suppose) },$$

$$\begin{aligned}
 2C - B &= 18x^2 - 89x + 105 = D \text{ (suppose),} \\
 \text{and } 17B + C &= 70x^3 - 306x^2 + 288x \\
 &= 2x(35x^2 - 153x + 144) = 2x E \text{ (suppose),} \\
 35D - 18E &= -361x + 1083 \\
 &= -361(x - 3) = -361 F \text{ (suppose),} \\
 D + 35F &= 18x^2 - 54x = 18x(x - 3) \\
 \text{The reqd H C F} &= x - 3
 \end{aligned}$$

II

- 1 See Art 3 (ii) Ch V, Page 49

$$\begin{array}{r}
 2 \quad x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 2x^{\frac{1}{6}} + 1 \\
 \underline{x^{\frac{1}{3}} - 2x^{\frac{1}{6}} + 1} \\
 x + 2x^{\frac{5}{6}} + 3x^{\frac{1}{2}} + 2x^{\frac{1}{3}} + x^{\frac{1}{6}} \\
 - 2x^{\frac{5}{6}} - 4x^{\frac{2}{3}} - 6x^{\frac{1}{2}} - 4x^{\frac{1}{3}} - 2x^{\frac{1}{6}} \\
 \hline
 x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 2x^{\frac{1}{6}} + 1 \\
 \hline
 x \qquad \qquad - 2x^{\frac{1}{2}} \qquad \qquad + 1
 \end{array}$$

- 3 Putting m for $ac + bd$ and n for $ad - bc$, the left-hand expression

$$\begin{aligned}
 &= (mx + ny)^2 + (my - nx)^2 \\
 &= m^2x^2 + n^2y^2 + 2mxy + m^2y^2 + n^2x^2 - 2mynx \\
 &= m^2x^2 + m^2y^2 + n^2y^2 + n^2x^2 \\
 &= m^2(x^2 + y^2) + n^2(x^2 + y^2) \\
 &= (m^2 + n^2)(x^2 + y^2) \\
 &= (a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2acbd)(x^2 + y^2) \\
 &= \{a^2(c^2 + d^2) + b^2(c^2 + d^2)\}(x^2 + y^2) \\
 &= (a^2 + b^2)(c^2 + d^2)(x^2 + y^2)
 \end{aligned}$$

- 4 $x^3 - 2x + 1 = (x - 1)^3$, the required cube
 $= \{(x - 1)^2\}^3 = (x - 1)^6$

$$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$$

$$\begin{array}{r}
 5 \quad x-1 \overline{) \begin{array}{l} x^5 - px^4 + ax^3 - cr^2 + px - 1 \\ x^5 - x^4 \\ \hline -x^4(p-1) + cr^2 - px + 1 \\ -x^4(p-1) + (p-1)x^3 \\ \hline (a-p+1)x^3 - cr^2 + px - 1 \\ (a-p+1)x^3 - (a-p+1)x^2 \\ \hline -(p-1)x^2 + px - 1 \\ -(p-1)x^2 + (p-1)x \\ \hline x - 1 \\ \hline r - 1 \end{array}} \quad \begin{array}{l} \text{Quotient} = x^4 - (p-1)x^3 \\ + (a-p+1)x^2 - (p-1)x \\ + 1 \end{array}
 \end{array}$$

$$\begin{array}{l}
 6 \quad (i) \quad ab - ac - b^2 + bc = a(b-c) - b(b-c) = (b-c)(a-b) \\
 (ii) \quad b^2 - 12ac - 4a^2 - 9c^2 = b^2 - (4a^2 + 12ac + 9c^2) \\
 \quad \quad \quad = b^2 - (2a + 3c)^2 = (b + 2a + 3c)(b - 2a - 3c)
 \end{array}$$

$$\begin{array}{l}
 7 \quad \text{1st. expression} = 2x^2 - 16 - 15x - 30 \\
 \quad \quad \quad = 2(x^2 - 8) - 15(x - 2) \\
 \quad \quad \quad = 2(x-2)(x^2 + 2x + 4) - 15(x-2) \\
 \quad \quad \quad = (x-2)(2x^2 + 4x + 8 - 15) \\
 \quad \quad \quad = (x-2)(2x^2 - 4x - 7), \\
 \text{and expression} = x^4 - 15x^2 + 28x - 12 \\
 \quad \quad \quad = x^4 - 15x^2 + 44 + 28x - 56 \\
 \quad \quad \quad = (x^2 - 11)(x^2 - 4) + 28x - 2 \\
 \quad \quad \quad = (x-2)(x^2 - 2x^2 - 11x - 22 - 28) \\
 \quad \quad \quad = (x-2)(x^2 + 2x^2 - 11x + 6),
 \end{array}$$

$$\text{the reqd H C F.} = x-2$$

$$\begin{array}{l}
 8 \quad \text{1st. expression} = x^2(x-b) + a^2x + b \\
 \quad \quad \quad = (x+b)(x^2 + a) \\
 \text{2nd expression} = x^2 - ax + bx - ab \\
 \quad \quad \quad = x(x-a) + b(x-a) \\
 \quad \quad \quad = (x-a)(x+b) \\
 \text{the reqd L C M} \\
 \quad \quad \quad = (x+b)(x-a)(x^2 + a) \\
 \quad \quad \quad = \{x^2 + (b-a)x - ab\}(x^2 + a) \\
 \quad \quad \quad = x^4 + (b-a)x^3 + (a-b)x^2 + (ab-a^2)x - a^2b
 \end{array}$$

III.

$$\begin{aligned} 1 \quad a^3 + b^3 &= (a+b)^3 - 3ab(a+b) \\ &= (8)^3 - 3 \cdot 5 \cdot 8 = 512 - 120 = 392 \end{aligned}$$

$$\begin{aligned} 2 \quad \text{The given expression} &= (7c)^2 - 2\{3(a+b)7c\} + \{3(a+b)\}^2 \\ &= \{7c - 3(a+b)\}^2 \\ &= \{7 \cdot 8 - 3(89 - 69)\}^2 \\ &= (56 - 3 \cdot 20)^2 = (-4)^2 = 16 \end{aligned}$$

$$\begin{aligned} 3 \quad \text{The dividend} &= x^3(y-z) + y^3z - y^3x + z^3x - z^3y \\ &= x^3(y-z) + yz(y^2-z^2) - x(y^3-z^3) \\ &= (y-z)\{x^3 + yz(y+z) - x(y^2+yz+z^2)\} \\ &= (y-z)\{x^3 + x^2y + x^2z - x^2(y+z) - x(y+z)^2 \\ &\quad + yz(y+z)\} \\ &= (y-z)\{x^2(x+y+z) - x(y+z)(x+y+z) \\ &\quad + yz(x+y+z)\} \\ &= (y-z)(x+y+z)(x^2 - xy - xz + yz), \end{aligned}$$

$$\text{and the divisor} = xy - xz + yz - z^2$$

$$= x(y-z) + (y+z)(y-z) = (y-z)(x+y+z);$$

$$\therefore \text{the quotient} = x^2 - xy - xz + yz$$

$$\begin{aligned} 4 \quad 4a - 3 + 16a^2 + 64a^3 &= (4a-1) + \{(4a)^2 - 1\} + \{(4a)^3 - 1\} \\ &= (4a-1)\{1 + (4a+1) + (16a^2 + 4a + 1)\} \\ &= (4a-1)(16a^2 + 8a + 3) \end{aligned}$$

$$\begin{aligned} 5 \quad (1+x+x^2)^2 - (1-x+x^2)^2 &= \{(1+x+x^2) + (1-x+x^2)\} \{(1+x+x^2) - (1-x+x^2)\} \\ &= (2+2x^2) \cdot 2x = 4x(1+x^2) \end{aligned}$$

$$6 \quad \text{The left-hand expression}$$

$$\begin{aligned} &= ns^2 - 2(a_1 + a_2 + a_3 + \dots + a_n)s \\ &\quad + a_1^2 + a_2^2 + a_3^2 + \dots + a_n^2 \\ &= ns^2 - 2 \cdot \frac{ns}{2} s + (a_1^2 + a_2^2 + a_3^2 + \dots + a_r^2) \\ &= ns^2 - ns^2 + (a_1^2 + a_2^2 + a_3^2 + \dots + a_r^2) \\ &= a_1^2 + a_2^2 + a_3^2 + \dots + a_r^2 \end{aligned}$$

7 The 1st expression

$$= (x^4 - x^2) - (p^4 - p^2) + (q^4 - q)$$

$$= (x^2 - 1)(x^2 - p^2 + q),$$

the 2nd expression

$$= (x^4 - x^2) - (q^4 - q^2) + (p^4 - p)$$

$$= (x^2 - 1)(x^2 - q^2 + p),$$

the reqd H C F $= x^2 - 1$

8 1st expression $= x^3 - 7x^2 + 10x + 6x - 12$

$$= x(x-2)(x-5) + 6(x-2)$$

$$= (x-2)(x^2 - 5x + 6)$$

$$= (x-2)(x-2)(x-3),$$

2nd expression $= x(3x^2 - 14x + 16)$

$$= x(3x^2 - 6x - 8x - 16)$$

$$= x(3x-8)(x-2),$$

the reqd L C M

$$= x(x-2)(x-2)(x-3)(3x-8)$$

$$= x(x^3 - 7x^2 + 16x - 12)(3x-8)$$

$$= 3x^6 - 29x^4 + 104x^3 - 164x^2 + 96x$$

IV

$$1 \quad a^5 - 2a^2b^{\frac{1}{2}} + 4a^{\frac{1}{2}}b^{\frac{3}{2}} - 8ab + 16a^{\frac{3}{2}}b^{\frac{1}{2}} - 32b^{\frac{5}{2}}$$

$$a^{\frac{1}{2}} + 2b^{\frac{1}{2}}$$

$$a^3 - 2a^{\frac{5}{2}}b^{\frac{1}{2}} + 4a^2b^{\frac{3}{2}} - 8a^{\frac{3}{2}}b + 16ab^{\frac{5}{2}} - 32a^{\frac{1}{2}}b^{\frac{3}{2}}$$

$$+ 2a^{\frac{5}{2}}b^{\frac{1}{2}} - 4a^2b^{\frac{3}{2}} + 8a^{\frac{3}{2}}b - 16ab^{\frac{5}{2}} + 32a^{\frac{1}{2}}b^{\frac{3}{2}} - 64b^{\frac{5}{2}}$$

$$a^3 \qquad \qquad \qquad - 64b^{\frac{5}{2}}$$

$$2 \quad (i) \quad a^3 - 3abc + (b^3 + c^3),$$

$$(ii) \quad a^2(b-c) - a(b^2 - c^2) + (b^3c - bc^3),$$

$$(iii) \quad a^3(b-c) - a(b^4 - c^4) + (b^4c - bc^4)$$

$$3 \quad (x+a)(x+b)(x+c) = \{x^2 + (a+b)x + ab\}(x+c)$$

$$= x^3 + (a+b+c)x^2 + (ab+ac+bc)x + abc$$

$$\begin{aligned}\text{Hence } (1-7)(1+8)(1-12) \\ &= 1^3 + (-7+8-12)1^2 + \{(-7 \cdot 8) + (-7)(-12) \\ &\quad + 8(-12)\}1 + 7(-8)(-12) \\ &= 1^3 - 111^2 - 681 + 672,\end{aligned}$$

the co-efficients of 1^2 and 1 are respectively -11 and -68

4 The left-hand expression

$$\begin{aligned}&= \{(ab+cd) + (ac+bd)\} \{(ab+cd) - (ac+bd)\} \\ &= (ab+cd)^2 - (ac+bd)^2 \\ &= a^2b^2 + 2abcd + c^2d^2 - (a^2c^2 + 2abcd + b^2d^2) \\ &= a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2\end{aligned}$$

5 The given expression

$$\begin{aligned}&= (a-b-c)^2 = \{(q+r+s) - (r+s-p) - (p+q-r)\}^2 \\ &= r^2\end{aligned}$$

6 The dividend $= a^3 + (2b)^3 + (3c)^3 - 3a(2b)(3c)$

$$\begin{aligned}&= (a+2b+3c)\{a^2+(2b)^2+(3c)^2 - a \cdot 2b - a \cdot 3c \\ &\quad - 2b \cdot 3c\} \\ &= (a+2b+3c)(a^2+4b^2+9c^2-2ab-3ac-6bc),\end{aligned}$$

the reqd quotient $= a+2b+3c$

7 The Highest Common Factor of two or more algebraical expressions is that Common factor which is resolvable into the greatest number of elementary factors

$$\text{1st expression} = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 1^2 a^4 c^6,$$

$$\text{2nd expression} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x y^2 a^3 b^4,$$

$$\text{3rd expression} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot y^3 a^1 b^2 c,$$

$$\text{the reqd H C F} = 2 \cdot 2 \cdot 3 \cdot a^1 = 12a^3$$

8 1st expression $= (x-y)(x+y),$

$$\text{2nd expression} = (x-y)(1-y),$$

$$\text{3rd expression} = (x-y)(x^2+xy+y^2),$$

the reqd L C M

$$= (1-y)^2 (x+y)(x^2+xy+y^2)$$

$$= (1-y)(x+y)(1-y^3)$$

$$= (x^2-y^2)(1^3-y^3) = x^5 - x^2y^2 - x^2y^3 + y^6$$

V.

- 1 The given expression

$$\begin{aligned}
 &= (7a)^2 + 2(7a)(9b) + (9b)^2 \\
 &= (7a + 9b)^2 = (322 - 333)^2 = (-11)^2 = 121
 \end{aligned}$$

- 2 The left-hand expression

$$\begin{aligned}
 &= s^2 - 2as + a^2 + s^2 - 2sb + b^2 + s^2 - 2sc + c^2 + s^2 \\
 &= 4s^2 - 2s(a + b + c) + (a^2 + b^2 + c^2) \\
 &= 4s^2 - 2s \cdot 2s + a^2 + b^2 + c^2 \\
 &= 4s^2 - 4s^2 + (a^2 + b^2 + c^2) = a^2 + b^2 + c^2
 \end{aligned}$$

- 3 Putting
- x
- for
- $5a - 7c$
- and
- y
- for
- $8c - 3a$
- , we get
- $x + y = 2a + c$
- , the given expression becomes

$$\begin{aligned}
 &= x^2 + y^2 + 3(x + y)xy = (x + y)^3 = (2a + c)^3 \\
 &= 8a^3 + 12a^2c + 6ac^2 + c^3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad a - b &= \left(a^{\frac{1}{4}} - b^{\frac{1}{4}}\right) \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) \\
 &= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) \left(a^{\frac{1}{4}} - b^{\frac{1}{4}}\right) \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right), \\
 \frac{a - b}{a^{\frac{1}{4}} - b^{\frac{1}{4}}} &= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) \\
 &= a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad (i) \quad \text{The given expression} &= 6a^4x^2 - 6a^2x^3 + a^3x - a^2x^2 \\
 &= 6a^2x^2(a - x) + a^2x(a - x) \\
 &= (6a^2x^2 + a^2x)(a - x) \\
 &= a^2x(6ax + 1)(a - x)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \text{The given expression} &= xjz^2 + z^2z + j^2z + xy \\
 &= xz(jz + 1) + j(jz + z) \\
 &= (jz + z)jz + j^2z
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \text{1st. expression} &= 2a^2 + a(b - c) - (3b^2 + 3bc + bc + c^2) \\
 &= 2a^2 + a(b - c) - (b + c)(3b + c) \\
 &= 2a^2 - 2a(b + c) + a(3b + c) - (b + c)(3b + c) \\
 &= 2a(a - b - c) + (3b + c)(a - b - c) \\
 &= (a - b - c)(2a + 3b + c),
 \end{aligned}$$

$$\begin{aligned}
 \text{2nd expression} &= 2a^2 - a(5b - 9c) - 4(3b^2 - 2bc - c^2) \\
 &= 2a^2 - a(5b - 9c) - 4(b - c)(3b + c)
 \end{aligned}$$

$$\begin{aligned}
 &= 2a^2 - 8a(b-c) + a(3b+c) - 4(b-c)(3b+c) \\
 &= 2a(a-4b+4c) + (3b+c)(a-4b+4c) \\
 &= (2a+3b+c)(a-4b+4c),
 \end{aligned}$$

the reqd H C F $= 2a + 3b + c$

7 See Art 3 (col) Chap X Page 155

$$\begin{aligned}
 8 \quad 1st \text{ expression} &= 61^2 - 91^2 - 21^2 + 51 - 3 \\
 &= 31^2(21-3) - (21-3)(1-1) \\
 &= (21-3)(31^2 - 1 + 1),
 \end{aligned}$$

$$\begin{aligned}
 2nd \text{ expression} &= 91^2 - 61^2 - 31^2 + 51 - 2 \\
 &= 31^2(31-2) - (31-2)(1-1) \\
 &= (31-2)(31^2 - 1 + 1),
 \end{aligned}$$

$$\begin{aligned}
 \text{the reqd L C M} &= (21-3)(31-2)(31^2-1+1) \\
 &= (61^2-131+6)(31^2-1+1) \\
 &= 181^2 - 451^2 + 371^2 - 191 + 6
 \end{aligned}$$

VI

1 The given expression

$$\begin{aligned}
 &= (8765943)^2 - (8765938)^2 \\
 &= (8765943 + 8765938) \times (8765943 - 8765938) \\
 &= 17531881 \times 5 = 87659405
 \end{aligned}$$

2 The given expression

$$\begin{aligned}
 &= (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3 \\
 &= (3a+4b)^3 = (87-92)^3 = (-5)^3 = -125
 \end{aligned}$$

3 $a + (b+c)$ $\frac{a^3 - 3abc}{a^3 + a^2b + a^2c} + \frac{(b^3 + c^3)}{a(b^2 + c^2 - bc)}$ Quotient $= \frac{a^2 - a(b+c)}{a(b^2 + c^2 - bc)}$

$$\begin{aligned}
 &\frac{-a^2(b+c) - 3abc + (b^3 + c^3)}{-a^2(b+c) - a(b^2 + c^2 + 2bc)} \\
 &\frac{a(b^2 + c^2 - bc) + (b^3 + c^3)}{a^3 + b^3 + c^3 - 3abc} \\
 &= (a+b+c)(a^2 + b^2 + c^2 - ab - ac - bc) \\
 &= \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc) \\
 &= \frac{1}{2}(a+b+c)\{(a^2 + b^2 - 2ab) + (a^2 + c^2 - 2ac) + (b^2 + c^2 - 2bc)\} \\
 &= \frac{1}{2}(a+b+c)\{(a-b)^2 + (c-a)^2 + (b-c)^2\}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad (a-b)(a-c)(b-c) &= \{a^2 - a(b+c) + bc\}(b-c) \\
 &= a^2(b-c) - a(b^2 - c^2) + b^2c - bc^2 \\
 &= a^2(b-c) + b^2(c-a) + c^2(a-b)
 \end{aligned}$$

Putting a for $(x+2y+3z)$, b for $(y+2z+3x)$ and c for $(z+2x+3y)$ we have

$$a-b = x-2x+y,$$

$$b-c = x-2y+z,$$

$$\text{and } c-a = y-2z+x,$$

the given expression becomes

$$\begin{aligned}
 &= a^2(b-c) + b^2(c-a) + c^2(a-b) + (b-c)(c-a)(a-b) \\
 &= (a-b)(a-c)(b-c) - (b-c)(a-c)(a-b) = 0
 \end{aligned}$$

$$\begin{aligned}
 5 \quad &(a^2 - b^2 - c^2 + d^2)^2 - 4(ad - bc)^2 \\
 &= (a^2 - b^2 - c^2 + d^2)^2 - (2ad - 2bc)^2 \\
 &= \{(a^2 - b^2 - c^2 + d^2) + (2ad - 2bc)\} \\
 &\quad \{(a^2 - b^2 - c^2 + d^2) - (2ad - 2bc)\} \\
 &= \{(a^2 + d^2 + 2ad) - (b^2 + c^2 + 2bc)\} \\
 &\quad \{(a^2 + d^2 - 2ad) - (b^2 + c^2 - 2bc)\} \\
 &= \{(a+d)^2 - (b+c)^2\} \{(a-d)^2 - (b-c)^2\} \\
 &= (a+d+b+c)(a+d-b-c)(a-d+b-c)(a-d-b+c) \\
 &= (a+b+c+d)(a-b-c+d)(a+b-d-c)(a-b+c-d)
 \end{aligned}$$

$$6 \quad (i) \quad a^2 - 2ab + b^2 + 2a - 2b = (a-b)^2 + 2(a-b) = (a-b)(a-b+2)$$

$$\begin{aligned}
 (ii) \quad 6a^2 - ab - b^2 + 6a - 3b &= (6a^2 - 3ab + 2ab - b^2) + 6a - 3b \\
 &= (3a+b)(2a-b) + 3(2a-b) \\
 &= (2a-b)(3a+b+3)
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad 15x^2 - 4xy - 4y^2 + 10x + 4y &= 15x^2 + 6xy - 10xy - 4y^2 + 10x + 4y \\
 &= 3x(5x+2y) - 2y(5x+2y) + 2(5x+2y) \\
 &= (5x+2y)(3x-2y+2)
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \text{The dividend} &= (2x-y)^2 a^4 - \{(x+y)^2 a^2 x^2 - 2(x+y)ax x^3 + x^6\} \\
 &= (2x-y)^2 a^4 - \{(x+y)ax - x^3\}^2 \\
 &= \{(2x-y)a^2 + (x+y)ax - x^3\} \{(2x-y)a^2 \\
 &\quad - (x+y)ax + x^3\},
 \end{aligned}$$

$$\text{the required quotient} = (2x-y)a^2 + (x+y)ax - x^3$$

$$\begin{aligned}
 8 \quad 1st \text{ expression} &= x^4 - a^2x^2 - b^2x^2 + a^2b^2 \\
 &= (x^2 - a^2)(x^2 - b^2) = (x+a)(x-a)(x+b)(x-b) \\
 2nd \text{ expression} &= x^4 - \{(a+b)^2x^2 + 2ab(a+b)x - a^2b^2\} \\
 &= x^4 - \{(a+b)x - ab\}^2 \\
 &= \{x^2 + (a+b)x - ab\} \{x^2 - (a+b)x + ab\} \\
 &= \{x^2 + (a+b)x - ab\} (x-a)(x-b), \\
 \text{the required H C F} &= (x-a)(x-b) \\
 &= x^2 - x(a+b) + ab
 \end{aligned}$$

VII

- 1 $2(xy + yz + zx) = (x+y+z)^2 - (x^2 + y^2 + z^2)$
 $= (8)^2 - 50 = 64 - 50 = 14,$
 $\therefore xy + yz + zx = 7$
- 2 Putting x for $(2a-3b)$, y for $(3b-5c)$ and z for $(5c-2a)$, we get $x+y+z=0$, and the left-hand expression

$$\begin{aligned}
 &= x^2 + y^2 + z^2 = (x+y+z)^2 - 2xy - 2yz - 2zx \\
 &= -2xy - 2yz - 2zx \\
 &= -2(2a-3b)(3b-5c) - 2(3b-5c)(5c-2a) \\
 &\quad - 2(5c-2a)(2a-3b) \\
 &= -2(3b-5c)(3b-2a) + 2(5c-2a)(5c-3b) + 2(2a-3b)(2a-5c)
 \end{aligned}$$
3. The dividend $= (x+a)\{(x-a)a^2 - ab(x+a) + b(x^2 - xa + a^2)\}$
 $= (x+a)\{a^2(x-a) - abx - a^2b + bx^2 - abx + a^2b\}$
 $= (x+a)\{a^2(x-a) + bx(x-2a)\},$
 the quotient $= x+a$
- 4 (i) $6x^2 + x - 15 = 6x^2 + 10x - 9x - 15$
 $= 2x(3x+5) - 3(3x+5) = (3x+5)(2x-3)$
 (ii) $35(x-y)^2 - 41(x-y) + 12$
 $= 35(x-y)^2 - 20(x-y) - 21(x-y) + 12$
 $= 5(x-y)\{7(x-y)-4\} - 3\{7(x-y)-4\}$
 $= (7x-7y-4)(5x-5y-3)$

$$(iii) \quad 11x^2 - 54xy^2 + 63y^4$$

$$= 11x^2 - 33xy^2 - 21xy^2 + 63y^4$$

$$= 11x(x - 3y^2) - 21y^2(x - 3y^2)$$

$$= (x - 3y^2)(11x - 21y^2)$$

$$5 \quad (x+y)(y+z)(z+x) = \{y^2 + y(x+z) + z\}(z+x)$$

$$= \{y(y+1+z) + z\}(z+x)$$

$$= y(z+1)(y+1+z) + rz(z+x)$$

$$= y(z+1)(1+y+z) + rz(x+y+z) - ryz$$

$$= (1+y+z)(yz + rz + rz) - ryz$$

$$6 \quad (a) \quad (x+y)(y+z)(z+x) = (x+y+z)(yz + rz + rz) - ryz$$

$$= 0 \times (yz + rz + rz) - ryz = -ryz$$

$$(b) \quad x^2 + y^2 + z^2 = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$$

$$= 0 \times (x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz = 3xyz$$

7 The given expression

$$= (a-b)\{c^2 + c(a+b) + ab\} + (b-c)$$

$$\{a^2 + a(b+c) + bc\} + (c-a)\{b^2 + b(c+a) + ca\}$$

$$= \{c^2(a-b) + a^2(b-c) + b^2(c-a)\}$$

$$+ \{a(a^2 - b^2) + a(b^2 - c^2) + b(c^2 - a^2)\}$$

$$+ \{ab(a-b) + bc(b-c) + ca(c-a)\}$$

$$= (a-b)(a-c)(b-c) + (a-b)(b-c)(c-a)$$

$$+ (a-b)(a-c)(b-c)$$

$$= 2(a-b)(a-c)(b-c) - (a-b)(b-c)(a-c)$$

$$= (a-b)(a-c)(b-c)$$

$$8 \quad 1st \text{ expression} = 35x^2 - 21x + 10x - 6$$

$$= 7x(5x-3) + 2(5x-3)$$

$$= (5x-3)(7x+2)$$

$$2nd \text{ expression} = 40x^2 - 24x - 5x + 3$$

$$= 8x(5x-3) - (5x-3)$$

$$= (8x-1)(5x-3)$$

$$reqd \text{ L.C.M.} = (5x-3)(7x+2)(8x-1)$$

$$= (7x+2)(40x^2 - 29x + 3)$$

$$= 280x^3 - 123x^2 - 37x + 6$$

VIII

$$1 \quad x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + xz) \\ = (15)^2 - 2(85) = 225 - 170 = 55$$

$$2 \quad \text{Putting } x \text{ for } (a+b-2c), y \text{ for } (b+c-2a) \text{ and} \\ z \text{ for } (c+a-2b), \text{ we get } x+y+z=0, \\ \text{and the left-hand expression} \\ = x^2 + y^2 + z^2 \\ = (x+y+z)(x^2+y^2+z^2-xy-xz-yz) + 3xyz \\ = 0 \times (x^2+y^2+z^2-xy-xz-yz) + 3xyz \\ = 3xyz = 3(a+b-2c)(b+c-2a)(c+a-2b)$$

$$3 \quad (ad-bc)(ad+bc) = a^2d^2 - b^2c^2 \\ = a^2d^2 + a^2c^2 - a^2c^2 - b^2c^2 \\ = a^2(d^2+c^2) - c^2(a^2+b^2) \\ = a^2 - c^2 = a^2 - c^2 = (a+c)(a-c)$$

$$4 \quad \text{1st expression} = 3x^2 - 9x - 2x + 6 = 3x(x-3) - 2(x-3) \\ = (x-3)(3x-2),$$

$$\text{2nd expression} = 2x^2 - x - 6x + 3 \\ = x(2x-1) - 3(2x-1) = (x-3)(2x-1),$$

$$\text{3rd expression} = 6x^2 - 3x - 4x + 2 \\ = 3x(2x-1) - 2(2x-1) \\ = (2x-1)(3x-2),$$

$$\text{the reqd L C M} = (x-3)(3x-2)(2x-1)$$

$$5 \quad \text{1st expression} = a^2x^2 + a^5 - 2abx^3 - 2a^3b + b^2x^2 + a^2b^2 \\ = a^2(x^2 + a^3) - 2ab(x^3 + a^2) + b^2(x^2 + a^2) \\ = (x^3 + a^3)(a^2 - 2ab + b^2) \\ = (x+a)(x^2 - ax + a^2)(a-b)^2,$$

$$\text{2nd expression} = 2a^2x^4 - 2b^2x^4 - 5a^4x^2 + 5a^2b^2x^2 + 3a^6 - 3a^4b^2 \\ = 2x^4(a^2 - b^2) - 5a^2x^2(a^2 - b^2) + 3a^4(a^2 - b^2) \\ = (a^2 - b^2)(2x^2 - 5a^2x^2 + 3a^4) \\ = (a-b)(a+b)(x^2 - a^2)(2x^2 - 3a^2) \\ = (a-b)(a+b)(x+a)(x-a)(2x^2 - 3a^2),$$

$$\text{The reqd H C F} = (x+a)(a-b)$$

6 The dividend

$$\begin{aligned}
&= \{(a\lambda + by) + (ax - by)\} \{(ax + by)^2 - (ax + by)(ax - by) \\
&\quad + (a\lambda - by)^2\} + \{(b\tau - ay) + (b\lambda + ay)\} \{(b\lambda - ay)^2 \\
&\quad - (b\tau - ay)(b\lambda + ay) + (b\tau + ay)^2\} \\
&= 2ax\{2(a^2\lambda^2 + b^2y^2) - (a^2\lambda^2 - b^2y^2)\} \\
&\quad + 2bx\{2(b^2x^2 + a^2y^2) - (b^2x^2 - a^2y^2)\} \\
&= 2ax(a^2\lambda^2 + 3b^2y^2) + 2bx(b^2x^2 + 3a^2y^2) \\
&= 2x(a^3\lambda^2 + 3ab^2y^2 + b^3x^2 + 3a^3by^2) \\
&= 2x\{\lambda^2(a^3 + b^3) + 3aby^2(a + b)\} \\
&= 2x(a + b)\{x^2(a^2 + b^2 - ab) + 3aby^2\} \\
&= 2x(a + b)\{\tau^2(a + b)^2 - 3ab\lambda^2 + 3aby^2\} \\
&= 2x(a + b)\{\lambda^2(a + b)^2 - 3ab\{\tau^2 - y^2\}\}, \\
\text{The reqd quotient} &= 2x(a + b)
\end{aligned}$$

$$7 \quad (a) \quad x^3 + \frac{1}{x^2} = \left(\tau + \frac{1}{x}\right)^3 - 2\lambda \frac{1}{x} = a^3 - 2$$

$$(b) \quad \tau^3 + \frac{1}{\tau^3} = \left(x + \frac{1}{x}\right)^3 - 3\lambda \frac{1}{x} \left(x + \frac{1}{x}\right) = a^3 - 3a$$

$$\begin{aligned}
(c) \quad \tau^4 + \frac{1}{\tau^4} &= \left(x^2 + \frac{1}{x^2}\right)^2 - 2x^2 \frac{1}{x^2} \\
&= (a^2 - 2)^2 - 2 = a^4 - 4a^2 + 4 - 2 = a^4 - 4a^2 + 2
\end{aligned}$$

8 The left-hand expression

$$\begin{aligned}
&= x^4 + \tau^2(y^2 + z^2) + y^2z^2 + 2x(x^2 + yz)(y + z) + 4x^2yz \\
&= (x^4 + 2x^2yz + y^2z^2) + x^2(y^2 + z^2 + 2yz) + 2x(x^2 + yz)(y + z) \\
&= (x^2 + yz)^2 + \{x(y + z)\}^2 + 2(x^2 + yz)(y + z) \\
&= (x^2 + yz + xy + \tau z)^2
\end{aligned}$$

IX

1 The left hand expression

$$\begin{aligned}
&= \{(a + b)^2 - c^2\} \{(a - b)^2 + c^2\} \\
&= (a^2 - b^2)^2 + c^2\{(a + b)^2 - (a - b)^2\} - c^4 \\
&= (a^2 - b^2)^2 + c^2(4ab) - c^4 \\
&= (a^2 - b^2)^2 + c^2(4ab - c^2)
\end{aligned}$$

$$\begin{aligned}
 2 \quad \left(x + \frac{2}{x}\right)^9 &= x^9 + 9x^7\left(\frac{2}{x}\right) + 36x^5\left(\frac{2}{x}\right)^2 \\
 &\quad + 84x^3\left(\frac{2}{x}\right)^3 + 36x\left(\frac{2}{x}\right)^4 + \left(\frac{2}{x}\right)^9 \\
 &= x^9 + 18x^6 + 144x^4 + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \text{Let } A &= x^5 + 11x - 12 \text{ and } B = x^5 + 11x^3 + 54, \\
 B - A &= 11x^3 - 11x + 66 = 11(x^3 - x + 6) = 11C \text{ (suppose)}, \\
 A + 2C &= x^5 + 2x^3 + 9x = x(x^4 + 2x^2 + 9) = xD \text{ (suppose)}, \\
 2D - 3C &= 2x^4 - 3x^3 + 4x^2 + 3x \\
 &= x(2x^3 - 3x^2 + 4x + 3) = xE \text{ (suppose)}, \\
 2E - C &= 3x^3 - 6x^2 + 9x = 3x(x^2 - 2x + 3) = 3xF \text{ (suppose)}, \\
 C - 2F &= x^3 - 2x^2 + 3x = x(x^2 - 2x + 3), \\
 \text{the reqd } H \ C \ F &= x^2 - 2x + 3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & \frac{a(b+c)+bc}{a^3(b^3-c^3)+a^2bc(b-c)} \cdot \frac{a^3(b^3-c^3)-a^2(b^3-c^3)+b^3c^2(b-c)}{a^3(b^3-c^3)+a^2bc(b-c)} \\
 &= \frac{-a^2(b-c)(b^2+c^2+2bc)+b^3c^2(b-c)}{a^3(b^3-c^3)+a^2bc(b-c)} \\
 &= \frac{-a^2(b-c)(b^2+c^2+2bc)-abc(b^3-c^3)}{a^3(b^3-c^3)+a^2bc(b-c)} \\
 &= \frac{abc(b^3-c^3)+b^3c^2(b-c)}{a^3(b^3-c^3)+a^2bc(b-c)} \\
 & \quad a^2(b-c)-a(b^2-c^2)+bc(b-c) \text{ is the Quotient}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \text{1st expression} &= x^3 - 5xy + 2xy - 10y^2 \\
 &= x(x - 5y) + 2y(x - 5y) = (x + 2y)(x - 5y), \\
 \text{2nd expression} &= x^2 + 7xy - 5xy - 35y^2 \\
 &= x(x + 7y) - 5y(x + 7y) = (x - 5y)(x + 7y), \\
 \text{3rd expression} &= x^2 - 5xy - 3xy + 15y^2 \\
 &= x(x - 5y) - 3y(x - 5y) = (x - 3y)(x - 5y), \\
 \text{the reqd } L \ C \ M &= (x - 5y)(x + 2y)(x - 3y)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \text{The left-hand expression} &= (x^3 - 2ax + a^2)(b - c) + (x^3 - 2bx + b^2)(c - a) \\
 &\quad + (x^3 - 2cx + c^2)(a - b) \\
 &= x^3\{(b - c) + (c - a) + (a - b)\} - 2x\{a(b - c) + b(c - a) \\
 &\quad + c(a - b)\} + a^2(b - c) + b^2(c - a) + c^2(a - b) \\
 &= x^3 \times 0 - 2x \times 0 + (a - b)(a - c)(b - c) = (a - b)(a - c)(b - c)
 \end{aligned}$$

$$\begin{aligned}
 7 \quad (a+b+c)^3 - a^3 - b^3 - c^3 &= (a+b)^3 + c^3 + 3a(a+b)(a+b+c) - a^3 - b^3 - c^3 \\
 &= a^3 + b^3 + 3ab(a+b) + 3c(a+b)(a+b+c) - a^3 - b^3 \\
 &= 3(a+b)\{ab + c(a+b+c)\} \\
 &= 3(a+b)\{(ab+ac) + c(b+c)\} = 3(a+b)(b+c)(a+c)
 \end{aligned}$$

Putting a for $y+z-x$, b for $x+y-z$ and c for $x+y-z$, we get $a+b+c=x+y+z$

$$\begin{aligned}
 \text{The left-hand expression} &= (a+b+c)^3 - a^3 - b^3 - c^3 \\
 &= 3(a+b)(b+c)(c+a) \\
 &= 3\{(y+z-x) + (x+y-z)\}\{(x+z-y) + (x+y-z)\} \\
 &\quad \{(x+y-z) + (y+z-x)\} \\
 &= 3(2x)(2x)(2y) = 24xyz
 \end{aligned}$$

$$\begin{aligned}
 8 \quad a^3 - b^3 + c^3 + 3abc &= a^3 + (-b)^3 + c^3 - 3a(-b)(c) \\
 &= (a-b+c)(a^2 + b^2 + c^2 + ab - ac + bc) \\
 &= (14278 - 12345 + 8067)(a^2 + b^2 + c^2 + ab - ac + bc) \\
 &= (12345 - 12345)(a^2 + b^2 + c^2 + ab - ac + bc) = 0
 \end{aligned}$$

X

$$\begin{aligned}
 1 \quad \text{The left-hand expression} &= \{(1+a^2)+2a\}(1+c^2) - \{(1+c^2)+2c\}(1+a^2) \\
 &= (1+a^2)(1+c^2) + 2a(1+c^2) - (1+c^2)(1+a^2) - 2c(1+a^2) \\
 &= 2a(1+c^2) - 2c(1+a^2) = 2(a+ac^2 - c - a^2c) \\
 &= 2\{(a-c) - ac(a-c)\} \\
 &= 2(a-c)(1-ac)
 \end{aligned}$$

$$\begin{aligned}
 2 \quad a^5 + b^5 &= (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4) \\
 &= 2\{(a^4 + 2a^3b^2 + b^4) - ab(a^2 + ab + b^2)\} \\
 &= 2\{(a^2 + b^2)^2 - 7\{(a+b)^2 - ab\}\} \\
 &= 2\{(a+b)^2 - 2ab\}^2 - 7(4-7) \\
 &= 2\{(4-14)^2 - 7(-3)\} \\
 &= 2\{(-10)^2 + 21\} = 2(100 + 21) = 242
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \text{The dividend} &= a - b + c - 3\left(\frac{1}{c^3} - \frac{1}{c^3}\right)\left\{a^3 - a^3\left(\frac{1}{c^3} + \frac{1}{c^3}\right) + b^3\frac{1}{c^3}\right\} \\
 &= \left(\frac{1}{a^3} - \frac{1}{c^3}\right) - \left(\frac{1}{c^3}\right)^3 - 3a^3\left(\frac{1}{c^3} - \frac{1}{c^3}\right) \\
 &\quad - 3a^3\left(\frac{1}{c^3} - \frac{1}{c^3}\right)^2 + 3b^3\frac{1}{c^3}\left(\frac{1}{c^3} + \frac{1}{c^3}\right) \\
 &= \left(\frac{1}{a^3} - \frac{1}{c^3}\right) - \left\{\left(\frac{1}{c^3}\right)^3 + \left(\frac{1}{c^3}\right)^3 - 3b^3\frac{1}{c^3}\left(\frac{1}{c^3} + \frac{1}{c^3}\right)\right\} \\
 &\quad - 3a^3\left(\frac{1}{c^3} + \frac{1}{c^3}\right)\left\{\frac{1}{a^3} - \left(\frac{1}{c^3} - \frac{1}{c^3}\right)\right\} \\
 &= \left(\frac{1}{a^3} - \frac{1}{c^3}\right) - \left(\frac{1}{c^3} - \frac{1}{c^3}\right)^3 - 3a^3\left(\frac{1}{c^3} - \frac{1}{c^3}\right) \\
 &\quad \left\{\frac{1}{c^3} - \left(\frac{1}{c^3} - \frac{1}{c^3}\right)\right\} \\
 &= \left\{\left(\frac{1}{a^3} - \left(\frac{1}{c^3} - \frac{1}{c^3}\right)\right)\right\}^3 = \left(\frac{1}{a^3} - \frac{1}{c^3} - \frac{1}{c^3}\right)^3;
 \end{aligned}$$

\therefore the required quotient

$$= \left(\frac{1}{a^3} - \frac{1}{c^3} - \frac{1}{c^3}\right)^3 = a^3 + c^3 - c^3 - 2a^3\frac{1}{c^3} - 2a^3\frac{1}{c^3} + 2b^3\frac{1}{c^3}$$

4. The given expression

$$\begin{aligned}
 &= 2(a^2 - b^2)(a^2 - a^2b^2 - b^2) - a^2(a^2 - b^2)(2ab - 3a^2 + 3b^2) \\
 &= (a^2 + b^2)(2a^2 - 2a^2b^2 - 2b^2 - 2a^2b^2 - 3a^2b - 3ab^2) \\
 &= (a^2 - b^2)(2a^2 - 2a^2b^2 + b^2 - 3a^2(a^2 - b^2)) \\
 &= (a^2 - b^2)(2(a^2 - b^2) - 3a^2(a^2 - b^2)) \\
 &= (a^2 - b^2)(a^2 - b^2)(2 - 3a^2 - 3b^2) \\
 &= (a^2 - b^2)(a + b)(a - b)(2a^2 - 1ab - c^2 - 2b^2) \\
 &= (a^2 - b^2)(a + b)(a - b)(2a - b)
 \end{aligned}$$

5 (i) The given expression

$$\begin{aligned}
 &= 6(a^2 - 2a^2b^2 - b^2) - 13a^2(a^2 - b^2) - 68a^2b^2 \\
 &= 6(a^2 - b^2)^2 - 13a^2(a^2 - b^2) - 68a^2b^2 \\
 &= 6(a^2 - b^2)^2 - 8a^2(a^2 - b^2) - 51a^2(a^2 + b^2) - 68a^2b^2 \\
 &= 2(a^2 - b^2)(3(a^2 - b^2) - 4ab^2 - 17a^2(3(a^2 - b^2) - 1ab^2)) \\
 &= (3a^2 - 3b^2 - 1ab^2)(2a^2 - 2b^2 - 17ab^2) \\
 &= (3a^2 - 1ab^2 - 3b^2)(2a^2 - 17ab^2 - 2b^2)
 \end{aligned}$$

(ii) The given expression

$$\begin{aligned}
 &= 12x^4 - 2x^2 - 1 - 37x(x^2 + 1) - 21x^2 \\
 &= 12x^4 - 1x^2 - 37x(x^2 + 1) - 21x^2 \\
 &= 12(x^2 - 1x^2 - 9x(x^2 - 1) - 28x(x^2 - 1) - 21x^2) \\
 &= 3(x^2 - 1)(4(x^2 - 1) - 3x^2) - 7x^2(4(x^2 - 1) - 3x^2) \\
 &= (4x^2 - 1) - 3x^2(3x^2 - 1) - 7x^2 \\
 &= (4x^2 - 3x - 1)(3x^2 - 7x - 3)
 \end{aligned}$$

(iii) The given expression

$$\begin{aligned}
 &= ab(x^4 + 2x^2 + 1) + (ac + b^2)x(x^2 + 1) + bcr^2 \\
 &= ab(r^2 + 1)^2 + (ac + b^2)x(r^2 + 1) + bcr^2 \\
 &= ab(x^2 + 1)^2 + acx(x^2 + 1) + b^2x(x^2 + 1) + bcr^2 \\
 &= a(r^2 + 1)\{b(x^2 + 1) + cx\} + bx\{b(x^2 + 1) + cx\} \\
 &= \{b(x^2 + 1) + cx\}\{a(x^2 + 1) + br\} \\
 &= (br^2 + cx + b)(ax^2 + br + a)
 \end{aligned}$$

6 Putting x for $a + b - 2c$, y for $c + a - 2b$ and z for $b + c - 2a$, we get $x - y = 3(b - c)$, $y - z = 3(a - b)$ and $z - x = 3(c - a)$,
 $3\{(b - c)(b + c - 2a)^2 + (c - a)(c + a - 2b)^2 + (a - b)(a + b - 2c)^2\}$

$$\begin{aligned}
 &= x^2(x - y) + y^2(y - z) + z^2(z - x) \\
 &= (x - y)(x - z)(y - z) \\
 &= 3(b - c) \cdot 3(a - b) \cdot 3(c - a) \\
 &= 3 \times 9(a - b)(a - c)(b - c),
 \end{aligned}$$

the left-hand expression $= 9(a - b)(a - c)(b - c)$

7 $a^3 + b^3 + c^3 - 3abc$

$$\begin{aligned}
 &= (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc) \\
 &= \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\} \\
 &= \frac{1}{2}(2658 + 2664 + 2678)\{(2658 - 2664)^2 \\
 &\quad + (2664 - 2678)^2 + (2678 - 2658)^2\} \\
 &= \frac{1}{2}(8000)\{(-6)^2 + (-14)^2 + (20)^2\} \\
 &= 4000(36 + 196 + 400) = 4000 \times 632 = 2528000
 \end{aligned}$$

8 $x^3 + y^3 + z^3 - 3xyz$

$$\begin{aligned}
 &= \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (z - x)^2\} \\
 &= \frac{1}{2}(a + b + c)\{(2b - 2a)^2 + (2c - 2b)^2 + (2a - 2c)^2\} \\
 &= \frac{1}{2}(a + b + c)\{4(a - b)^2 + 4(b - c)^2 + 4(c - a)^2\} \\
 &= 4 \cdot \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\} \\
 &= 4(a^3 + b^3 + c^3 - 3abc)
 \end{aligned}$$

Exercise 56

$$1 \quad \frac{2a^2b^3}{4a^3b^4} = \frac{2a^2b^3}{2a^3b^3 \cdot 2b} = \frac{1}{2b}, \quad 2 \quad \frac{6x^2y^3}{8xy^4} = \frac{2xy^3}{2xy^3 \cdot 4y} = \frac{3x}{4y}$$

$$3 \quad \frac{4a^2xy^2}{10ax^3y^2} = \frac{2axy^2 \cdot 2a}{2axy^2 \cdot 5x} = \frac{2a}{5x} \quad 4 \quad \frac{15x^3y^3z^4}{25x^2y^4z^3} = \frac{5x^3y^3z^3 \cdot 3xz}{5x^2y^3z^3 \cdot 5y^1} = \frac{3xz}{5y^1}$$

$$5 \quad \frac{18a^2bc^4d^6}{27a^3b^2c^4d^4} = \frac{9a^2bc^4d^4 \cdot 2d}{9a^2bc^4d^4 \cdot 3ab} = \frac{2d}{3ab}$$

$$6 \quad \frac{16x^2a^4y^3z^6}{40a^3z^4x^3y^4} = \frac{8x^2a^3y^3z^4 \cdot 2az}{8x^2a^3y^3z^4 \cdot 5xy} = \frac{2az}{5xy}$$

$$7 \quad \frac{70a^2b^3c^4a^7}{105c^4d^2a^3b^3} = \frac{35a^2b^3c^4d^2 \cdot 2d^3}{35a^2b^3c^4d^2 \cdot 5a} = \frac{2d^3}{5a}$$

$$8 \quad \frac{39m^2n^5p^3q^6}{65p^2m^3n^4q^6} = \frac{13m^2n^4p^3q^5 \cdot 3npq}{13m^2n^4p^2q^5 \cdot 5m} = \frac{3npq}{5m}$$

$$9 \quad \frac{x^2 - a^2}{x^2 + ax} = \frac{(x-a)(x+a)}{x(x+a)} = \frac{x-a}{x}$$

$$10 \quad \frac{x^2 - 3x}{9x - x^3} = \frac{x(x-3)}{x(9-x^2)} = \frac{x-3}{(3-x)(3+x)} = \frac{-(3-x)}{(3-x)(3+x)} = -\frac{1}{3+x}$$

$$11 \quad \frac{4x^2 - 9a^2}{4x^2 + 6ax} = \frac{(2x+3a)(2x-3a)}{2x(2x+3a)} = \frac{2x-3a}{2x}$$

$$12 \quad \frac{3a^2 - 12ab}{48b^2 - 3a^2} = \frac{3a(a-4b)}{3(16b^2 - a^2)} = \frac{a(a-4b)}{(4b-a)(4b+a)} \\ = \frac{-a(4b-a)}{(4b-a)(4b+a)} = -\frac{a}{4b+a}$$

$$13 \quad \frac{3ax - 12a^2}{x^2 - 16a^2} = \frac{3a(x-4a)}{(x-4a)(x+4a)} = \frac{3a}{x+4a}$$

$$14 \quad \frac{2x^4 - 4a^2x^2}{x^4 - 4a^2x^2 + 4a^4} = \frac{2x^2(x^2 - 2a^2)}{(x^2 - 2a^2)^2} = \frac{2x^2}{x^2 - 2a^2}$$

$$15 \quad \frac{4x^2 + 8x}{x^2 + 5x + 6} = \frac{4x(x+2)}{(x+2)(x+3)} = \frac{4x}{x+3}$$

$$16 \quad \frac{x^2 + 2x - 8}{x^2 + x - 12} = \frac{(x+4)(x-2)}{(x+4)(x-3)} = \frac{x-2}{x-3}$$

$$17 \quad \frac{x^2 + 2x - 15}{x^2 + 9x + 20} = \frac{(x+5)(x-3)}{(x+5)(x+4)} = \frac{x-3}{x+4}$$

$$18 \quad \frac{a^2 - 3ab - 4b^2}{a^2 - 4ab - 5b^2} = \frac{(a-4b)(a+b)}{(a-5b)(a+b)} = \frac{a-4b}{a-5b}$$

$$19 \quad \frac{a^4 - a^3b + a^2b^2}{a^3 + b^3} = \frac{a^2(a^2 - ab + b^2)}{(a+b)(a^2 - ab + b^2)} = \frac{a^2}{a+b}$$

$$20 \quad \frac{1-7x+12x^2}{1-8x+15x^2} = \frac{(1-3x)(1-4x)}{(1-3x)(1-5x)} = \frac{1-4x}{1-5x}$$

$$21 \quad \frac{x^2 - 6xy + 5y^2}{x^2 + 2xy - 35y^2} = \frac{(x-5y)(x-y)}{(x-5y)(x+7y)} = \frac{x-y}{x+7y}$$

$$22 \quad \frac{1-9a^2+14a^4}{1-4a^2-21a^4} = \frac{(1-7a^2)(1-2a^2)}{(1-7a^2)(1+3a^2)} = \frac{1-2a^2}{1+3a^2}$$

$$23 \quad \frac{x^4 - 8x^2 - 65}{x^4 + x^2 - 20} = \frac{(x^2-13)(x^2+5)}{(x^2-4)(x^2+5)} = \frac{(x^2-13)}{(x^2-4)}$$

$$24 \quad \frac{3a^3x+9a^2x^2+27ax^3}{a^3-27x^3} = \frac{3ax(a^2+3ax+9x^2)}{(a-3x)(a^2+3ax+9x^2)} = \frac{3ax}{a-3x}$$

$$25 \quad \frac{2x^2-x-6}{3x^2-2x-8} = \frac{2x^2-4x+3x-6}{3x^2-6x+4x-8} = \frac{(x-2)(2x+3)}{(x-2)(3x+4)} = \frac{2x+3}{3x+4}$$

$$26 \quad \frac{3x^2-5ax+2a^2}{3x^2+ax-2a^2} = \frac{3x^2-3ax-2ax+2a^2}{3x^2+3ax-2ax-2a^2} = \frac{(x-a)(3x-2a)}{(x+a)(3x-2a)} \\ = \frac{x-a}{x+a}$$

$$27 \quad \frac{3x^2+16ax+5a^2}{3x^2+22ax+7a^2} = \frac{3x^2+15ax+ax+5a^2}{3x^2+21ax+ax+7a^2} \\ = \frac{(x+5a)(3x+a)}{(x+7a)(3x+a)} = \frac{x+5a}{x+7a}$$

$$28 \quad \frac{6x^2-7x-20}{9x^2+6x-8} = \frac{6x^2-15x+8x-20}{9x^2+12x-6x-8} = \frac{(2x-5)(3x+4)}{(3x+4)(3x-2)} = \frac{2x-5}{3x-2}$$

$$29 \quad \frac{2x^2+3ax-20a^2}{3x^2+5ax-28a^2} = \frac{2x^2+8ax-5ax-20a^2}{3x^2+12ax-7ax-28a^2} \\ = \frac{(x+4a)(2x-5a)}{(x+4a)(3x-7a)} = \frac{2x-5a}{3x-7a}$$

$$30 \quad \frac{10-17ax+3a^2x^2}{5-26ax+5a^2x^2} = \frac{10-2ax-15ax+3a^2x^2}{5-ax-25ax+5a^2x^2} \\ = \frac{(5-ax)(2-3ax)}{(5-ax)(1-5ax)} = \frac{2-3ax}{1-5ax}$$

$$31 \quad \frac{x^2-(a-b)x-ab}{x^2+bx^2+ax+ab} = \frac{(x-a)(x+b)}{(x+b)(x^2+a)} = \frac{x-a}{x^2+a}$$

$$\begin{aligned} 32 \quad \frac{6ac + 10bc + 9a^2 + 15b^2}{6c^2 + 9a^2 - 2c - 3} &= \frac{2c(3a + 5b) + 3(3a + 5b)}{3c(2c + 3) - (2c + 3)} \\ &= \frac{(3a + 5b)(2c + 3)}{(2c + 3)(3c - 1)} = \frac{3a + 5b}{3c - 1} \end{aligned}$$

$$\begin{aligned} 33 \quad \frac{8bx + 12ab + 6by + 9a^2}{12bx + 8ab + 9a^2 + 6ay} &= \frac{4b(2x + 3a) + 3y(2a + 3a)}{4a(3x + 2a) + 3y(3x + 2a)} \\ &= \frac{(2x + 3a)(4b + 3y)}{(3x + 2a)(4b + 3y)} = \frac{2x + 3a}{3x + 2a} \end{aligned}$$

$$\begin{aligned} 34 \quad \frac{2a^2 + ab - b^2}{a^2 + a^2b - a - b} &= \frac{2a^2 + 2ab - ab - b^2}{a^2(a + b) - (a + b)} \\ &= \frac{(a + b)(2a - b)}{(a + b)(a^2 - 1)} = \frac{2a - b}{a^2 - 1} \end{aligned}$$

$$\begin{aligned} 35 \quad \frac{1^2 + 4x^2 + 1 - 6}{x^2 + x - 2} &= \frac{1^2 + 3x^2 + 1^2 + 3x - 2x - 6}{x^2 + x - 2} \\ &= \frac{1^2(1 + 3) + 1(1 + 3) - 2(1 + 3)}{x^2 + x - 2} \\ &= \frac{(1 + 3)(x^2 + x - 2)}{x^2 + x - 2} = 1 + 3 \end{aligned}$$

$$\begin{aligned} 36 \quad \frac{a^2 - b^2 - 2bc - c^2}{a^2 + 2ab + b^2 - c^2} &= \frac{a^2 - (b^2 + 2bc + c^2)}{(a^2 + 2ab + b^2) - c^2} \\ &= \frac{a^2 - (b + c)^2}{(a + b)^2 - c^2} \\ &= \frac{(a + b + c)(a - b - c)}{(a + b + c)(a + b - c)} = \frac{a - b - c}{a + b - c} \end{aligned}$$

$$\begin{aligned} 37 \quad \frac{x^2 - 7x + 6}{x^3 + 2x^2 - 13x + 10} &= \frac{x^2 - x - 6x + 6}{x^3 - x^2 + 3x^2 - 3x - 10x + 10} \\ &= \frac{x(x^2 - 1) - 6(x - 1)}{x^2(x - 1) + 3x(x - 1) - 10(x - 1)} \\ &= \frac{(x - 1)(x^2 + x - 6)}{(x - 1)(x^2 + 3x - 10)} = \frac{x^2 + 3x - 6}{x^2 + 5x - 2x - 10} \\ &= \frac{(x + 3)(x - 2)}{(x + 5)(x - 2)} = \frac{x + 3}{x + 5} \end{aligned}$$

$$\begin{aligned}
 38 \quad \frac{a^3 + 2a^2b - 2ab^2 + 3b^3}{a^3 - 5a^2b + 5ab^2 - 4b^3} &= \frac{(a^3 + b^3) + (2a^2b - 2ab^2 + 2b^3)}{(a^3b + b^3) - (5a^2b - 5ab^2 + 5b^3)} \\
 &= \frac{(a+b)(a^2 - ab + b^2) + 2b(a^2 - ab + b^2)}{(a+b)(a^2 - ab + b^2) - 5b(a^2 - ab + b^2)} \\
 &= \frac{(a^2 - ab + b^2)(a+b+2b)}{(a^2 - ab + b^2)(a+b-5b)} = \frac{a+3b}{a-4b}
 \end{aligned}$$

$$\begin{aligned}
 39 \quad \frac{x^4 + (2b^2 - a^2)x^2 + b^4}{x^4 + 2ax^3 + a^2x^2 - b^4} &= \frac{x^4 + 2b^2x^2 + b^4 - a^2x^2}{x^4 + 2ax^3 + a^2x^2 - b^4} \\
 &= \frac{(x^2 + b^2)^2 - (ax)^2}{x^2(x+a)^2 - b^4} = \frac{(x^2 + b^2 + ax)(x^2 + b^2 - ax)}{(x^2 + ax + b^2)(x^2 + ax - b^2)} \\
 &= \frac{x^2 + ax + b^2}{x^2 - b^2 + ax}
 \end{aligned}$$

$$\begin{aligned}
 40 \quad \frac{3x^3 + 4x^2y - 7xy^2 + 2y^3}{2x^3 + 9x^2y + 8xy^2 - 5y^3} &= \frac{3x^3 - 2x^2y + 6x^2y^3 - 4xy^2 - 3xy^2 + 2y^3}{2x^3 + 5x^2y + 4x^2y + 10xy^2 - 2xy^2 - 5y^3} \\
 &= \frac{x^2(3x - 2y) + 2xy(3x - 2y) - y^2(3x - 2y)}{x^2(2x + 5y) + 2xy(2x + 5y) - y^2(2x + 5y)} \\
 &= \frac{(3x - 2y)(x^2 + 2xy - y^2)}{(2x + 5y)(x^2 + 2xy - y^2)} = \frac{3x - 2y}{2x + 5y}
 \end{aligned}$$

$$\begin{aligned}
 41 \quad \frac{1 + 3x - x^3 - 3x^3}{1 - x + 2x^2 + x^3 + 3x^4} &= \frac{(1 + 3x) - x^3(1 + 3x)}{1 + x + x^2 - 2x - 2x^2 - 2x^3 + 3x^2 + 3x^3 + 3x^4} \\
 &= \frac{(1 + 3x)(1 - x^3)}{(1 + x + x^2)(1 - 2x + 3x^2)} = \frac{(1 + 3x)(1 - x)}{1 - 2x + 3x^2} = \frac{1 + 2x - 3x^2}{1 - 2x + 3x^2}
 \end{aligned}$$

$$\begin{aligned}
 42 \quad \frac{x^4 - x^3 - x + 1}{x^4 - 2x^3 - x^2 - 2x + 1} &= \frac{x^3(x-1) - (x-1)}{x^4 + x^3 + x^2 - 3x^3 - 3x^2 - 3x + x^2 + x + 1} \\
 &= \frac{(x-1)(x^3-1)}{x^4(x^2+x+1) - 3x(x^2+x+1) + (x^2+x+1)} \\
 &= \frac{(x-1)(x-1)(x^2+x+1)}{(x^2+x+1)(x^2-3x+1)} = \frac{(x-1)^2}{x^2-3x+1}
 \end{aligned}$$

43 The H C F of the numerator and denominator of the given fraction can be found as follows —

$$\begin{array}{r}
 x^4 - 9x^2 + 30x - 25 \\
 -(x^4 + x^2 + 25) \\
 \hline
 -10x^2 + 30x - 50 \\
 \hline
 x^2 - 3x + 5 \\
 \hline
 x^4 + x^2 + 25 \\
 -(x^4 - 3x + 5)x^2 \\
 \hline
 3x^2 - 4x^2 + 25 \\
 \hline
 -(x^2 - 3x + 5)3x \\
 \hline
 5x^2 - 15x + 25 \\
 \hline
 -(x^2 - 3x + 5)5
 \end{array}$$

Thus the H C F required $= x^2 - 3x + 5$

$$\begin{aligned}
 \text{Hence, the required result} &= \frac{(x^4 + x^2 + 25) - (x^2 - 3x + 5)}{(x^4 - 9x^2 + 30x - 25) - (x^2 - 3x + 5)} \\
 &= \frac{x^2 + 3x + 5}{x^2 + 3x - 5}
 \end{aligned}$$

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$$\begin{array}{r}
 4x^3 - 12ax^2 + 19a^2x - 15a^3 \\
 -(2x^3 + 3ax^2 + 5a^2x - 21a^3)2 \\
 \hline
 -9a^3 - 18ax^2 + 9a^2x + 27a^3 \\
 \hline
 2x^3 - ax - 3a^3 \\
 \hline
 2x^3 + 3ax^2 + 5a^2x - 21a^3 \\
 -(2x^3 - ax - 3a^3)x \\
 \hline
 4ax^2 + 8a^2x - 21a^3 \\
 \hline
 (2x^2 - ax - 3a^2)2x \\
 \hline
 5a^2 \mid 10a^2x - 15a^3 \\
 \hline
 2x - 3a \\
 \hline
 2x^2 - ax - 3a^2 \\
 \hline
 (2x - 3a)x \\
 \hline
 2ax - 3a^2 \\
 \hline
 (2x - 3a)a
 \end{array}$$

Thus the H C F of the numerator and denominator $= 2x - 3a$.

$$\begin{aligned}
 \text{The required result} &= \frac{(2x^3 + 3ax^2 + 5a^2x - 21a^3) - (2x - 3a)}{(4x^3 - 12ax^2 + 19a^2x - 15a^3) - (2x - 3a)} \\
 &= \frac{x^2 + 3ax + 7a^2}{2x^2 - 3ax + 5a^2}
 \end{aligned}$$

$$\begin{aligned}
 45 \quad \frac{2x^4 + x^3 - 3x^2 + 2x + 3}{3x^4 + x^3 - 4x^2 + 3x + 4} &= \frac{(2x^4 + 3x^3 - 2x^3 - 3x^3) + (2x + 3)}{(3x^4 + 4x^3 - 3x^3 - 4x^2) + (3x + 4)} \\
 &= \frac{(2x + 3)(x^3 - x^2) + (2x + 3)}{(3x + 4)(x^3 - x^2) + (3x + 4)} \\
 &= \frac{(2x + 3)(x^3 - x^2 + 1)}{(3x + 4)(x^3 - x^2 + 1)} = \frac{2x + 3}{3x + 4}
 \end{aligned}$$

$$\begin{aligned}
 46 \quad \frac{9x^3 - 7a^2x - 2a^3}{9x^3 + 6ax^2 - 5a^2x - 2a^3} &= \frac{9x^3 - 9a^2x + 2a^2x - 2a^3}{(9x^3 + 6ax^2 + a^2x) - (6a^2x + 2a^3)} \\
 &= \frac{9x(x^2 - a^2) + 2a^2(x - a)}{x(3x + a)^2 - 2a^2(3x + a)} \\
 &= \frac{(x - a)(9x^2 + 9ax + 2a^2)}{(3x + a)(3x^2 + ax - 2a^2)} \\
 &= \frac{(x - a)(9x^2 + 3ax + 6ax + 2a^2)}{(3x + a)(3x^2 + 3ax - 2a^2 - 2a^2)} \\
 &= \frac{(x - a)(3x + a)(3x + 2a)}{(3x + a)(x + a)(3x - 2a)} \\
 &= \frac{(x - a)(3x + 2a)}{(x + a)(3x - 2a)} = \frac{3x^2 - ax - 2a^2}{3x^2 + ax - 2a^2}
 \end{aligned}$$

$$\begin{aligned}
 47 \quad \frac{2|2a^3 - 16a^2b + 44ab^2 - 42b^3}{a^3 - 8a^2b + 22ab^2 - 21b^3} &= \frac{a^3 - 8a^2b + 22ab^2 - 21b^3}{(a - 3b)a^2} \\
 \frac{3|3a^3 + 6a^2b - 24ab^2 - 63b^3}{a^3 + 2a^2b - 8ab^2 - 21b^3} &= \frac{-5a^2b + 22ab^2 - 21b^3}{-(a - 3b)(-5ab)} \\
 \frac{-(a^3 - 8a^2b + 22ab^2 - 21b^3)}{10ab|10a^2b - 30ab^2} &= \frac{7ab^2 - 21b^3}{-(a - 3b)7b} \\
 &= \frac{a - 3b}{a - 3b}
 \end{aligned}$$

The H C F of the numerator and denominator = $a - 3b$

The required result

$$= \frac{(2a^3 - 16a^2b + 44ab^2 - 42b^3) - (a - 3b)}{(3a^3 + 6a^2b - 24ab^2 - 63b^3) - (a - 3b)} = \frac{2(a^3 - 5ab + 7b^2)}{3(a^2 + 5ab + 7b^2)}$$

$$\begin{aligned}
 48 \quad \frac{3|9x^4 + 30x^3 + 12x^2 - 6x - 45}{3x^4 + 10x^3 + 4x^2 - 2x - 15} &= \frac{4|8x^4 + 28x^3 + 16x^2 - 4x - 48}{2x^4 + 7x^3 + 4x^2 - x - 12} \\
 &= \frac{3|6x^4 + 21x^3 + 12x^2 - 3x - 36}{-(3x^4 + 10x^3 + 4x^2 - 2x - 15)2} \\
 &= \frac{x^3 + 4x^2 - x - 6}{x^3 + 4x^2 - x - 6}
 \end{aligned}$$

$$\begin{array}{r}
 3x^4 + 10x^3 + 14x^2 - 2x - 15 \\
 -(x^3 + 4x^2 + x - 6)3x \\
 \hline
 -2x^3 + x^2 + 16x - 15 \\
 -(x^3 + 4x^2 + x - 6) \times (-2) \\
 \hline
 9x^2 + 18x - 27 \\
 9(x^2 + 2x - 3) \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 x^3 + 4x^2 + x - 6 \\
 -(x^2 + 2x - 3)x \\
 \hline
 2x^3 + 4x - 6 \\
 -(x^2 + 2x - 3)2 \\
 \hline
 0
 \end{array}$$

Thus the H C F of the numerator and denominator $= x^2 + 2x - 3$

The required result

$$= \frac{3(3x^4 + 10x^3 + 4x^2 - 2x - 15) - (x^2 + 2x - 3)}{4(2x^4 + 7x^3 + 4x^2 - x - 12) - (x^2 + 2x - 3)} = \frac{3(3x^2 + 4x + 5)}{4(2x^2 + 3x + 4)}$$

$$\begin{array}{r}
 49 \quad a | 4a^6 - 6a^4b + 3a^2b^3 - ab^4 \\
 \quad \quad 4a^4 - 6a^2b + 3ab^3 - b^4 \\
 \hline
 a^2 | 6a^4 - 9a^2b + a^2b^3 - a^2b^4 \\
 \quad \quad 6a^4 - 9a^2b + a^2b^3 + 3ab^3 - b^4 \\
 \hline
 2 \\
 \quad \quad 12a^4 - 18a^2b + 2a^2b^3 + 6ab^3 - 2b^4 \\
 \hline
 -(4a^4 - 6a^2b + 3ab^3 - b^4)3 \\
 \quad \quad b^2 | 2a^4b^3 - 3ab^5 + b^4 \\
 \quad \quad \quad 2a^3 - 3ab + b^2 \\
 \hline
 a | 4a^4 - 6a^2b + 3ab^3 - b^4 \\
 \quad \quad 4a^4 - 6a^2b + b^3 \cdot 2a^2 \\
 \hline
 -2a^2b^2 + 3ab^3 - b^4 \\
 \hline
 -(2a^2 - 3ab + b^2) \times (-b^2) \\
 \hline
 0
 \end{array}$$

Thus the H C F of the numerator and denominator $= a(2a^2 - 3ab + b^2)$,

the required result

$$\begin{aligned}
 &= \frac{a^2(6a^4 - 9a^2b + a^2b^3 + 3ab^3 - b^4) - b(2a^2 - 3ab + b^2)}{a(4a^4 - 6a^2b + 3ab^3 - b^4) - a(2a^2 - 3ab + b^2)} \\
 &= \frac{a(3a^2 - b^2)}{2a^2 - b^2}
 \end{aligned}$$

$$50 \quad 4x | 24x^5 + 16x^4y - 28x^3y^2 - 24x^2y^3 - 12xy^4 \\
 \quad \quad 6x^4 + 4x^3y - 7x^2y^2 - 6xy^3 - 3y^4$$

$$\begin{array}{r}
5y' \overline{) 45x^4y + 30x^3y^2 - 15x^2y^3 - 20xy^4 - 10y^5} \\
\underline{9x^4 + 6x^3y - 3x^2y^2 - 4xy^3 - 2y^4} \\
2 \\
\underline{18x^4 + 12x^3y - 6x^2y^2 - 8xy^3 - 4y^4} \\
-(6x^4 + 4x^3y - 7x^2y^2 - 6xy^3 - 3y^4) 3 \\
\underline{5y^2 \overline{) 15x^2y^2 + 10xy^3 + 5y^4}} \\
\underline{3x^2 + 2xy + y^2} \\
6x^2 + 4xy - 7x^2y^2 - 6xy^3 - 3y^4 \\
\underline{-(3x^2 + 2xy + y^2) 2x^2} \\
-9x^2y^2 - 6xy^3 - 3y^4 \\
\underline{-(3x^2 + 2xy + y^2) \times (-3y^2)}
\end{array}$$

Thus the H C F of the numerator and denominator
 $= 3x^2 + 2xy + y^2,$

the reqd result

$$\begin{aligned}
&= \frac{4x(6x^4 + 4x^3y - 7x^2y^2 - 6xy^3 - 3y^4) - (3x^2 + 2xy + y^2)}{5y(9x^4 + 6x^3y - 3x^2y^2 - 4xy^3 - 2y^4) - (3x^2 + 2xy + y^2)} \\
&= \frac{4x(2x^2 - 3y^4)}{5y(3x^2 - 2y^2)}
\end{aligned}$$

Exercise 57

- 1 The L C M of the denominators $= 4bdf$

$$\begin{aligned}
\frac{a}{2b} &= \frac{a \times 2df}{2b \times 2df} = \frac{2adf}{4bdf}, & \frac{3c}{4d} &= \frac{3c \times bf}{4d \times bf} = \frac{3bcf}{4bdf}, \\
\text{and } \frac{c}{f} &= \frac{c \times 4bd}{f \times 4bd} = \frac{4bdc}{4bdf}
\end{aligned}$$

- 2 The L C M of the denominators $= 12abc$,

$$\begin{aligned}
\frac{x^2}{2bc} &= \frac{x^2 \times 6a}{2bc \times 6a} = \frac{6ax^2}{12abc}, & \frac{y^3}{3ca} &= \frac{y^3 \times 4b}{3ca \times 4b} = \frac{4by^3}{12abc}, \\
\text{and } \frac{z^2}{4ab} &= \frac{z^2 \times 3c}{4ab \times 3c} = \frac{3cz^2}{12abc}
\end{aligned}$$

- 3 The L C M of the denominators $= 60x^3y^3$,

$$\begin{aligned}
\frac{ab}{4xy^3} &= \frac{ab \times 15x^2}{4xy^3 \times 15x^2} = \frac{15abx^2}{60x^3y^3}, \\
\frac{bc}{6x^2y} &= \frac{bc \times 10xy}{6x^2y \times 10xy} = \frac{10bcxy}{60x^3y^3}, \\
\frac{ca}{10x^3} &= \frac{ca \times 6y^3}{10x^3 \times 6y^3} = \frac{6acy^3}{60x^3y^3}
\end{aligned}$$

- 4 The L C M of the denominators
- $= a(a^2 - b^2)$
- ,

$$\frac{a}{a-b} = \frac{a \times a(a+b)}{(a-b) \times a(a+b)} = \frac{a^2(a+b)}{a(a^2-b^2)},$$

$$\frac{a}{a+b} = \frac{a \times a(a-b)}{(a+b) \times a(a-b)} = \frac{a^2(a-b)}{a(a^2-b^2)},$$

$$\text{and } \frac{c}{a(a+b)} = \frac{c \times (a-b)}{a(a+b) \times (a-b)} = \frac{c(a-b)}{a(a^2-b^2)}$$

- 5 The L C M of the denominators
- $= a(a^2 - 4b^2)$
- ,

$$\frac{x^2}{a^2+2ab} = \frac{x^2 \times (a-2b)}{a(a+2b) \times (a-2b)} = \frac{x^2(a-2b)}{a(a^2-4b^2)},$$

$$\text{and } \frac{y^2}{a-2b} = \frac{y^2 \times a(a+2b)}{(a-2b) \times a(a+2b)} = \frac{ay^2(a+2b)}{a(a^2-4b^2)}$$

- 6 The L C M of the denominators
- $= a(a-b)$
- ,

$$\frac{2a}{a-b} = \frac{2a \times a}{(a-b) \times a} = \frac{2a^2}{a(a-b)},$$

$$\text{and } \frac{a-c}{ab-a^2} = \frac{(a-c) \times (-1)}{a(b-a) \times (-1)} = \frac{c-a}{a(a-b)}$$

- 7 The L C M of the denominators
- $= a^2 - b^2$
- ,

$$\frac{2a}{a-b} = \frac{2a(a+b)}{(a-b) \times (a+b)} = \frac{2a(a+b)}{a^2-b^2},$$

$$\frac{3b}{b-a} = \frac{3b \times \{-(b+a)\}}{(b-a) \times -(b+a)} = \frac{-3b(a+b)}{a^2-b^2},$$

$$\text{and } \frac{4c}{a+b} = \frac{4c \times (a-b)}{(a+b) \times (a-b)} = \frac{4c(a-b)}{a^2-b^2}$$

- 8 The L C M of the denominators
- $= a^2b^2c^2(a^2-x^2)$
- ,

$$\frac{2x}{a^2(a+x)} = \frac{2x \times b^2c^2(a-x)}{a^2(a+x) \times b^2c^2(a-x)} = \frac{2b^2c^2x(a-x)}{a^2b^2c^2(a^2-x^2)},$$

$$\frac{3y}{b^2(a-x)} = \frac{3y \times a^2c^2(a+x)}{b^2(a-x) \times a^2c^2(a+x)} = \frac{3a^2c^2y(a+x)}{a^2b^2c^2(a^2-x^2)},$$

$$\text{and } \frac{4z}{c^2(a^2-x^2)} = \frac{4z \times a^2b^2}{c^2(a^2-x^2) \times a^2b^2} = \frac{4a^2b^2z}{a^2b^2c^2(a^2-x^2)}$$

- 9 The L C M of the denominators
- $= xy(4x^2 - 9y^2)$
- ,

$$\frac{a^2}{2xy-3y^2} = \frac{a^2 \times x(2x+3y)}{y(2x-3y) \times x(2x+3y)} = \frac{a^2x(2x+3y)}{xy(4x^2-9y^2)},$$

$$\frac{b^2}{2x^2+3xy} = \frac{b^2 \times y(2x-3y)}{x(2x+3y) \times y(2x-3y)} = \frac{b^2 y(2x-3y)}{xy(4x^2-9y^2)}$$

and $\frac{c^2}{4x^2y-9xy^2} = \frac{c^2}{xy(4x^2-9y^2)} = \frac{c^2}{xy(4x^2-9y^2)}$

10 The L C M of the denominators

$$= (x^2+x+1)(x^2-x+1) = x^4+x^2+1,$$

$$\frac{a^2}{x^2+x+1} = \frac{a^2 \times (x^2-x+1)}{(x^2+x+1) \times (x^2-x+1)} = \frac{a^2(x^2-x+1)}{x^4+x^2+1},$$

and $\frac{b^2}{x^2-x+1} = \frac{b^2 \times (x^2+x+1)}{(x^2-x+1) \times (x^2+x+1)} = \frac{b^2(x^2+x+1)}{x^4+x^2+1}$

11 $x^2-x-2=(x-2)(x+1)$, and $x^2+x-6=(x-2)(x+3)$

Thus the L C M of the denominators

$$= (x-2)(x+1)(x+3) = x^3+2x^2-5x-6,$$

$$\frac{3}{x^2-x-2} = \frac{3(x+3)}{(x^2-x-2) \times (x+3)} = \frac{3(x+3)}{x^3+2x^2-5x-6},$$

and $\frac{4}{x^2+x-6} = \frac{4(x+1)}{(x^2+x-6) \times (x+1)} = \frac{4(x+1)}{x^3+2x^2-5x-6}$

12 $a^3+8b^3=(a+2b)(a^2-2ab+4b^2)$

The L C M of the denominators

$$= a(a^2+8b^2) = a^4+8ab^3,$$

$$\frac{a-2b}{a(a^2-2ab+4b^2)} = \frac{(a-2b) \times (a+2b)}{a(a^2-2ab+4b^2) \times (a+2b)} = \frac{a^2-4b^2}{a^4+8ab^3},$$

and $\frac{bc}{a^2+8b^3} = \frac{bc \cdot a}{(a^2+8b^3) \times a} = \frac{abc}{a^4+8ab^3}$

13 $a^3-27b^3=(a-3b)(a^2+3ab+9b^2)$,

the L C M of the denominators $= a^3-27b^3$,

$$\frac{a}{a-3b} = \frac{a(a^2+3ab+9b^2)}{(a-3b)(a^2+3ab+9b^2)} = \frac{a(a^2+3ab+9b^2)}{a^3-27b^3},$$

$$\frac{b}{a^2+3ab+9b^2} = \frac{b(a-3b)}{(a^2+3ab+9b^2) \times (a-3b)} = \frac{b(a-3b)}{a^3-27b^3},$$

and $\frac{c}{a^3-27b^3} = \frac{c}{a^3-27b^3}$

$$14 \quad a^2 + b^2 - c^2 - 2ab = a^2 + b^2 - 2ab - c^2 \\ = (a-b)^2 - c^2 = (a-b+c)(a-b-c),$$

the L C M of the denominators $= ab(a^2 + b^2 - c^2 - 2ab)$,

$$\frac{a}{b(a-b-c)} = \frac{a \times a(a-b+c)}{b(a-b-c) \times a(a-b+c)} = \frac{a^2(a-b+c)}{ab(a^2 + b^2 - c^2 - 2ab)},$$

$$\frac{b}{a(a-b+c)} = \frac{b \times b(a-b-c)}{a(a-b+c) \times b(a-b-c)} = \frac{b^2(a-b-c)}{ab(a^2 + b^2 - c^2 - 2ab)},$$

$$\text{and } \frac{c}{a^2 + b^2 - c^2 - 2ab} = \frac{c \times ab}{(a^2 + b^2 - c^2 - 2ab) \times ab} = \frac{abc}{ab(a^2 + b^2 - c^2 - 2ab)}$$

15 The L C M of the denominators $= (a-b)(b-c)(c-a)$,

$$\frac{c-a}{(a-b)(b-c)} = \frac{(c-a) \times (c-a)}{(a-b)(b-c) \times (c-a)} = \frac{(c-a)^2}{(a-b)(b-c)(c-a)},$$

$$\frac{b-a}{(a-c)(b-c)} = \frac{(b-a) \times \{-(a-b)\}}{(a-c)(b-c) \times \{-(a-b)\}} = \frac{(a-b)^2}{(a-b)(b-c)(c-a)},$$

$$\text{and } \frac{b-c}{(c-a)(a-b)} = \frac{(b-c) \times (b-c)}{(c-a)(a-b) \times (b-c)} = \frac{(b-c)^2}{(a-b)(b-c)(c-a)}$$

EXERCISE 58

$$1 \quad \text{The given expression} = \frac{(a+b)b}{ab} + \frac{(a-b)a}{ab} \\ = \frac{b(a+b) + a(a-b)}{ab} = \frac{a^2 + b^2}{ab}$$

$$2 \quad \text{The given expression} = \frac{z(1-y)}{xyz} + \frac{x(y-z)}{xyz} + \frac{y(z-x)}{xyz} \\ = \frac{z(1-y) + x(y-z) + y(z-x)}{xyz} = \frac{0}{xyz} = 0$$

$$3 \quad \text{The given expression} = \frac{a}{a-1} + \frac{x(-1)}{(x-a)(-1)} = \frac{a-x}{a-x} = 1$$

$$4 \quad \text{The given expression} = \frac{(a+b)(a+b)}{(a-b)(a+b)} - \frac{(a-b)(a-b)}{(a+b)(a-b)} \\ = \frac{(a+b)^2 - (a-b)^2}{a^2 - b^2} = \frac{4ab}{a^2 - b^2}$$

$$\begin{aligned}
 5 \quad \text{The given expression} &= \frac{2(a^2 + b^2)}{2(a^2 - b^2)} - \frac{(a-b)(a-b)}{2(a+b)(a-b)} \\
 &= \frac{2(a^2 + b^2) - (a-b)^2}{2(a^2 - b^2)} = \frac{(a+b)^2}{2(a^2 - b^2)} \\
 &= \frac{a+b}{2(a-b)}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \text{The given expression} &= \frac{4x^2 + 9y^2}{4x^2 - 9y^2} - \frac{(2x-3y)(2x-3y)}{(2x+3y)(2x-3y)} \\
 &= \frac{4x^2 + 9y^2 - (2x-3y)^2}{4x^2 - 9y^2} = \frac{12xy}{4x^2 - 9y^2}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \text{The given expression} &= \frac{a(a-b)}{(a+b)(a-b)} - \frac{b(a+b)}{(a+b)(a-b) \times (a+b)} \\
 &= \frac{a(a-b) - b(a+b)}{(a+b)^2(a-b)} = \frac{a^2 - 2ab - b^2}{(a+b)^2(a-b)}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \text{The given expression} &= \frac{(a^2 + ab + b^2)(a-b)}{(a+b)(a-b)} + \frac{(a^2 - ab + b^2)(a+b)}{(a-b)(a+b)} \\
 &= \frac{(a^3 - b^3) + (a^3 + b^3)}{a^2 - b^2} = \frac{2a^3}{a^2 - b^2}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \text{The given expression} &= \frac{b-c}{(a-b)(a-c) \times (b-c)} + \frac{a-b}{(a-c)(b-c) \times (a-b)} \\
 &= \frac{(b-c) + (a-b)}{(a-b)(a-c)(b-c)} = \frac{a-c}{(a-b)(a-c)(b-c)} = \frac{1}{(a-b)(b-c)}
 \end{aligned}$$

$$\begin{aligned}
 10 \quad \text{The given expression} &= \frac{1}{(x-2)(x-1)} + \frac{1}{(x-2)(x-3)} \\
 &= \frac{x-3}{(x-2)(x-1) \times (x-3)} + \frac{x-1}{(x-2)(x-3) \times (x-1)} \\
 &= \frac{(x-3) + (x-1)}{(x-1)(x-2)(x-3)} = \frac{2(x-2)}{(x-1)(x-2)(x-3)} \\
 &= \frac{2}{x^2 - 4x + 3}
 \end{aligned}$$

11 The given expression

$$\begin{aligned}
 &= \frac{1}{(1+5)(1+2)} + \frac{1}{(1+5)(1+8)} \\
 &= \frac{1+8}{(1+5)(1+2) \times (1+8)} + \frac{1+2}{(1+5)(1+8) \times (1+2)} \\
 &= \frac{(1+8)(1+2)}{(1+2)(1+5)(1+8)} = \frac{2(1+5)}{(1+2)(1+5)(1+8)} \\
 &= \frac{2}{x^3 + 10x + 16}
 \end{aligned}$$

12 The given expression

$$\begin{aligned}
 &= \frac{4x^2 - 6xy + 9y^2}{(2x+3y)(4x^2-6xy+9y^2)} - \frac{(2x-3y)^2}{8x^3+27y^3} \\
 &= \frac{4x^2 - 6xy + 9y^2 - (4x^2 - 12xy + 9y^2)}{8x^3+27y^3} = \frac{6xy}{8x^3+27y^3}
 \end{aligned}$$

13 The given expression

$$\begin{aligned}
 &= \frac{(a+b)(a+b)}{(a-b)(a+b)} - \frac{(a-b)(a-b)}{(a+b)(a-b)} + \frac{2ab(-1)}{(b^2-a^2) \times (-1)} \\
 &= \frac{(a+b)^2 - (a-b)^2 - 2ab}{a^2-b^2} = \frac{4ab - 2ab}{a^2-b^2} = \frac{2ab}{a^2-b^2}
 \end{aligned}$$

14 The given expression

$$\begin{aligned}
 &= \frac{a-2b}{(a+2b)(a-2b)} + \frac{a+2b}{(a-2b)(a+2b)} + \frac{2a \times (-1)}{(4b^2-a^2) \times (-1)} \\
 &= \frac{(a-2b) + (a+2b) - 2a}{a^2-4b^2} = \frac{0}{a^2-4b^2} = 0
 \end{aligned}$$

15 We have $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{(x+y)^2 + (x-y)^2}{x^2-y^2} = \frac{2(x^2+y^2)}{x^2-y^2}$

Hence the required value $= \frac{2(x^2+y^2)}{x^2-y^2} - \frac{2(x^2-y^2)}{x^2+y^2}$

$$= \frac{2(x^2+y^2)^2 - 2(x^2-y^2)^2}{x^4-y^4} = \frac{8x^2y^2}{x^4-y^4}$$

16 We have $\frac{a-2x}{a+2x} - \frac{a+2x}{a-2x} = \frac{(a-2x)^2 - (a+2x)^2}{a^2-4x^2} = \frac{-8ax}{a^2-4x^2}$

$$\begin{aligned}
 \text{Hence the required value} &= \frac{-8ax}{a^3-4x^3} + \frac{8ax}{a^3+4x^3} \\
 &= 8ax \left(\frac{1}{a^3-4x^3} - \frac{1}{a^3+4x^3} \right) \\
 &= 8ax \left(\frac{(a^3-4x^3)-(a^3+4x^3)}{a^6-16x^6} \right) \\
 &= \frac{(8ax) \times (-8x^3)}{a^6-16x^6} = \frac{-64ax^4}{a^6-16x^6}
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \text{We have } \frac{3x+1}{x-3} - \frac{x-3}{3x+9} &= \frac{3(3x+1)(x+3) - (x-3)^2}{3(x^2-9)} \\
 &= \frac{8x^2+36}{3(x^2-9)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence the required value} &= \frac{8x^2+36}{3(x^2-9)} - \frac{5x^2+24x}{2x^2-18} \\
 &= \frac{8x^2+36x}{3(x^2-9)} - \frac{5x^2+24x}{2(x^2-9)} \\
 &= \frac{16x^2+72x-15x^2-72x}{6(x^2-9)} \\
 &= \frac{x^2}{6(x^2-9)}
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= \frac{4a-b}{1-4ab} - \frac{4a+b}{1+4ab} + \frac{4b(1-8a^2)}{1-16a^2b^2} \\
 &= \frac{(4a-b)(1+4ab) - (4a+b)(1-4ab) + 4b(1-8a^2)}{1-16a^2b^2} \\
 &= \frac{(4a+16a^2b-b-4a^2b^2) - (4a-16a^2b+b-4ab^2) + 4b-32a^2b}{1-16a^2b^2} \\
 &= \frac{2b}{1-16a^2b^2}
 \end{aligned}$$

$$19 \quad \text{We have } \frac{x}{x-2a} + \frac{x}{x+2a} = \frac{x(x+2a+x-2a)}{x^2-4a^2} = \frac{2x^2}{x^2-4a^2}$$

$$\begin{aligned}
 \text{Hence the required value} &= 2x^2 \left(\frac{1}{x^2-4a^2} + \frac{1}{x^2+4a^2} \right) \\
 &= 2x^2 \left(\frac{x^2+4a^2+x^2-4a^2}{x^4-16a^4} \right) \\
 &= \frac{4x^4}{x^4-16a^4}
 \end{aligned}$$

20 We have $\frac{b}{a-b} + \frac{b}{a+b} = b \left(\frac{a+b+a-b}{a^2-b^2} \right) = \frac{2ab}{a^2-b^2}$

Again $\frac{2ab}{a^2-b^2} + \frac{2ab}{a^2+b^2} = 2ab \left(\frac{a^2+b^2+a^2-b^2}{a^4-b^4} \right) = \frac{4a^2b}{a^4-b^4}$

Hence the required value $= \frac{4a^2b}{a^4-b^4} + \frac{4a^2b}{a^4+b^4}$
 $= 4a^2b \left(\frac{a^4+b^4+a^4-b^4}{a^8-b^8} \right) = \frac{8a^4b}{a^8-b^8}$

21 We have $\frac{x}{3x-y} + \frac{y}{3x+y} = x \left(\frac{3x+y+3x-y}{9x^2-y^2} \right) = \frac{6x^2}{9x^2-y^2}$

Hence the required value $= \frac{6x^2}{9x^2-y^2} + \frac{6y^2}{9x^2+y^2}$
 $= 6x^2 \left(\frac{9x^2+y^2+9x^2-y^2}{81x^4-y^4} \right)$
 $= \frac{6x^2 \cdot 18x^2}{81x^4-y^4} = \frac{108x^4}{81x^4-y^4}$

22 We have $\frac{1}{x-3a} - \frac{1}{x+6a} = \frac{2(1+3a)-(1-3a)}{2(x^2-9a^2)} = \frac{x+9a}{2(x^2-9a^2)}$

Hence the required value

$$\begin{aligned} &= \frac{x+9a}{2(x^2-9a^2)} - \frac{x-9a}{2(x^2+9a^2)} \\ &= \frac{(x+9a)(x^2+9a^2) - (x-9a)(x^2-9a^2)}{2(x^4-81a^4)} \\ &= \frac{18ax(x+a)}{2(x^4-81a^4)} = \frac{9ax(x+a)}{x^4-81a^4} \end{aligned}$$

23 We have $\frac{a}{b} + \frac{b}{a} + 2 = \frac{a^2+b^2+2ab}{ab} = \frac{(a+b)^2}{ab}$

Hence the given expression

$$\begin{aligned} &= \frac{(a^2+b^2)^2}{ab(a-b)^2} - \left(\frac{a}{b} + \frac{b}{a} + 2 \right) \\ &= \frac{(a^2+b^2)^2}{ab(a-b)^2} - \frac{(a+b)^2}{ab} \\ &= \frac{(a^2+b^2)^2 - (a^2-b^2)^2}{ab(a-b)^2} = \frac{4a^2b^2}{ab(a-b)^2} = \frac{4ab}{(a-b)^2} \end{aligned}$$

24 The given expression

$$\begin{aligned}
 &= \frac{x+1-(x-1)}{x^2-1} + \frac{x+2-(x-2)}{x^2-4} \\
 &= \frac{2}{x^2-1} + \frac{4}{x^2-4} = \frac{2x^2-8+4x^2-4}{x^4-5x^2+4} = \frac{6x^2-12}{x^4-5x^2+4}
 \end{aligned}$$

25 The given expression

$$\begin{aligned}
 &= \left(\frac{1}{x-a} + \frac{1}{x+a} \right) - 2 \left(\frac{1}{2x+a} + \frac{1}{2x-a} \right) \\
 &= \left(\frac{x+a+x-a}{x^2-a^2} \right) - 2 \left(\frac{2x-a+2x+a}{4x^2-a^2} \right) \\
 &= \frac{2x}{x^2-a^2} - \frac{8x}{4x^2-a^2} = \frac{8x^3-2a^2x-8x^3+8a^2x}{(x^2-a^2)(4x^2-a^2)} \\
 &= \frac{6a^2x}{4x^4-5a^2x^2+a^4}
 \end{aligned}$$

26 The given expression

$$\begin{aligned}
 &= 3 \left(\frac{1}{a-x} + \frac{1}{a+x} \right) - \left(\frac{1}{x+3a} - \frac{1}{x-3a} \right) \\
 &= 3 \left(\frac{a+x+a-x}{a^2-x^2} \right) - \left(\frac{x-3a-x-3a}{x^2-9a^2} \right) \\
 &= \frac{6a}{a^2-x^2} + \frac{6a}{x^2-9a^2} \\
 &= 6a \left(\frac{x^2-9a^2+a^2-x^2}{(a^2-x^2)(x^2-9a^2)} \right) \\
 &= 6a \left(\frac{-8a^2}{(a^2-x^2)(x^2-9a^2)} \right) = \frac{48a^3}{(x^2-a^2)(x^2-9a^2)} \\
 &= \frac{48a^3}{x^4-10a^2x^2+9a^4}
 \end{aligned}$$

27 The given expression

$$\begin{aligned}
 &= \left(\frac{2}{x-1} - \frac{1}{x+1} - \frac{3}{x^2-1} \right) - \frac{x}{x^2+1} \\
 &= \frac{2x+2-x+1-3}{x^2-1} - \frac{x}{x^2+1} \\
 &= \frac{2}{x^2-1} - \frac{x}{x^2+1} = \frac{x(x^2+1-x^2+1)}{x^4-1} = \frac{2x}{x^4-1}
 \end{aligned}$$

28 The given expression

$$\begin{aligned}
 &= \frac{a-c}{(a-b)(1-a)} - \frac{b-c}{(a-b)(1-b)} \\
 &= \frac{(a-c)(1-b) - (b-c)(1-a)}{(a-b)(1-a)(1-b)} \\
 &= \frac{ax - ab - c1 + bc - (b1 - ab - c2 + ac)}{(a-b)(1-a)(1-b)} \\
 &= \frac{2(a-b) - c(a-b)}{(a-b)(1-a)(1-b)} \\
 &= \frac{(1-c)(a-b)}{(a-b)(1-a)(1-b)} = \frac{1-c}{(1-a)(1-b)}
 \end{aligned}$$

29 The given expression

$$\begin{aligned}
 &= \frac{1}{(1-1)(1-2)} + \frac{1}{(1-2)(1-3)} + \frac{2}{(1-3)(1-5)} \\
 &= \frac{x-3+1-1}{(1-1)(1-2)(1-3)} + \frac{2}{(1-3)(1-5)} \\
 &= \frac{2(1-2)}{(1-1)(1-2)(1-3)} + \frac{2}{(1-3)(1-5)} \\
 &= 2 \left\{ \frac{1}{(1-1)(1-3)} + \frac{1}{(1-3)(1-5)} \right\} \\
 &= 2 \left(\frac{x-5+1-1}{(1-1)(1-3)(1-5)} \right) \\
 &= \frac{22(x-3)}{(x-1)(1-3)(1-5)} = \frac{4}{(1-1)(1-5)} = \frac{4}{1^2-61+5}
 \end{aligned}$$

30 The given expression

$$\begin{aligned}
 &= \frac{1}{(1+a)(1+4a)} + \frac{1}{(1+4a)(1+7a)} + \frac{2}{(1+7a)(1+13a)} \\
 &= \frac{1+7a+1+a}{(1+a)(1+4a)(1+7a)} + \frac{2}{(1+7a)(1+13a)} \\
 &= \frac{2(x+4a)}{(1+a)(1+4a)(1+7a)} + \frac{2}{(1+7a)(1+13a)} \\
 &= 2 \left\{ \frac{1}{(1+a)(1+7a)} + \frac{1}{(1+7a)(1+13a)} \right\} \\
 &= 2 \left\{ \frac{x+13a+x+a}{(1+a)(1+7a)(1+13a)} \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2(2+7a)}{(r+a)(r+7a)(r+13a)} = \frac{4}{(x+a)(x+13a)} \\
 &= \frac{4}{r^2+14ax+13a^2}
 \end{aligned}$$

31 The given expression

$$\begin{aligned}
 &= \frac{1}{(x+1)(r+2)} + \frac{1}{(r+2)(x+3)} + \frac{2x}{(x+1)(x+3)} \\
 &= \frac{(x+3) + (1+1) + 2x(x+2)}{(x+1)(x+2)(x+3)} \\
 &= \frac{2(r+2) + 2x(x+2)}{(x+1)(x+2)(x+3)} \\
 &= \frac{2(r+2)(1+r)}{(x+1)(r+2)(r+3)} = \frac{2}{x+3}
 \end{aligned}$$

32 The given expression

$$\begin{aligned}
 &= \frac{(1+r+x^2) - (1-x+r^2) - 2x}{1+x^2+x^4} \\
 &= \frac{2x-2r}{1+x^2+x^4} = 0
 \end{aligned}$$

33 The given expression

$$\begin{aligned}
 &= \frac{(1-x+x^2) - (1+x+r^2)}{1+x^2+r^4} + \frac{2x}{1-x^2+x^4} \\
 &= \frac{-2x}{1+x^2+x^4} + \frac{2x}{1-x^2+x^4} \\
 &= 2x \left\{ \frac{-(1-x^2+x^4) + (1+x^2+x^4)}{1+r^4+1^8} \right\} \\
 &= \frac{4x^3}{1+x^3+r^8}
 \end{aligned}$$

34 The given expression

$$\begin{aligned}
 &= \frac{(x^2+2x+4) - (x-2)^3}{r^3-8} + \frac{6r}{x^3+8} \\
 &= \frac{(r^2+2r+4) - (r^2-4r+4)}{x^3-8} + \frac{6r}{x^3+8} \\
 &= 6r \left(\frac{1}{r^3-8} + \frac{1}{r^3+8} \right) \\
 &= 6r \frac{r^3+8+r^3-8}{r^6-64} = \frac{12r^4}{r^6-64}
 \end{aligned}$$

35 The given expression

$$\begin{aligned}
 &= \frac{(2x^2 + 6ax + 9a^2) - (2x^2 - 6ax + 9a^2)}{4x^4 + 81a^4} + \frac{12ax}{4x^4 - 81a^4} \\
 &= 12ax \left(\frac{1}{4x^4 + 81a^4} + \frac{1}{4x^4 - 81a^4} \right) \\
 &= 12ax \frac{4x^4 - 81a^4 + 4x^4 + 81a^4}{16x^8 - 6561a^8} = \frac{96ax^5}{16x^8 - 6561a^8}
 \end{aligned}$$

Exercise 59

1 The required product

$$= \frac{2 \times a^2 \times 9 \times b^2 \times 8 \times c^2}{3 \times a \times b \times 16 \times a \times c \times 9 \times b \times c} = \frac{2 \times 9 \times 8 \times a^2 b^2 c^2}{3 \times 16 \times 9 \times a^2 b^2 c^2} = \frac{1}{3}$$

2 The required product

$$= \frac{4 \times a^2 \times b^2 \times 9 \times c^2 \times 4 \times d^2}{3 \times c^2 \times 16 \times d^2 \times 27 \times b^2} = \frac{4 \times 9 \times 4 \times a^2 b^2 c^2 d^2}{3 \times 16 \times 27 \times b^2 c^2 d^2} = \frac{a^2}{9}$$

3 The required product

$$= \frac{x^2 \times y^2 \times z^2}{j \times x \times x \times i \times x \times y} = \frac{1^2 j^2 z^2}{1 \cdot y^2 z^2} = yz$$

4 The required product

$$= \frac{7 \times 4 \times a^2 b^2 c^2 \times 1^2 j^2 z^2}{12 \times 21 \times a^2 b^2 c^2 \times 1 j^2 z^2} = \frac{7^2 y^2 z^2}{9 a^2 b^2 c^2}$$

5 The required product

$$= \frac{12 \times 35 \times m^2 \times n^2 \times x^2 \times y \times z}{7 \times 95 \times m^2 \times n \times 1 \times y \times z} = \frac{5n^2 x^2}{8my}$$

6 The given expression

$$= \frac{\tau+1}{x-1} \times \frac{(x+2)(\tau-1)}{x(x+1)} = \frac{\tau+2}{\tau}$$

7 The given expression

$$= \frac{(a+3b)(a-3b)}{a^2 a + 3b} \times \frac{3a^2}{a(a-3b)} = 3$$

8 The given expression

$$= \frac{(a-b)(a^2 + ab + b^2)}{a(a+b)} \times \frac{(a+b)^2}{a^2 + ab + b^2} = \frac{(a-b)(a+b)}{a} = \frac{a^2 - b^2}{a}$$

9 The given expression

$$= \frac{(a+2x)(a^2-2ax+4x^2)}{a^2(a-2x)} \times \frac{(a-2x)^2}{a^2-2ax+4x^2}$$

$$= \frac{(a+2x)(a-2x)}{a^2} = \frac{a^2-4x^2}{a^2}$$

10 The given expression

$$= \frac{(x+1)(x+3)}{(x+2)(x-2)} \times \frac{(x-1)(x-2)}{(x-3)(x+3)}$$

$$= \frac{(x+1)(x-1)}{(x+2)(x-3)} = \frac{x^2-1}{x^2-x-6}$$

11 The given expression

$$= \frac{(x-2)(x-5)}{(x+3)(x-5)} \times \frac{(x+3)(x-6)}{(x-2)(x-6)} = 1$$

12 The given expression

$$= \frac{(x-1)(x-3)}{(x-1)(x-5)} \times \frac{(x-2)(x-5)}{(x-2)(x-3)} = 1$$

13 The given expression

$$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)^2} \times \frac{(a-b)}{a(a+b)} = \frac{a^2+b^2}{a}$$

14 The given expression

$$= \frac{(2x-1)(x-2)}{(3x+1)(x-2)} \times \frac{x(3x+1)}{2(2x-1)} = \frac{x}{2}$$

15 The given expression

$$= \frac{(x+2)(x-8)}{(x+3)(x-7)} \times \frac{(x-4)(x-7)}{(x-4)(x-8)} = \frac{x+2}{x+3}$$

16 The given expression

$$= \frac{(a+x)(a-x)}{a+b} \times \frac{(a+b)(a-b)}{x(a+x)} \times \frac{a^2}{a-x} = \frac{a^2(a-b)}{x}$$

17 The given expression

$$= \frac{x^3-ax+a^2}{a^2} \times \frac{x^2+ax+a^2}{a^2} = \frac{x^4+a^2x^2+a^4}{a^4}$$

$$= \frac{x^4}{a^4} + \frac{x^2}{a^2} + 1$$

18 The given expression

$$\begin{aligned}
 &= \frac{8ab+9x^2}{6b^2} \times \frac{8ab+9x^2}{12a^2} \\
 &= \frac{(8ab+9x^2)^2}{72abx^2} = \frac{64a^2b^2+144abx^2+81x^4}{72abx^2} \\
 &= \frac{8ab}{9x^2} + 2 + \frac{9x^2}{8ab}
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= \frac{a^2+b^2}{ab} \times \frac{c^2+d^2}{cd} - \frac{a^2-b^2}{ab} \times \frac{c^2-d^2}{cd} \\
 &= \frac{1}{abcd} \{a^2(c^2+d^2-c^2+d^2) + b^2(c^2+d^2+c^2-d^2)\} \\
 &= \frac{1}{abcd} (2a^2d^2 + 2b^2c^2) = 2 \left(\frac{a^2d^2}{abcd} + \frac{b^2c^2}{abcd} \right) = 2 \left(\frac{ad}{bc} + \frac{bc}{ad} \right)
 \end{aligned}$$

20 The given expression

$$= \frac{(2z-1)(z-3)}{(2z-1)(z+4)} \times \frac{(3z-1)(z+4)}{(3z-1)(z+3)} \times \frac{(2z-5)(z+3)}{(2z-5)(z-3)} = 1$$

21 The given expression

$$\begin{aligned}
 &= \frac{b^2-(a-c)^2}{(a+c)^2-b^2} \times \frac{(b-c)^2-a^2}{(a-c)^2-b^2} \\
 &= \frac{(b+a-c)(b-a+c)}{(a+c+b)(a+c-b)} \times \frac{(b-c+a)(b-c-a)}{(a-c+b)(a-c-b)} \\
 &= \frac{(a-c+b)(a-b-c)(a+b-c)(a-b+c)}{(a+b+c)(a-b+c)(a+b-c)(a-b-c)} \\
 &= \frac{a+b-c}{a+b+c}
 \end{aligned}$$

22 The given expression

$$\begin{aligned}
 &= \frac{c^2-(a-b)^2}{b^2-(a-c)^2} \times \frac{(a-c)^2-b^2}{(a-b)^2-c^2} \\
 &= \frac{-\{(a-b)^2-c^2\} \times (a-c)^2-b^2}{-\{(a-c)^2-b^2\} \times (a-b)^2-c^2} \\
 &= \frac{-1}{-1} = 1
 \end{aligned}$$

Exercise 60

- 1 The given expression

$$= \frac{4a^2bc}{15x^2z} \times \frac{25x^2yz}{8ab^2c} = \frac{a \times 5 \times z}{3 \times 1 \times 2 \times b} = \frac{5ax}{6by}$$

- 2 The given expression

$$= \frac{a^2 + ab}{a-b} \times \frac{a^2 - b^2}{ab} = \frac{a(a+b)}{a-b} \times \frac{(a+b)(a-b)}{ab} = \frac{(a+b)^2}{b}$$

- 3 The given expression

$$= \frac{x^2 - 49}{x^2 - 25} \times \frac{x+5}{x+7} = \frac{(x+7)(x-7)}{(x+5)(x-5)} \times \frac{(x+5)}{(x+7)} = \frac{x-7}{x-5}$$

- 4 The given expression

$$\begin{aligned} &= \frac{a^2 - b^2}{a^2 + 2ab + b^2} \times \frac{a+b}{a^2 + b^2} \\ &= \frac{(a^2 + b^2)(a+b)(a-b)}{(a+b)^2} \times \frac{a+b}{a^2 + b^2} = (a-b) \end{aligned}$$

- 5 The given expression

$$\begin{aligned} &= \frac{m^2 - 9n^2}{m^2 + 5mn + 6n^2} \times \frac{m^2 - n^2}{m^2 - 2mn - 3n^2} \\ &= \frac{(m+3n)(m-3n)}{(m+3n)(m+2n)} \times \frac{(m+n)(m-n)}{(m-3n)(m+n)} = \frac{m-n}{m+2n} \end{aligned}$$

- 6 The given expression

$$\begin{aligned} &= \frac{m^2 - n^2}{m+n} \times \frac{m^2 + mn + n^2}{m^2 + mn + n^2} \\ &= \frac{(m-n)(m^2 + mn + n^2)}{m+n} \times \frac{(m+n)(m-n)}{m^2 + mn + n^2} = (m-n)^2 \end{aligned}$$

- 7 The given expression

$$\begin{aligned} &= \left(\frac{2x+y-x-1}{x+y} \right) - \left(\frac{x+y-1}{x+y} \right) \\ &= \left(\frac{x}{x+y} \right) - \left(\frac{x}{x+y} \right) = 1 \end{aligned}$$

8 The given expression

$$\begin{aligned}
 &= \left\{ \frac{a^2 - ab + ab + b^2}{(a + b)(a - b)} \right\} - \left\{ \frac{a^2 + ab - ab + b^2}{(a - b)(a + b)} \right\} \\
 &= \left(\frac{a^2 + b^2}{a^2 - b^2} \right) - \left(\frac{a^2 + b^2}{a^2 - b^2} \right) = 1
 \end{aligned}$$

9 The given expression

$$\begin{aligned}
 &= \frac{(x+y)^2 + (x-y)^2}{x^2 - y^2} - \frac{(x+y)^2 - (x-y)^2}{x^2 - y^2} \\
 &= \frac{2(x^2 + y^2)}{x^2 - y^2} - \frac{4xy}{x^2 - y^2} \\
 &= \frac{2(x^2 + y^2)}{x^2 - y^2} \times \frac{x^2 - y^2}{4xy} = \frac{x^2 + y^2}{2xy}
 \end{aligned}$$

10 The given expression

$$\begin{aligned}
 &= \frac{(x+2)(x-2)}{(x-3)(x+6)} - \frac{(x+2)(x-7)}{(x-6)(x+6)} \\
 &= \frac{(x+2)(x-2)}{(x-3)(x+6)} \times \frac{(x-6)(x+6)}{(x+2)(x-7)} \\
 &= \frac{(x-2)(x-6)}{(x-3)(x-7)} = \frac{x^2 - 8x + 12}{x^2 - 10x + 21}
 \end{aligned}$$

11 The given expression

$$\begin{aligned}
 &= \frac{p^3 + q^3 - 2pq}{p^3 + q^3} - \frac{p^3 - q^3 - 3pq(p-q)}{p^3 - q^3} \\
 &= \frac{(p-q)^3}{p^3 + q^3} - \frac{(p-q)^3}{p^3 - q^3} = \frac{(p-q)^3}{p^3 + q^3} \times \frac{p-q}{(p-q)^3} = \frac{1}{p^3 + q^3}
 \end{aligned}$$

12 The given expression

$$\begin{aligned}
 &= \frac{(a+b)^3}{(a-b)^3} - \frac{(a+b)^3}{(a-b)^3} = \frac{(a+b)^3}{(a-b)^3} \times \frac{(a-b)^3}{(a+b)^3} \\
 &= (a+b)(a-b) = a^2 - b^2
 \end{aligned}$$

13 The given expression

$$\begin{aligned}
 &= \frac{(x+y)(x^2 - xy + y^2)}{(x^3 + xy + y^3)} - \frac{x^3 - 2y + y^3}{(x-y)(x^2 + xy + y^2)} \times \frac{xy}{(x+y)(x-y)} \\
 &= \frac{(x+y)(x^2 - xy + y^2)}{x^3 + xy + y^3} \times \frac{(x-y)(x^2 + xy + y^2)}{x^3 - xy + y^3} \times \frac{xy}{(x+y)(x-y)} \\
 &= xy
 \end{aligned}$$

14 The given expression

$$\begin{aligned}
 &= \frac{a\{(a-b)^2 + 4ab\}}{b(a+b)} - \frac{(a+b)a-b}{ab} \times \frac{b\{(a+b)^2 - 4ab\}}{a(a-b)} \\
 &= \frac{a(a+b)^2}{b(a+b)} \times \frac{ab}{(a+b)(a-b)} \times \frac{b(a-b)^2}{a(a-b)} = ab
 \end{aligned}$$

15 The given expression

$$\begin{aligned}
 &= \frac{(x-6)(x+5)}{(x+6)(x-6)} - \frac{(x+5)(x-2)}{(x-2)(x+4)} - \frac{(x+4)}{2x(x+6)} \\
 &= \frac{x+5}{x+6} \times \frac{x+4}{x+5} \times \frac{2x(x+6)}{x+4} = 2x
 \end{aligned}$$

16 The given expression

$$\begin{aligned}
 &= \frac{(x+12)(x-9)}{(x+8)(x-8)} - \frac{(x-6)(x+12)}{(x+8)(x-7)} - \frac{(x-9)(x-7)}{(x-8)(x-6)} \\
 &= \frac{(x+12)(x-9)}{(x+8)(x-8)} \times \frac{(x+8)(x-7)}{(x-6)(x+12)} \times \frac{(x-8)(x-6)}{(x-9)(x-7)} \\
 &= 1
 \end{aligned}$$

17 The given expression

$$\begin{aligned}
 &= \frac{(x^2+y^2)^2 - (x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)} - \frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)} \\
 &= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy} = \frac{xy}{x^2+y^2}
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= \frac{1}{a-b} \left\{ \frac{(a+b)^3 + (a^3+b^3)}{a+b} \right\} - \frac{a-b}{a+b} \left\{ \frac{a^3-ab+b^3 - (a^3+ab+b^3)}{a^2-ab+b^2} \right\} \\
 &= \frac{2(a^3+ab+b^3)}{(a-b)(a+b)} \times \frac{(a+b)(a^2-ab+b^2)}{(a-b)(-2ab)} \\
 &= \frac{(a^3+ab+b^3)(a^2-ab+b^2)}{-(a-b)^2ab} = -\frac{a^4+a^2b^2+b^4}{ab(a-b)^2}
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= \frac{(a^2+b^2)(a+b)(a-b)}{a^3+b^3} - \frac{(a-b)^2}{(a^2-ab+b^2)} \times \frac{a}{(a+b)^2} \\
 &= \frac{(a^3+b^3)(a-b)}{a^3-ab+b^3} \times \frac{a^2-ab+b^2}{(a-b)^2} \times \frac{a}{a^2+b^2} = \frac{a}{a-b}
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= \frac{(a-b)(a^2+ab+b^2)}{a^2+b^2} - \frac{a^2+ab+b^2}{(a+b)(a^2-ab+b^2)} \times \frac{a^2+b^2}{a^2-ab+b^2} \\
 &= \frac{(a-b)(a^2+ab+b^2)}{a^2+b^2} \times \frac{(a+b)(a^2-ab+b^2)}{a^2+ab+b^2} \times \frac{a^2+b^2}{a^2-ab+b^2} \\
 &= (a-b)(a+b) = a^2-b^2
 \end{aligned}$$

21 The given expression

$$\begin{aligned}
 &= \frac{\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a^2}{b^2} + 1 + \frac{b^2}{a^2}\right)}{\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a}{b} + \frac{b}{a} - 1\right)} \times \frac{\left(\frac{1}{b} - \frac{1}{a}\right) \times ab}{\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{ab}\right) \times ab} \\
 &= \frac{\left(\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}\right) - 1}{\frac{a}{b} + \frac{b}{a} - 1} \times \frac{a-b}{\frac{b}{a} + \frac{a}{b} + 1} \\
 &= \frac{\left(\frac{a}{b} + \frac{b}{a}\right)^2 - 1}{\frac{a}{b} + \frac{b}{a} - 1} \times \frac{a-b}{\frac{b}{a} + \frac{a}{b} + 1} \\
 &= \frac{\left(\frac{a}{b} + \frac{b}{a} + 1\right)\left(\frac{a}{b} + \frac{b}{a} - 1\right) \times (a-b)}{\left(\frac{a}{b} + \frac{b}{a} - 1\right)\left(\frac{b}{a} + \frac{a}{b} + 1\right)} = a-b
 \end{aligned}$$

Exercise 61

$$1 \quad \frac{a}{b + \frac{c}{d + \frac{e}{f}}} = \frac{a}{b + \frac{c}{\frac{df+e}{f}}} = \frac{a}{b + \frac{cf}{df+e}} = \frac{a}{\frac{bdf+bc+cf}{df+e}} = \frac{adf+ae}{bdf+bc+cf}$$

$$2 \quad \frac{\frac{x}{x - \frac{x-1}{1 - \frac{1}{x+1}}}}{1 - \frac{1}{x+1}} = \frac{x}{1 - \frac{x-1}{\frac{x+1-1}{x+1}}} = \frac{x}{1 - \frac{x-1}{x}} = \frac{x}{\frac{x^2-x^2+1}{x}} = \frac{x}{\frac{1}{x}} = x^2$$

$$\begin{aligned}
 3 \quad a^2 + \frac{b^4}{a^2 - \frac{a^3 + b^3}{a + \frac{b^4}{a - b}}} &= a^2 + \frac{b^4}{a^2 - \frac{a^3 + b^3}{a - b}} \\
 &= a^2 + \frac{b^4}{a^2 - \frac{(a^3 + b^3)(a - b)}{a^2 - ab + b^2}} \\
 &= a^2 + \frac{b^4}{a^2 - (a + b)(a - b)} = a^2 + \frac{b^4}{a^2 - (a^2 - b^2)} \\
 &= a^2 + \frac{b^4}{b^2} = a^2 + b^2
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \frac{m}{m^2 - \frac{1}{m + \frac{1}{m+1}}} &= \frac{m}{m^2 - \frac{(m^2 - 1)(m + 1)}{m^2 + m + 1}} \\
 &= \frac{m}{m^2 - (m - 1)(m + 1)} = \frac{m}{m^2 - m^2 + 1} = m
 \end{aligned}$$

5 The given expression

$$\begin{aligned}
 &= \frac{\frac{(x-y)^2}{x+y}}{\frac{(x+y)^2 - (x-y)^2}{x+y} - 1} = \frac{(x-y)^2}{\frac{(x+y)^2 - 1}{4xy} - 1} \\
 &= \frac{(x-y)^2 \times 4xy}{(x+y)^2 - 4xy} = \frac{(x-y)^2 \times 4xy}{(x-y)^2} = 4xy
 \end{aligned}$$

6 The given expression

$$\begin{aligned}
 &= \frac{x^2(x+2)}{4(x+2) + \frac{2(x^2+4)(x-4)(x+2)}{(x+2)^2 + (x-2)^2}} \\
 &= \frac{x^2(x+2)}{4(x+2) + \frac{2(x^2+4)(x^2-4)(x+2)}{2(x^2+4)}} \\
 &= \frac{x^2(x+2)}{(x+2)\{4 + (x^2-4)\}} = \frac{x^2}{x^2} = 1
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \frac{a^3 + b^3}{a - b} &= \frac{(a^3 - b^3) + 2b^3}{a - b} = \frac{a^3 - b^3}{a - b} + \frac{2b^3}{a - b} \\
 &= a^2 + ab + b^2 + \frac{2b^3}{a - b}
 \end{aligned}$$

$$8 \quad \frac{a^3 - b^3}{a+b} = \frac{(a^3 + b^3) - 2b^3}{a+b} = \frac{a^3 + b^3}{a+b} - \frac{2b^3}{a+b}$$

$$= a^2 - ab + b^2 - \frac{2b^3}{a+b}$$

$$9 \quad \frac{a^2 + 2ab + 3b^2}{a+b} = \frac{(a^2 + 2ab + b^2) + 2b^2}{a+b} = \frac{(a+b)^2}{a+b} + \frac{2b^2}{a+b}$$

$$= a+b + \frac{2b^2}{a+b}$$

$$10 \quad \frac{a^2 + 2ab - 3b^2}{a+b} = \frac{(a^2 + 2ab + b^2) - 4b^2}{a+b} = \frac{(a+b)^2}{a+b} - \frac{4b^2}{a+b}$$

$$= a+b - \frac{4b^2}{a+b}$$

$$11 \quad \frac{x^6}{x^2 + y^2} = \frac{(x^6 + y^6) - y^6}{x^2 + y^2} = \frac{x^6 + y^6}{x^2 + y^2} - \frac{y^6}{x^2 + y^2}$$

$$= x^4 - x^2 y^2 + y^4 - \frac{y^6}{x^2 + y^2}$$

$$12 \quad \frac{x^6}{x^2 - y^2} = \frac{(x^6 - y^6) + y^6}{x^2 - y^2} = \frac{x^6 - y^6}{x^2 - y^2} + \frac{y^6}{x^2 - y^2}$$

$$= x^4 + x^2 y^2 + y^4 + \frac{y^6}{x^2 - y^2}$$

$$13 \quad \frac{x^2 y z + x y^2 z + x y z^2}{x^2 y^2 z^2} = \frac{x^2 y z}{x^2 y^2 z^2} + \frac{x y^2 z}{x^2 y^2 z^2} + \frac{x y z^2}{x^2 y^2 z^2}$$

$$= \frac{1}{y z} + \frac{1}{z x} + \frac{1}{x y}$$

$$14 \quad \frac{x y^2 z^2 + y z^2 x^2 + z x^2 y^2}{x^2 y^2 z^2} = \frac{x y^2 z^2}{x^2 y^2 z^2} + \frac{y z^2 x^2}{x^2 y^2 z^2} + \frac{z x^2 y^2}{x^2 y^2 z^2}$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$15 \quad \frac{3a-6}{(a-1)(a-2)(a-3)} = \frac{(a-1) + (a-2) + (a-3)}{(a-1)(a-2)(a-3)}$$

$$= \frac{a-1}{(a-1)(a-2)(a-3)} + \frac{a-2}{(a-1)(a-2)(a-3)}$$

$$+ \frac{a-3}{(a-1)(a-2)(a-3)}$$

$$= \frac{1}{(a-2)(a-3)} + \frac{1}{(a-1)(a-3)} + \frac{1}{(a-1)(a-2)}$$

$$\begin{aligned}
 16 \quad \frac{3x^2-14}{(x+1)(x+2)(x+3)} &= \frac{(x^2-1)+(x^2-4)+(x^2-9)}{(1+1)(1+2)(x+3)} \\
 &= \frac{x^2-1}{(1+1)(x+2)(1+3)} + \frac{x^2-4}{(x+1)(1+2)(1+3)} \\
 &\quad + \frac{x^2-9}{(1+1)(x+2)(1+3)} \\
 &= \frac{x-1}{(x+2)(1+3)} + \frac{x-2}{(x+1)(1+3)} + \frac{x-3}{(x+1)(x+2)}
 \end{aligned}$$

$$\begin{array}{r}
 17 \quad 1+x \overline{) 1-x \left(1-2x+2x^2-2x^3 \text{ (Quotient)} \right.} \\
 \underline{-2x} \\
 -2x-2x^2 \\
 \underline{2x^2} \\
 2x^2+2x^3 \\
 \underline{-2x^3} \\
 -2x^3-2x^4 \\
 \underline{2x^4} \text{ (Remainder)}
 \end{array}$$

$$\text{Thus } \frac{1-x}{1+x} = 1-2x+2x^2-2x^3+\frac{2x^4}{1+x}$$

$$\begin{array}{r}
 18 \quad x^2-a^2 \overline{) a - \frac{a^3}{x^2} \left(\frac{a}{x^2} + \frac{a^3}{x^4} + \frac{a^5}{x^6} \text{ (Quotient)} \right.} \\
 \underline{-a^2} \phantom{+ \frac{a^5}{x^2}} \\
 \frac{a^2}{x^2} \phantom{+ \frac{a^5}{x^2}} \\
 \underline{\frac{a^2}{x^2} - \frac{a^5}{x^4}} \phantom{+ \frac{a^5}{x^2}} \\
 \frac{a^5}{x^4} \phantom{+ \frac{a^5}{x^2}} \\
 \underline{\frac{a^5}{x^4} - \frac{a^7}{x^6}} \phantom{+ \frac{a^5}{x^2}} \\
 \frac{a^7}{x^6} \text{ (Remainder)}
 \end{array}$$

$$\text{Thus } \frac{a}{x^2-a^2} = \frac{a}{x^2} + \frac{a^3}{x^4} + \frac{a^5}{x^6} + \frac{a^7}{x^6(x^2-a^2)}$$

$$\begin{array}{r}
 19 \quad x^3 + a^3 \bigg) a^3 - a^6 \left(\frac{a^3}{x^3} - \frac{a^6}{x^6} + \frac{a^9}{x^9} \right) \text{ (Quotient)} \\
 \underline{a^3 + \frac{a^6}{x^3}} \\
 \quad \quad \quad a^6 \\
 \quad \quad \quad \underline{- \frac{a^6}{x^3}} \\
 \quad \quad \quad \quad \quad \quad \frac{a^6}{x^3} - \frac{a^9}{x^6} \\
 \quad \quad \quad \quad \quad \quad \underline{\frac{a^9}{x^6}} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \frac{a^9}{x^6} + \frac{a^{12}}{x^9} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{\frac{a^{12}}{x^9}} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad - \frac{a^{12}}{x^9} \text{ (Remainder)}
 \end{array}$$

Thus $\frac{a^3}{x^3+a^3} = \frac{a^3}{x^3} - \frac{a^6}{x^6} + \frac{a^9}{x^9} - \frac{a^{12}}{x^{12}(x^3+a^3)}$

Again $\frac{a^7}{x^3+a^3} = \frac{a^3}{a^3+x^3}$

[illegible]

Thus we get $\frac{x^3}{x^3+a^3} = 1 - \frac{x^3}{a^3} + \frac{x^6}{a^6} - \frac{x^9}{a^9} + \frac{x^{12}}{a^9(x^3+a^3)}$

$$\begin{array}{r}
 20 \quad x+a \overline{) x^4 - 1} \left(x^3 - ax^2 + a^2x - a^3 \text{ (Quotient)} \right) \\
 \underline{1^4 + a1^3} \\
 -ax^3 - 1 \\
 \underline{-ax^3 - a^2x^2} \\
 a^2x^2 - 1 \\
 \underline{a^2x^2 + a^3x} \\
 -a^3x - 1 \\
 \underline{-a^3x - a^4} \\
 a^4 - 1 \text{ (Remainder)}
 \end{array}$$

$$\text{Thus } \frac{x^4 - 1}{x + a} = x^3 - ax^2 + a^2x - a^3 + \frac{a^4 - 1}{x + a}$$

21 The given expression

$$\begin{aligned}
 &= \frac{-(x+2a)(x+2b) + (x-2a)(x-2b) + 4ab}{x^2 - 4b^2} \\
 &= \frac{-\{x^2 + 2x(a+b) + 4ab\} + \{x^2 - 2x(a+b) + 4ab\} + 4ab}{x^2 - 4b^2} \\
 &= \frac{-4x(a+b) + 4ab}{x^2 - 4b^2} \left[x = \frac{ab}{a+b}, \quad x(a+b) = ab \right] \\
 &= \frac{-4ab + 4ab}{x^2 - 4b^2} = 0
 \end{aligned}$$

22 The left-hand expression

$$\begin{aligned}
 &= \left(\frac{x+a}{x-a} - 1 \right) + \left(\frac{1+b}{x-b} - 1 \right) + \left(\frac{1+c}{x-c} - 1 \right) \\
 &= \frac{2a}{x-a} + \frac{2b}{x-b} + \frac{2c}{x-c} \\
 &= 2 \left\{ \frac{a(x-b)(x-c) + b(1-a)(1-c) + c(x-a)(x-b)}{(x-a)(1-b)(x-c)} \right\} \\
 &= 2 \left[\frac{x^2(a+b+c) - \{a(b+c) + b(a+c) + c(a+b)\} + 3abc}{(1-a)(x-b)(x-c)} \right] \\
 &= 2 \left[\frac{1 - 2(ab+bc+ca) - 1 - 2(ab+bc+ca) + 3abc}{(1-a)(x-b)(x-c)} \right] \\
 &\quad \quad \quad \left[\text{since } 1(a+b+c) = 2(ab+bc+ca) \right] \\
 &= \frac{2 \cdot 3abc}{(x-a)(1-b)(1-c)} = \frac{6abc}{(x-a)(x-b)(x-c)}
 \end{aligned}$$

23 The left-hand expression

$$\begin{aligned}
 &= \left(\frac{a+2x}{a-2x} + 1 \right) + \left(\frac{b+2x}{b-2x} + 1 \right) + \left(\frac{c+2x}{c-2x} + 1 \right) \\
 &= \frac{2a}{a-2x} + \frac{2b}{b-2x} + \frac{2c}{c-2x} \\
 &= \left[\frac{a(b-2x)(c-2x) + b(a-2x)(c-2x) + c(a-2x)(b-2x)}{(a-2x)(b-2x)(c-2x)} \right] \\
 &= 2 \left[\frac{3abc - 2x\{a(b+c) + b(c+a) + c(a+b)\} + 4x^2(a+b+c)}{(a-2x)(b-2x)(c-2x)} \right] \\
 &= 2 \left[\frac{3abc - 2x\{2(ab+bc+ca)\} + 4x^2\{a+b+c\}}{(a-2x)(b-2x)(c-2x)} \right] \\
 &\quad \quad \quad [\text{since } ab+bc+ca=x(a+b+c)] \\
 &= \frac{6abc}{(a-2x)(b-2x)(c-2x)}
 \end{aligned}$$

24 The given expression

$$\begin{aligned}
 &= \left(\frac{x^2 - (b+c)x}{x^2 - (b+c)x + bc} - 1 \right) + \left(\frac{x^2 - (c+a)x}{x^2 - (c+a)x + ac} - 1 \right) \\
 &\quad \quad \quad + \left(\frac{x^2 - (a+b)x}{x^2 - (a+b)x + ab} - 1 \right) + 3 \\
 &= -\frac{bc}{(x-b)(x-c)} - \frac{ac}{(x-c)(x-a)} - \frac{ab}{(x-a)(x-b)} + 3 \\
 &= 3 - \frac{\{bc(x-a) + ac(x-b) + ab(x-c)\}}{(x-a)(x-b)(x-c)} \\
 &= 3 - \frac{\{x(ab+bc+ca) - 3abc\}}{(x-a)(x-b)(x-c)} \\
 &= 3 - \frac{\{3abc - 3abc\}}{(x-a)(x-b)(x-c)} \\
 &\quad \quad \quad [\text{since } x(ab+bc+ca) = 3abc] \\
 &= 3
 \end{aligned}$$

25 The given expression

$$\begin{aligned}
 &= \frac{x(x+1) - y^2}{y(y+1) - x^2} = \frac{\frac{a-b}{a+b} \left(\frac{a-b}{a+b} + 1 \right) - \frac{(a+b)^2}{(a-b)^2}}{\frac{a+b}{a-b} \left(\frac{a+b}{a-b} + 1 \right) - \frac{(a-b)^2}{(a+b)^2}}
 \end{aligned}$$

$$\begin{aligned}
& \frac{(a-b)2a}{(a+b)^2} - \frac{(a+b)^2}{(a-b)^2} \\
&= \frac{2a(a+b)}{(a-b)^2} - \frac{(a-b)^2}{(a+b)^2} \\
&= \frac{2a(a-b)^3 - (a+b)^4}{2a(a+b)^3 - (a-b)^4} \\
&= \frac{2a(a^3 - 3a^2b + 3ab^2 - b^3) - (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)}{2a(a^3 + 3a^2b + 3ab^2 + b^4) - (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)} \\
&= \frac{a^4 - 10a^3b - 6ab^3 - b^4}{a^4 + 10a^3b + 6ab^3 - b^4}
\end{aligned}$$

26 The given expression

$$\begin{aligned}
&= \frac{(x^2 - 2ab)(x^2 + 5ab)}{(x^2 + 2ab)(x^2 + 5ab)} \times \frac{a^3 + b^3 + 2ab}{a^3 + b^3 - 2ab} \\
&= \frac{(x^2 - 2ab)}{(x^2 + 2ab)} \times \frac{a^3 + b^3 + 2ab}{a^3 + b^3 - 2ab} \\
&= \frac{a^3 + b^3 - 2ab}{a^3 + b^3 + 2ab} \times \frac{a^3 + b^3 + 2ab}{a^3 + b^3 - 2ab} = 1
\end{aligned}$$

27 The given expression

$$\begin{aligned}
&= \frac{(x^2 - 3ab)(y^2 + 6ab)}{(y^2 + 6ab)(y^2 + 3ab)} \times \frac{a^3 - b^3}{a^3 + b^3} \\
&= \frac{x^2 - 3ab}{y^2 + 3ab} \times \frac{a^3 - b^3}{a^3 + b^3} = \frac{(a+b)^2 - 3ab}{(a-b)^2 + 3ab} \times \frac{a^3 - b^3}{a^3 + b^3} \\
&= \frac{a^2 + b^2 - ab}{a^2 + b^2 + ab} \times \frac{(a-b)(a^2 + b^2 + ab)}{(a+b)(a^2 + b^2 - ab)} \\
&= \frac{a-b}{a+b}
\end{aligned}$$

28 The given expression

$$\begin{aligned}
&= \frac{(x^2 + 2ab)(x^2 - ab)}{(x^2 + 2ab)(y^2 + ab)} - \frac{a^2 + ab + b^2}{a^2 - ab + b^2} \\
&= \frac{x^2 - ab}{y^2 + ab} - \frac{a^2 + ab + b^2}{a^2 - ab + b^2} \\
&= \frac{(a+b)^2 - ab}{(a-b)^2 + ab} - \frac{a^2 + ab + b^2}{a^2 - ab + b^2} \\
&= \frac{a^2 + ab + b^2}{a^2 - ab + b^2} - \frac{a^2 + ab + b^2}{a^2 - ab + b^2} = 0
\end{aligned}$$

29 The given expression

$$\begin{aligned}
&= \frac{x^{12} - x^0 + x^0 + x^6 + x^3 - 1 - x^3 - 1}{x^0 - 1} \\
&= \frac{x^{12} + x^6 - 2}{x^0 - 1} = \frac{(x^6 - 1)(x^6 + 2)}{x^0 - 1} = x^6 + 2
\end{aligned}$$

30 The given expression

$$\begin{aligned}
 &= \frac{(x+a-b)(1-a+b)}{(x+a+b)(1-a+b)} + \frac{(a+1-b)(a-1+b)}{(1+a+b)(1+a-b)} \\
 &\quad + \frac{(b+x-a)(b-1+a)}{(1+a+b)(a+b-x)} \\
 &= \frac{1+a-b}{1+a+b} + \frac{a-1+b}{1+a+b} + \frac{1-a+b}{1+a+b} \\
 &= \frac{x+a-b+a-1+b+1-a+b}{1+a+b} = \frac{1+a+b}{1+a+b} = 1
 \end{aligned}$$

31 The given expression

$$\begin{aligned}
 &= \frac{(a+2b+b)(a+2b-b)}{(a+b+2b)(a+b-2b)} + \frac{(a-b+2b)(a-b-2b)}{(a-2b+b)(a-2b-b)} \\
 &\quad + \frac{(2a+3b+b)(2a+3b-b)}{(2a+b+3b)(2a+b-3b)} \\
 &= \frac{(a+3b)(a+b)}{(a+3b)(a-b)} + \frac{(a+b)(a-3b)}{(a-b)(a-3b)} + \frac{(2a+4b)(2a-2b)}{(2a+4b)(2a-2b)} \\
 &= \frac{a+b}{a-b} + \frac{a+b}{a-b} + \frac{a+b}{a-b} = \frac{3(a+b)}{(a-b)}
 \end{aligned}$$

32 The given expression

$$\begin{aligned}
 &= \frac{(x^3+1-1)(x^3-x+1)}{(1^3+1+1)(x^3-x+1)} + \frac{(1+x^3-1)(x-x^3+1)}{\{x(1+1)+1\}\{x(x+1)-1\}} \\
 &\quad + \frac{\{x(1-1)+1\}\{1(x-1)-1\}}{(1^3+1+1)(x^3-1-1)} \\
 &= \frac{x^3+x-1}{1^3+1+1} + \frac{1+1-x^3}{1^3+x+1} + \frac{x^3-1+1}{x^2+1+1} \\
 &= \frac{x^3+x-1+1+x-x^3+x^3-x+1}{1^3+1+1} = \frac{x^3+1+1}{1^3+x+1} = 1
 \end{aligned}$$

33 The left-hand expression

$$\begin{aligned}
 &= \frac{2ab-a^2-b^2+c^2}{2ab} = \frac{c^2-(a-b)^2}{2ab} \\
 &= \frac{(c+a-b)(c-a+b)}{2ab} \\
 &= \frac{(a+b+c-2b)(a+b+c-2a)}{2ab} \\
 &= \frac{(2s-2b)(2s-2a)}{2ab} = \frac{2(s-a)(s-b)}{ab}
 \end{aligned}$$

34 The given expression

$$\begin{aligned}
&= \frac{b-c}{(a+b-c)(a-b+c)} + \frac{c-a}{(b+c-a)(b-c+a)} \\
&\quad + \frac{a-b}{(c+a-b)(c-a+b)} \\
&= \frac{(b-c)(b+c-a) + (c-a)(c+a-b) + (a-b)(a+b-c)}{(a+b-c)(a-b+c)(b+c-a)} \\
&= \frac{b^2 - c^2 - a(b-c) + c^2 - a^2 - b(c-a) + (a^2 - b^2) - c(a-b)}{(a+b-c)(a-b+c)(b+c-a)} \\
&= \frac{(b^2 - c^2 + c^2 - a^2 + a^2 - b^2) - \{a(b-c) + b(c-a) + c(a-b)\}}{(a+b-c)(b+c-a) + (c+a-b)} \\
&= 0
\end{aligned}$$

35 The given expression

$$\begin{aligned}
&= \frac{a(a+b)(a+b-c) + a(b+c)(b+c-a) + b(c+a)(c+a-b)}{2abc} \\
&= \frac{a(a+b)^2 - c^2(a+b) + a(b+c)^2 - a^2(b+c) + b(c+a)^2 - b^2(c+a)}{2abc} \\
&= \frac{a^2(c-b-c+b) + b^2(c-c-a+a) + c^2(-a-b+a+b) + 6abc}{2abc} \\
&= \frac{6abc}{2abc} = 3
\end{aligned}$$

36 The given expression

$$\begin{aligned}
&= \frac{(x^2 + x^3 - y^3)z(x+y)(x^2 + y^2 - z^2) + x(y+z)(y^3 + z^3 - x^3) + y(z+x)(z^3 + x^3 - y^3)}{2xyz} \\
&\quad + \frac{z^3(yz + y^2z) + y^3(xz + x^2z) + x^3(zy + xy^2)}{2xyz} \\
&= \frac{x^3(z-y-z+y) + y^3(z+x-z-x) + z^3(-1-y+x+y)}{2xyz} \\
&= \frac{2(x^2yz) + 2(xy^2z) + 2(xyz^2)}{2xyz} \\
&= \frac{2xyz(x+y+z)}{2xyz} = x+y+z
\end{aligned}$$

37 The given expression

$$\begin{aligned}
 & \frac{b(c+a)(c^3+a^3-b^3)}{2abc} \\
 &= \frac{c(a+b)(a^3+b^3-c^3)+a(b+c)(b^3+c^3-a^3)+}{2abc} \\
 & \quad + a^3(bc+bc)+b^3(ac+ac)+c^3(ab+ab) \\
 &= \frac{a^3(c-b-c+b)+b^3(c+a-c-a)+c^3(-a-b+a+b)}{2abc} \\
 &= \frac{2a^3b+c+2ab^3c^3}{2abc} = a^2+b^2+c^2
 \end{aligned}$$

38 The given expression

$$\begin{aligned}
 &= \frac{(b+c)(b^3+c^3-a^3)}{2bc} + \frac{(c+a)(a^3+c^3-b^3)}{2ac} + \frac{(a+b)(a^3+b^3-c^3)}{2ab} \\
 &= \frac{a(b+c)(b^3+c^3-a^3)+b(c+a)(a^3+c^3-b^3)+c(a+b)(a^3+b^3-c^3)}{2abc} \\
 & \quad + a^3(bc+bc)+b^3(ac+ac)+c^3(ab+ab) \\
 &= \frac{a^3(-b-b-c+b+c)+b^3(a-c-a+c)+c^3(a+b-a-b)}{2abc} \\
 &= \frac{2a^2bc+2ab^2c+2abc^3}{2abc} = a+b+c
 \end{aligned}$$

39 The required value

$$\begin{aligned}
 &= \frac{a-b}{r-c} + \frac{b-c}{r-a} + \frac{c-a}{r-b} + \frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(r-c)} \\
 & \quad + (c-a)\{x^2-x(c+a)+ca\} \\
 &= \frac{(a-b)\{x^2-r(a+b)+ab\}+(b-c)\{x^2-x(b+c)+bc\}}{(1-a)(r-b)(r-c)} \\
 & \quad + \frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(r-c)} \\
 & \quad + ab(a-b)+bc(b-c)+ca(c-a) \\
 &= \frac{x^2\{(a-b)+(b-c)+(c-a)\}-x\{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)\}+}{(r-a)(r-b)(r-c)} + \\
 & \quad \frac{(a-b)(b-c)(c-a)}{(x-a)(r-b)(r-c)} \\
 &= \frac{ab(a-b)+bc(b-c)+ca(c-a)}{(1-a)(x-b)(x-c)} + \frac{(a-b)(b-c)(c-a)}{(r-a)(x-b)(r-c)} \\
 &= \frac{-(a-b)(b-c)(c-a)}{(r-a)(x-b)(x-c)} + \frac{(a-b)(b-c)(c-a)}{(r-a)(1-b)(r-c)} = 0
 \end{aligned}$$

40 The left-hand expression

$$\begin{aligned}
 &= \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2} \\
 &\quad + 2 \left\{ \frac{1}{(b-c)(c-a)} + \frac{1}{(b-c)(a-b)} + \frac{1}{(c-a)(a-b)} \right\} \\
 &= \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2} \\
 &\quad + 2 \left\{ \frac{(a-b) + (b-c) + (c-a)}{(a-b)(b-c)(c-a)} \right\} \\
 &= \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2} + 2 \times 0 \\
 &= \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2}
 \end{aligned}$$

41 The left-hand expression

$$\begin{aligned}
 &= \frac{1}{1-x} - \left(\frac{1}{1-x} - \frac{1}{1+x} \right) + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - \frac{2x}{1-x^2} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - 2x \left\{ \frac{(1+x^2) - (1-x^2)}{(1-x^2)(1+x^2)} \right\} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - \frac{4x^2}{1-x^4} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - 4x^3 \left\{ \frac{(1+x^4) - (1-x^4)}{1-x^8} \right\} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - \frac{8x^7}{1-x^8} + \frac{8x^7}{1+x^8} \\
 &= \frac{1}{1-x} - 8x^7 \left\{ \frac{(1+x^8) - (1-x^8)}{1-x^{16}} \right\} \\
 &= \frac{1}{1-x} - \frac{16x^8}{1-x^{16}}
 \end{aligned}$$

42 The given expression

$$= \frac{-(b-c) - (c-a) - (a-b)}{(a-b)(b-c)(c-a)} = 0$$

43 The given expression

$$\begin{aligned}
 &= \frac{-a^2(b-c) - b^2(c-a) - c^2(a-b)}{(a-b)(b-c)(c-a)} \\
 &= \frac{-\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}}{(a-b)(b-c)(c-a)} \\
 &= \frac{-\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)} = 1
 \end{aligned}$$

44 The given expression

$$\begin{aligned}
 &= \frac{-(x^2 + yz)(1-x) - (y^2 + zx)(x-1) - (z^2 + xy)(1-y)}{(1-y)(y-z)(z-x)} \\
 &\quad - \{x^2(y-z) + y^2(z-x) + z^2(1-y)\} \\
 &= \frac{-\{yz(y-z) + zx(x-1) + xy(1-y)\}}{(x-y)(y-z)(z-x)} \\
 &= \frac{-\{-(1-y)(y-z)(z-x)\} - \{-(1-y)(y-z)(z-x)\}}{(1-y)(y-z)(z-x)} \\
 &= \frac{2(1-y)(y-z)(z-x)}{(1-y)(y-z)(z-x)} = 2
 \end{aligned}$$

45 The given expression

$$\begin{aligned}
 &= \frac{-bc(b-c) - ca(c-a) - ab(a-b)}{(a-b)(b-c)(c-a)} \\
 &= \frac{-\{bc(b-c) + ca(c-a) + ab(a-b)\}}{(a-b)(b-c)(c-a)} \\
 &= \frac{-\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)} = 1
 \end{aligned}$$

46 The given expression

$$\begin{aligned}
 &= \frac{(x^2 - yz)(y+z) - (y^2 + zx)(1-x) - (z^2 + xy)(1-y)}{(x-y)(1-x)(y+z)} \\
 &= \frac{x^2(y+z-x-y) - y^2(z+1-x-1) - z^2(y-1+x-y)}{(1-y)(1-x)(y+z)} = 0
 \end{aligned}$$

47 The given expression

$$\begin{aligned}
 &= \frac{-yz(y-z) - xz(z-x) - xy(1-y)}{xyz(x-y)(y-z)(z-x)} \\
 &= \frac{-\{yz(y-z) + xy(z-x) + xz(1-y)\}}{xyz(x-y)(y-z)(z-x)} \\
 &= \frac{(x-y)(y-z)(z-x)}{xyz(x-y)(y-z)(z-x)} = \frac{1}{xyz}
 \end{aligned}$$

48 The given expression

$$\begin{aligned}
 &= \frac{-(a-b)(x-c)(b-c) - (c-a)(x-c)(c-a) - (x-a)(1-b)(a-b)}{(a-b)(b-c)(c-a)(x-a)(1-b)(1-c)} \\
 &\quad + \{x^2 - x(a+b) + ab\}(a-b) \\
 &= \frac{[x^2 - x(b+c) + bc](b-c) + [x^2 - x(a+c) + ac](c-a)}{(a-b)(b-c)(c-a)(1-b)(1-c)} \\
 &\quad - \{bc(b-c) + ac(c-a) + ab(a-b)\} \\
 &= \frac{-[x^2\{(b-c) + (c-a) + (a-b)\} - \{b^2 - c^2 + (c^2 - a^2) + (a^2 - b^2)\}]}{(a-b)(b-c)(c-a)(x-a)(1-b)(1-c)} \\
 &\quad - \{-(a-b)(b-c)(c-a)\} \\
 &= \frac{1}{(a-b)(b-c)(c-a)(1-b)(1-c)} \\
 &= \frac{1}{(1-a)(1-b)(x-c)}
 \end{aligned}$$

49 The given expression

$$\begin{aligned}
 &= \frac{-a(b-c)(x-b)(1-c) - b(c-a)(1-a)(1-c) - c(a-b)(1-a)}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &\quad + c(a-b)\{x^2 - x(a+b) + ab\} \\
 &= \frac{-[a(b-c)\{x^2 - x(b+c) + bc\} + b(c-a)\{x^2 - x(a+c) + ac\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &\quad + c(a-b)\{x^2 - x(a+b) + ab\} \\
 &= \frac{-[x^2\{a(b-c) + b(c-a) + c(a-b)\} - \{a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &\quad + c(a-b)(b-c)(c-a) \\
 &= \frac{x}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &= \frac{x}{(1-a)(1-b)(x-c)}
 \end{aligned}$$

50 The given expression

$$\begin{aligned}
 &= \frac{c^2(a-b)(1-a)(1-b)}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &\quad - \{a^2(b-c)(1-b)(1-c) + b^2(c-a)(1-a)(1-c) + \\
 &\quad + c^2(a-b)(1-a)(1-b)\} \\
 &= \frac{-[a^2(b-c)\{x^2 - x(b+c) + bc\} + b^2(c-a)\{x^2 - x(a+c) + ac\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &\quad + c^2(a-b)\{x^2 - x(a+b) + ab\} \\
 &\quad + abc\{a(b-c) + b(c-a) + c(a-b)\} \\
 &= \frac{-[x^2\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} - \{a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-[1^2\{-(a-b)(b-c)(c-a)\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &= \frac{x^2}{(1-a)(1-b)(1-c)}
 \end{aligned}$$

51 The given expression

$$\begin{aligned}
 &= \frac{-(c^2+hc+\lambda)(a-b)(1-a)(x-b) - (a^2+ha+\lambda)(b-c)(1-b)(1-c) - (b^2+hb+\lambda)(c-a)(1-a)(x-c)}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &= \frac{(a-c)+ac + (c^2+hc+\lambda)(a-b)\{1^2-1(a+b)+ab\} - [(a^2+ha+\lambda)(b-c)\{1^2-1(b+c)+bc\} + (b^2+hb+\lambda)(c-a)\{1^2-x \\
 &\quad (1-a)+(a-b)\} + \lambda\{bcb-c)+ac(c-a)+ab(a-b)\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &= \frac{(c^3-a^3)+(a^3-b^3)+abc\{a(b-c)+b(c-a)+c(a-b)\} + abch\{(b-c)+ \\
 &\quad c^3(a^3-b^3)\} - h1\{ab(b^2-c^2)+b(c^2-a^2)+c(a^3-b^3)\} - 1\lambda\{(b^2-c^2)+ \\
 &\quad x^2h\{-(b-c)+(c-a)+(a-b)\} - 1\{a^2(b^2-c^2)+b^2(c^2-a^2)+ \\
 &\quad -[x^2\{a^2(b-c)+b^2(c-a)+c^2(a-b)+1^2h\{a(b-c)+b(c-a)+c(a-b)\} + \\
 &\quad (a-b)(b-c)(c-a)(1-a)(x-b)(1-c) + \lambda\{-(a-b)(b-c)(c-a)\}]}{(a-b)(b-c)(c-a)(1-a)(1-b)(1-c)} \\
 &= \frac{-[1^2\{-(a-b)(b-c)(c-a)\} - h1\{(a-b)(b-c)(c-a) \\
 &\quad (a-b)(b-c)(c-a)(x-a)(x-b)(1-c) \\
 &= \frac{(a-b)(b-c)(c-a)(x^3+hx+\lambda)}{(a-b)(b-c)(c-a)(x-a)(x-b)(1-c)} \\
 &= \frac{1^3+hx+\lambda}{(1-a)(1-b)(1-c)}
 \end{aligned}$$

52 The left-hand expression

$$\begin{aligned}
 &= \frac{a^2(c-b)}{bc} + \frac{b^2(a-c)}{ac} + \frac{c^2(b-a)}{ab} \\
 &= \frac{a(c-b)}{bc} + \frac{b(a-c)}{ac} + \frac{c(b-a)}{ab} \\
 &= \frac{-a^2(b-c) - b^2(c-a) - c^2(a-b)}{abc} \\
 &= \frac{-a^2(b-c) - b^2(c-a) - c^2(a-b)}{abc} \\
 &= \frac{a^2(b-c) + b^2(c-a) + c^2(a-b)}{a^2(b-c) + b^2(c-a) + c^2(a-b)} \\
 &= \frac{(a+b+c)(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)} = a+b+c
 \end{aligned}$$

53 The left-hand expression

$$= \frac{(ab + bc + ca)(a - b)(a - c)(b - c)}{(a - b)(a - c)(b - c)} \quad \left[\begin{array}{l} \text{See page 127} \\ \text{Example 41} \end{array} \right]$$

$$= ab + bc + ca$$

54 The left-hand expression

$$= \frac{a\{a^2 + a(b + c) + bc\}}{(a - b)(a - c)} + \frac{b\{b^2 + b(a + c) + ac\}}{(b - a)(b - c)} + \frac{c\{c^2 + c(a + b) + ab\}}{(c - a)(c - b)}$$

$$= \frac{-abc(c - a) - c^2(a - b) - c^2(a^2 - b^2) - abc(b - c) - a^2(b - c) - a^2(b^2 - c^2) - abc(a - b) - b^2(c - a) - b^2(c^2 - a^2)}{(a - b)(b - c)(c - a)}$$

$$= \frac{c^2(a^2 - b^2) + abc\{(b - c) + (c - a) + (a - b)\}}{(a - b)(b - c)(c - a)}$$

$$= \frac{-\{a^2(b - c) + b^2(c - a) + c^2(a - b)\} + \{a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)\}}{(a - b)(b - c)(c - a)}$$

$$= a + b + c$$

55 The left-hand expression

$$= \frac{-b^2c^2(b^2 - c^2) - a^2c^2(c^2 - a^2) - a^2b^2(b^2 - a^2)}{abc(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}$$

$$= \frac{-\{b^2c^2(b^2 - c^2) + a^2c^2(c^2 - a^2) + a^2b^2(b^2 - a^2)\}}{abc(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}$$

$$= \frac{(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{abc(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}$$

$$= \frac{1}{abc}$$

56 The given expression

$$= \frac{-ab(x^2 - 2cx + c^2)(a + b) - bc(x^2 - 2ax + a^2)(b - c) - ac(x^2 - 2bx + b^2)(c - a)}{(a - b)(b - c)(c - a)}$$

$$+ 2abcx\{(b - c) + (c - a) + (a - b)\} - abc\{a(b - c) + b(c - a) + c(a - b)\}$$

$$= \frac{-x^2\{bcb - c) + ac(c - a) + ab(a - b)\} - x\{a(b - c) + b(c - a) + c(a - b)\}}{(a - b)(b - c)(c - a)}$$

$$= \frac{-x^2\{-1(a - b)(b - c)(c - a)\}}{(a - b)(b - c)(c - a)}$$

$$= x^2$$

EXERCISE 62

- 1 $4x + 3 = 2x + 5$ 2 $3x + 2 = x + 6$
 or $4x - 2x = 5 - 3$ or $3x - x = 6 - 2$
 or $2x = 2,$ or $2x = 4,$
 $x = 1$ $x = 2$
- 3 $5x - 6 = 2x + 3$ 4 $15x - 9 = 11x - 25$
 or $5x - 2x = 3 + 6$ or $15x - 11x = -25 + 9$
 or $3x = 9,$ or $4x = -16,$
 $x = 3$ $x = -4$
- 5 $4(x - 3) = 2(x - 6)$ 6 $2(x - 15) = 5(x - 11) + 4$
 or $4x - 12 = 2x - 12$ or $2x - 30 = 5x - 55 + 4 = 5x - 51$
 or $4x - 2x = -12 + 12$ or $2x - 5x = -51 + 30$
 or $2x = 0,$ or $-3x = -21;$
 $x = 0$ $x = 7$
- 7 $19 - 3x = 5x + 35$
 or $-3x - 5x = 35 - 19$
 or $-8x = 16;$
 $x = -2$
- 8 $3(x - 2) + 7(2x - 3) = 5(1 - 2x) - 59$
 or $3x - 6 + 14x - 21 = 5 - 10x - 59$
 or $17x - 27 = -10x - 54$
 or $17x + 10x = -54 + 27$
 or $27x = -27,$ $x = -1$
- 9 $13x - 4(5x - 8) + 17 = 0$
 or $13x - 20x + 32 + 17 = 0$
 or $-7x + 49 = 0$
 or $-7x = -49,$ $x = 7$
- 10 $14(x - 4) + 3(x + 5) = 6(7 - 2x) + 4$
 or $14x - 56 + 3x + 15 = 42 - 12x + 4$
 or $17x - 41 = 46 - 12x$
 or $17x + 12x = 46 + 41$
 or $29x = 87,$ $x = 3$

- 11 $8(2x-7)-9(3x-14)=15$,
 or $16x-56-27x+126=15$
 or $-11x+70=15$
 or $-11x=15-70=-55$, $x=5$.
- 12 $3x-13(2x-13)=4x-20$
 or $3x-26x+169=4x-20$
 or $-23x+169=4x-20$
 or $-23x-4x=-20-169$
 or $-27x=-189$, $x=7$
- 13 $49+13(5x+27)=8(5+x)-3x$
 or $49+65x+351=40+8x-3x$
 or $65x+400=40+5x$
 or $65x-5x=40-400$
 or $60x=-360$, $x=-6$
- 14 $16-5(7x-2)=13(x-2)+4(13-x)$
 or $16-35x+10=13x-26+52-4x$
 or $26-35x=9x+26$
 or $-35x-9x=26-26$
 or $-44x=0$, $x=0$
- 15 $8x+5(x+7)+9(2x+23)-3(x+6)=0$
 or $8x+5x+35+18x+207-3x-18=0$
 or $8x+5x+18x-3x=-35-207+18$
 or $28x=-224$, $x=-8$
- 16 $(x-7)(4x-29)=(2x-5)(2x-17)+1$
 or $4x^2-57x+203=4x^2-44x+85+1$
 or $-57x+203=-44x+86$
 or $-57x+44x=86-203$
 or $-13x=-117$, $x=9$.
- 17 $(3x+2)(2x-6)=(4-3x)(1-2x)-10$
 or $6x^2-14x-12=4-11x+6x^2-10$
 or $-14x-12=-11x-6$
 or $-14x+11x=-6+12$
 or $-3x=6$, $x=-2$

$$18 \quad (3x+5)(6x-7) = (3x+2)(9x-13) - (3x+1)(3x-1)$$

$$\text{or } 18x^2 + 9x - 35 = 27x^2 - 21x - 26 - 9x^2 + 1$$

$$\text{or } 18x^2 + 9x - 35 = 18x^2 - 21x - 25$$

$$\text{or } 9x + 21 = -25 + 35$$

$$\text{or } 30x = 10, \quad x = \frac{1}{3}$$

$$19 \quad (x+2)(2x+5) = 2(x+1)^2 + 13$$

$$\text{or } 2x^2 + 9x + 10 = 2x^2 + 4x + 2 + 13$$

$$\text{or } 9x + 10 = 4x + 15$$

$$\text{or } 9x - 4x = 15 - 10$$

$$\text{or } 5x = 5, \quad x = 1$$

$$20 \quad (x+1)(4x-7) - (x-1)(x+5) = 3(x+2)^2 + 5$$

$$\text{or } 4x^2 - 3x - 7 - (x^2 + 4x - 5) = 3x^2 + 12x + 12 + 5$$

$$\text{or } 3x^2 - 3x - 4x - 7 + 5 = 3x^2 + 12x + 17$$

$$\text{or } -3x - 4x - 12x = 17 + 7 - 5$$

$$\text{or } -19x = 19, \quad x = -1$$

$$21 \quad 3(2-4)^2 + 5(1-3)^2 = (2x-5)(4x-1) + 24$$

$$\text{or } 3(x^2 - 8x + 16) + 5(x^2 - 6x + 9) = 8x^2 - 22x + 5 + 24$$

$$\text{or } 3x^2 - 24x + 48 + 5x^2 - 30x + 45 = 8x^2 - 22x + 29$$

$$\text{or } -24x - 30x + 22x = 29 - 48 - 45$$

$$\text{or } -32x = -64, \quad x = 2$$

$$22 \quad (6x+9)^2 + (8x-7)^2 = (10x+3)^2 - 71$$

$$\text{or } 36x^2 + 108x + 81 + 64x^2 - 112x + 49$$

$$= 100x^2 + 60x + 9 - 71$$

$$\text{or } 108x - 112x - 60x = 9 - 71 - 81 - 49$$

$$\text{or } -64x = -192, \quad x = 3$$

$$23 \quad 5(x+1)^2 + 7(x+3)^2 = 12(x+2)^2$$

$$\text{or } 5(x^2 + 2x + 1) + 7(x^2 + 6x + 9) = 12(x^2 + 4x + 4)$$

$$\text{or } 5x^2 + 10x + 5 + 7x^2 + 42x + 63 = 12x^2 + 48x + 48$$

$$\text{or } 10x + 42x - 48x = 48 - 5 - 63$$

$$\text{or } 4x = -20, \quad x = -5$$

- 24 $(3x-14)^2 + (4x-19)^2 - (5x-23)^2 = 22$
 or $9x^2 - 84x + 196 + 16x^2 - 152x + 361 - (25x^2 - 230x + 529)$
 $= 22$
 or $-84x - 152x + 230x = 22 - 196 - 361 + 529$
 or $-6x = -6$, $x = 1$
- 25 $(5x-8)^2 + (12x-7)^2 = (13x-10)^2 + 37$
 or $25x^2 - 80x + 64 + 144x^2 - 168x + 49$
 $= 169x^2 - 260x + 100 + 37$
 or $-80x - 168x + 260x = 100 + 37 - 64 - 49$
 or $12x = 24$, $x = 2$
- 26 $(x-1)^3 + (x+1)^3 = 2x(x^2-1) + 4$
 or $2(x^3 + 3x) = 2(x^3 - x + 2)$
 or $x^3 + 3x = x^3 - x + 2$
 or $3x + x = 2$
 or $4x = 2$, $x = \frac{1}{2}$
27. $(x-2)^3 + 2x^3 + (x+2)^3 = 4x^2(x+2)$
 or $2(x^3 + 12x) + 2x^3 = 4x^2(x+2)$
 or $2x^3 + 24x + 2x^3 = 4x^3 + 8x^2$
 or $24x = 8x^2$
 or $24 = 8x$, $x = 3$
- 28 $(x+2)(x+3)(x+4) + 96 = x^2(x+9) + 5(3x+13)$
 or $x^3 + (2+3+4)x^2 + (6+8+12)x + 24 + 96$
 $= x^3 + 9x^2 + 15x + 65$
 or $9x^2 + 26x + 120 = 9x^2 + 15x + 65$
 or $26x - 15x = 65 - 120$
 or $11x = -55$, $x = -5$
- 29 $3(x^2-14) = (x+1)^2 + (x-2)^2 + (x-5)^2$
 or $3(x^2-14) = x^2 + 2x + 1 + x^2 - 4x + 4 + x^2 - 10x + 25$
 or $3(x^2-14) = 3(x^2-4x+10)$
 or $x^2-14 = x^2-4x+10$
 or $4x = 10+14 = 24$, $x = 6$

$$30. \quad a(x-a)=b(x-b)$$

$$\text{or } ax-a^2=bx-b^2$$

$$\text{or } ax-bx=a^2-b^2$$

$$\text{or } x(a-b)=(a+b)(a-b), \quad x=a+b$$

$$31 \quad 3(x-a)+5(2x-3a)=8a$$

$$\text{or } 3x-3a+10x-15a=8a$$

$$\text{or } 13x=8a+3a+15a=26a, \quad x=2a$$

$$32 \quad (x+a)(x+b)-(a+b)^2=(x-a)(x-b)$$

$$\text{or } x^2+x(a+b)+ab-(a+b)^2=x^2-x(a+b)+ab$$

$$\text{or } 2x(a+b)=(a+b)^2, \quad x=\frac{1}{2}(a+b)$$

$$33 \quad a^2(x-a)+b^2(x-b)=abx$$

$$\text{or } x(a^2+b^2)-(a^3+b^3)=abx$$

$$\text{or } x(a^2+b^2-ab)=a^3+b^3 \\ = (a+b)(a^2-ab+b^2), \quad x=a+b$$

$$34 \quad m^2(x-m)+n^2(x+n)+mnx=0$$

$$\text{or } x(m^2+n^2+mn)-(m^3-n^3)=0$$

$$\text{or } x(m^2+mn+n^2)=(m^3-n^3) \\ = (m-n)(m^2+mn+n^2), \quad x=m-n$$

$$35 \quad b(x-2a)+a(x-2b)=(a-b)^2$$

$$\text{or } x(a+b)-4ab=(a-b)^2$$

$$\text{or } x(a+b)=(a-b)^2-4ab \\ = (a+b)^2, \quad x=a+b$$

$$36 \quad a(4x-a) \times b(4x-b)-2ab=0$$

$$\text{or } 4x(a+b)-(a^2+b^2+2ab)=0$$

$$\text{or } 4x(a+b)=(a+b)^2, \quad x=\frac{1}{4}(a+b)$$

$$37 \quad x(x-a)+x(x-b)-2(x-a)(x-b)=0$$

$$\text{or } 2x^2-x(a+b)-2\{x^2-x(a+b)+ab\}=0$$

$$\text{or } x(a+b)-2ab=0$$

$$\text{or } x(a+b)=2ab, \quad x=\frac{2ab}{a+b}$$

$$\begin{aligned}
 38. \quad & (x+3a)(x-3b)+3(x-3a)(x+3b)=4(x-3a)(x-3b) \\
 \text{or} \quad & x^2+3x(a-b)-9ab+3\{x^2-3x(a-b)-9ab\} \\
 & \qquad \qquad \qquad =4\{x^2-3x(a+b)+9ab\} \\
 \text{or} \quad & 3x\{(a-b)-3(a-b)+4(a+b)\}=9ab(4+3+1) \\
 \text{or} \quad & 3x(2a+6b)=9ab \quad 8 \\
 \text{or} \quad & x(2a+6b)=3ab \quad 8 \\
 \text{or} \quad & 2x(a+3b)=3ab \quad 8 \\
 \text{or} \quad & x(a+3b)=12ab, \qquad x=\frac{12ab}{a+3b}
 \end{aligned}$$

$$\begin{aligned}
 39 \quad & (2b+2c-x)^2+(2b-2c+x)^2=(2b+2d-x)^2 \\
 & \qquad \qquad \qquad + (2b-2d+x)^2 \\
 \text{or} \quad & (2b+2c)^2-2x(2b+2c)+x^2+(2b-2c)^2+2x(2b-2c)+x^2 \\
 & \qquad \qquad \qquad = (2b+2d)^2-2x(2b+2d) \\
 & \qquad \qquad \qquad + x^2+(2b-2d)^2+2x(2b-2d)+x^2 \\
 \text{or} \quad & 2(4b^2+4c^2)-2x(4c)=2(4b^2+4d^2)-2x(4d) \\
 \text{or} \quad & -8x(c-d)=-8(c^2-d^2) \\
 \text{or} \quad & x(c-d)=(c+d)(c-d), \qquad x=c+d
 \end{aligned}$$

$$\begin{aligned}
 40 \quad & (x-a)^3+(x-b)^3+(x-c)^3=3(x-a)(x-b)(x-c) \\
 \text{or} \quad & x^3-3ax^2+3a^2x-a^3+x^3-3bx^2+3b^2x-b^3 \\
 & \qquad \qquad \qquad +x^3-3cx^2+3c^2x-c^3 \\
 & \qquad \qquad \qquad =3\{x^3-x^2(a+b+c)+x(ab+ac+bc)-abc\} \\
 \text{or} \quad & 3x^2\{(a+b+c)-a-b-c\}+3x(a^2+b^2+c^2-ab-ac-bc) \\
 & \qquad \qquad \qquad =a^3+b^3+c^3-3abc \\
 \text{or} \quad & 3x(a^2+b^2+c^2-ab-ac-bc)=(a+b+c)(a^2+b^2+c^2 \\
 & \qquad \qquad \qquad -ab-ac-bc), \qquad x=\frac{1}{3}(a+b+c)
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & (x+a)^3+(x+b)^3+(x+c)^3=3(x+a)(x+b)(x+c) \\
 \text{or} \quad & x^3+3ax^2+3a^2x+a^3+x^3+3bx^2+3b^2x+b^3+x^3+3cx^2 \\
 & \qquad \qquad \qquad +3c^2x+c^3=3\{x^3+x^2(a+b+c)+x(ab+ac+bc)+abc\} \\
 \text{or} \quad & 3x^2\{a+b+c-(a+b+c)\}+3x(a^2+b^2+c^2-ab-ac-bc) \\
 & \qquad \qquad \qquad =3abc-(a^3+b^3+c^3) \\
 \text{or} \quad & 3x(a^2+b^2+c^2-ab-ac-bc)=-(a^3+b^3+c^3-3abc) \\
 & \qquad \qquad \qquad =-(a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\
 & \qquad \qquad \qquad x=-\frac{1}{3}(a+b+c)
 \end{aligned}$$

$$\begin{aligned}
 42 \quad & \frac{x}{2} + 5 = \frac{x}{3} + 7, \\
 \text{or} \quad & \left(\frac{x}{2} - \frac{x}{3} \right) = 7 - 5 \\
 \text{or} \quad & \frac{x}{6} = 2, \quad x = 12
 \end{aligned}
 \qquad
 \begin{aligned}
 43 \quad & -\frac{x}{a} + a = \frac{x}{b} + b, \\
 \text{or} \quad & \frac{x}{a} - \frac{x}{b} = b - a \\
 \text{or} \quad & x \left(\frac{b-a}{ab} \right) = b - a, \\
 & x = ab,
 \end{aligned}$$

$$\begin{aligned}
 44 \quad & \frac{x}{6} - \frac{x}{5} = \frac{x}{15} - \frac{x}{3} + 7 \\
 \text{or} \quad & -\frac{x}{30} = -x \left(\frac{4}{15} \right) + 7 \\
 \text{or} \quad & x \left(\frac{4}{15} - \frac{1}{30} \right) = 7 \\
 \text{or} \quad & x \left(\frac{8-1}{30} \right) = 7 \\
 \text{or} \quad & x \left(\frac{7}{30} \right) = 7, \quad x = 30
 \end{aligned}$$

$$\begin{aligned}
 45 \quad & \frac{x}{2} - \frac{x}{3} + \frac{x}{4} = 2 - \frac{x}{6} + \frac{5x}{12} \\
 \text{or} \quad & x \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{6} - \frac{5}{12} \right) = 2 \\
 \text{or} \quad & x \left(\frac{6-4+3+2-5}{12} \right) = 2 \\
 \text{or} \quad & x \left(\frac{2}{12} \right) = 2,
 \end{aligned}$$

$$\begin{aligned}
 46 \quad & \frac{a}{bx} - \frac{b}{ax} = a^2 - b^2 \\
 \text{or} \quad & \frac{a^2 - b^2}{abx} = a^2 - b^2 \\
 \text{or} \quad & abx = 1, \quad x = \frac{1}{ab}
 \end{aligned}$$

$$\begin{aligned}
 47 \quad & \frac{1}{2}(x+1) + \frac{1}{3}(x+2) + \frac{1}{4}(x+3) = 16 \\
 \text{or} \quad & \frac{1}{12}\{(6x+6) + (4x+8) + (3x+9)\} = 16 \\
 \text{or} \quad & 13x + 23 = 192 \\
 \text{or} \quad & 13x = 192 - 23 = 169, \quad x = 13
 \end{aligned}$$

$$48 \quad \frac{x-6}{5} + \frac{x-4}{3} = 8 - \frac{x-2}{7}$$

or $\frac{1}{15}\{(3x-18) + (5x-20)\} = \frac{1}{7}(56-x+2)$
 or $7(8x-38) = 15(58-x)$
 or $56x + 15x = 870 + 266$
 or $71x = 1136, \quad x = 16$

$$49 \quad \frac{x}{10} + \frac{2x-13}{9} = 8 - \frac{4x-35}{15}$$

or $\frac{1}{90}(9x+20x-130) = \frac{1}{15}(120-4x+35)$
 or $29x-130 = 6(155-4x)$
 or $29x+24x = 930+130$
 or $53x = 1060, \quad x = 20$

$$50 \quad \frac{x+7}{2} + \frac{x+13}{5} + \frac{x+17}{7} = \frac{x+27}{4}$$

or $\frac{35(x+7) + 14(x+13) + 10(x+17)}{70} = \frac{x+27}{4}$
 or $70(x+7) + 28(x+13) + 20(x+17) = 35(x+27)$
 or $70x+490+28x+364+20x+340 = 35x+945$
 or $70x+28x+20x-35x = 945-490-364-340$
 or $83x = -249, \quad x = -3$

$$51 \quad 6\frac{1}{3} - \frac{x-7}{3} = \frac{4x-2}{5}$$

or $\frac{19-x+7}{3} = \frac{4x-2}{5}$
 or $5(26-x) = 3(4x-2)$
 or $130-5x = 12x-6$
 or $-5x-12x = -6-130$
 or $-17x = -136, \quad x = 8$

$$52 \quad \frac{x-1}{3} - \frac{x-9}{2} + \frac{3x-2(x-2)}{7} = 4\frac{1}{2}$$

or $\frac{x-1}{3} - \frac{x-9}{2} + \frac{3x-2x+4}{7} = \frac{9}{2}$
 or $14(x-1) - 21(x-9) + 6(x+4) = 189$
 or $14x-14-21x+189+6x+24 = 189$
 or $14x-21x+6x = 14-24, \quad x = 10$

$$53 \quad \frac{2x-9}{27} + \frac{x}{18} - \frac{x-3}{4} = 8\frac{1}{3} - x$$

$$\text{or } 4(2x-9) + 6x - 27(x-3) = 36 \times 25 - 108x$$

$$\text{or } 8x - 36 + 6x - 27x + 81 = 900 - 108x$$

$$\text{or } 8x + 6x - 27x + 108x = 900 + 36 - 81$$

$$\text{or } 95x = 855, \quad x = 9$$

$$54 \quad \frac{9x+7}{2} - \left(x - \frac{x-2}{7}\right) = 36$$

$$\text{or } 7(9x+7) - 2(7x - x + 2) = 14 \times 36$$

$$\text{or } 63x + 49 - 12x - 4 = 504$$

$$\text{or } 63x - 12x = 504 - 49 + 4$$

$$\text{or } 51x = 459, \quad x = 9$$

$$55 \quad \frac{7x+9}{4} - \left(x - \frac{2x-1}{9}\right) = 7$$

$$\text{or } 9(7x+9) - 4(9x - 2x + 1) = 36 \times 7$$

$$\text{or } 63x + 81 - 28x - 4 = 252$$

$$\text{or } 63x - 28x = 252 - 81 + 4$$

$$\text{or } 35x = 175, \quad x = 5$$

$$56 \quad \frac{x+7}{3} - 5\frac{1}{2} = \frac{2x+5}{7} + \frac{10-5x}{8}$$

$$\text{or } 56(x+7) - 23 \times 42 = 24(2x+5) + 21(10-5x)$$

$$\text{or } 56x + 392 - 966 = 48x + 120 + 210 - 105x$$

$$\text{or } 56x - 48x + 105x = 120 + 210 - 392 + 966$$

$$\text{or } 113x = 904, \quad x = 8$$

$$57 \quad x - \left(3x - \frac{2x-5}{10}\right) = \frac{1}{6}(2x-57) - \frac{5}{3}$$

$$\text{or } 30x - 3(30x - 2x + 5) = 5(2x - 57) - 50$$

$$\text{or } 30x - 84x - 15 = 10x - 285 - 50$$

$$\text{or } 30x - 84x - 10x = -285 - 50 + 15$$

$$\text{or } -64x = -320, \quad x = 5$$

$$58 \quad \frac{4x-21}{7} + 7\frac{5}{8} + \frac{7x-28}{3} = x + 3\frac{3}{4} - \frac{9-7x}{8} + 1\frac{1}{2}$$

$$\text{or } x\left(\frac{4}{7} + \frac{7}{3}\right) + 7\frac{5}{8} - 3 - 9\frac{1}{2} = x\left(1 + \frac{7}{8}\right) + 3\frac{3}{4} - \frac{9}{8} + 1\frac{1}{2}$$

$$\text{or } x\left(\frac{12+49}{21}\right) + 7\frac{5}{8} - 12\frac{1}{3} = 1\frac{5}{8}x + 3\frac{5}{8} - 1\frac{1}{3}$$

$$\text{or } x\left(\frac{61}{21} - \frac{15}{8}\right) = 3\frac{5}{8} - 1\frac{1}{3} - 7\frac{5}{8} + 12\frac{1}{3}$$

$$\text{or } x\left(\frac{488-315}{168}\right) = -4 + 11 + \frac{1}{3} - \frac{1}{8}$$

$$\text{or } x \frac{173}{168} = 7\frac{5}{24} = \frac{173}{24}, \quad x = \frac{168}{24} = 7$$

$$59 \quad \frac{1}{2}\left(x - \frac{a}{3}\right) - \frac{1}{3}\left(x - \frac{a}{4}\right) + \frac{1}{4}\left(x - \frac{a}{5}\right) = 0$$

$$\text{or } 6\left(x - \frac{a}{3}\right) - 4\left(x - \frac{a}{4}\right) + 3\left(x - \frac{a}{5}\right) = 0$$

$$\text{or } 6x - 2a - 4x + a + 3x - \frac{3a}{5} = 0$$

$$\text{or } 6x - 4x + 3x = 2a - a + \frac{3a}{5}$$

$$\text{or } 5x = 8\frac{a}{5}, \quad x = \frac{8a}{25}$$

$$60 \quad \frac{x-3}{7} - \frac{\frac{1}{2}x-3}{3} = \frac{\frac{1}{6}x+2}{2} - \frac{x-6}{3} + \frac{x}{8}$$

$$\text{or } x\left(\frac{1}{7} - \frac{1}{6} - \frac{1}{12} + \frac{1}{3} - \frac{1}{8}\right) = 1 + 2 + \frac{3}{4} - 1$$

$$\text{or } x\left(\frac{24-28-14+56-21}{24 \times 7}\right) = \frac{17}{7}$$

$$\text{or } x \frac{17}{24 \times 7} = \frac{17}{7}, \quad x = 24.$$

$$61. \quad \frac{1}{8}(x-2) - \frac{1}{4}(x-4) = \frac{1}{12}(2x-3) - 2\frac{3}{4}$$

$$\text{or } x\left(\frac{1}{8} - \frac{1}{4} - \frac{1}{8}\right) = \frac{1}{4} - \frac{11}{4} - \frac{1}{4} + \frac{3}{2}$$

$$\text{or } x\left(\frac{21-24-28}{8 \times 7 \times 3}\right) = \frac{7-77-7-16}{7 \times 4}$$

$$\text{or } x \frac{-31}{8 \times 7 \times 3} = \frac{-93}{7 \times 4}$$

$$\text{or } \frac{x}{6} = 3, \quad x = 18$$

$$62 \quad \frac{a-1}{a} + \frac{2a-x}{2a} = \frac{3a-x}{3a}$$

$$\text{or} \quad 1 - \frac{x}{a} + 1 - \frac{x}{2a} = 1 - \frac{x}{3a}$$

$$\text{or} \quad x \left(-\frac{1}{a} - \frac{1}{2a} + \frac{1}{3a} \right) = -1$$

$$\text{or} \quad x \left(\frac{-6-3+2}{6a} \right) = -1$$

$$\text{or} \quad x \frac{-7}{6a} = -1, \quad x = \frac{6a}{7}$$

$$63 \quad \frac{2x-13}{9} - \frac{x-1}{11} = \frac{x}{8} + \frac{x}{7} - 9$$

$$\text{or} \quad x \left(\frac{2}{9} - \frac{1}{11} - \frac{1}{8} - \frac{1}{7} \right) = -9 - \frac{1}{11} + \frac{13}{9}$$

$$\text{or} \quad x \left(\frac{22-9}{99} - \frac{7+8}{56} \right) = \frac{-891-9+143}{99}$$

$$\text{or} \quad x \left(\frac{13}{99} - \frac{15}{56} \right) = \frac{-757}{99}$$

$$\text{or} \quad x \left(\frac{728-1485}{99 \times 56} \right) = \frac{-757}{99}$$

$$\text{or} \quad x \frac{-757}{99 \times 56} = \frac{-757}{99},$$

$$64 \quad \frac{2x-3}{6} + \frac{3x-8}{11} = \frac{4x+15}{33} + \frac{1}{2}$$

$$\text{or} \quad x \left(\frac{1}{3} + \frac{3}{11} - \frac{4}{33} \right) = \frac{1}{2} + \frac{15}{33} + \frac{1}{11}$$

$$\text{or} \quad x \left(\frac{11+9-4}{33} \right) = 1 + \frac{15+24}{33}$$

$$\text{or} \quad x \frac{16}{33} = 1 + \frac{13}{11}$$

$$\text{or} \quad x \frac{16}{33} = \frac{24}{11}, \quad x = \frac{24}{11} \times \frac{33}{16} = \frac{9}{2} = 4\frac{1}{2}$$

$$65 \quad \frac{4x+3}{9} + \frac{13x}{108} = \frac{8x+19}{18}$$

$$\text{or} \quad 12(4x+3) + 13x = 6(8x+19)$$

$$\text{or} \quad 48x + 36 + 13x = 48x + 114$$

$$\text{or} \quad 13x = 114 - 36 = 78, \quad x = 6$$

66

$$\frac{x^2-21}{4} - \frac{x-31}{5} = \frac{2x^2-3}{8} - \frac{x-51}{3}$$

or $\frac{x^2}{4} - \frac{5}{8} - \frac{x}{5} + \frac{7}{10} = \frac{x^2}{4} - \frac{3}{8} - \frac{x}{3} + \frac{11}{6}$

or, $x(-\frac{1}{5} + \frac{1}{3}) = -\frac{3}{8} + \frac{11}{6} + \frac{5}{8} - \frac{7}{10}$

or $x\left(\frac{-3+5}{15}\right) = \frac{-45+220+75-84}{120}$

or $x \frac{2}{15} = \frac{166}{120}, \therefore x = \frac{166}{120} \times \frac{15}{2} = \frac{249}{8} = 10\frac{3}{8}$

67

$$\frac{a-x^2}{bx} - \frac{b-x}{c} = \frac{c-x}{b} - \frac{b-x^2}{cx}$$

or $c(a-x^2) - bx(b-x) = cx(c-x) - b(b-x^2)$

or $ac - cx^2 - b^2x + bx^2 = c^2x - cx^2 - b^2 + bx^2$

or $ac - b^2x = c^2x - b^2$

or $-b^2x - c^2x = -b^2 - ac$

or $x(b^2 + c^2) = b^2 + ac, \quad x = \frac{ac + b^2}{b^2 + c^2}$

68

$$\frac{x+21}{15} + \frac{x+31}{25} = \frac{x+41}{55}$$

or $\frac{x+21}{3} + \frac{x+31}{5} = \frac{x+41}{11}$

or $x(\frac{1}{3} + \frac{1}{5} - \frac{1}{11}) = \frac{25}{66} - \frac{5}{3} - \frac{2}{3}$

or $x\left(\frac{55+33-15}{15 \times 11}\right) = \frac{25-55-44}{66}$

or $x \frac{73}{165} = -\frac{74}{66} = -\frac{37}{33}$

$x = -\frac{37}{33} \times \frac{165}{73} = -\frac{185}{73} = -2\frac{39}{73}$

69

$$\frac{11x-13}{25} + \frac{19x+3}{7} - \frac{5x-251}{4} = 281 - \frac{17x+4}{21}$$

or $84(11x-13) + 300(19x+3) - 525(5x-251) = 300 \times 197 - 100(17x+4)$

or $x(924 + 5700 - 2625 + 1700) = 59100 - 400$

- 13300 - 900 + 1092

or $5699x = 45592, \quad \therefore x = 8$

$$70 \quad \frac{x-2\frac{5}{8}}{2} - \frac{2-6x}{13} = x - \frac{5x-1(10-3x)}{39}$$

or $39(x-2\frac{5}{8}) - 6(2-6x) = 78x - 2(5x-\frac{1}{3}+x)$

or $x(39+36-78+10+\frac{1}{3}) = 5+12+1\frac{1}{3}$

or $1\frac{1}{3} = 1\frac{1}{3}, \quad x=11$

$$71. \quad \frac{3x-4(1+x)}{4} + \frac{1-\frac{1}{2}x}{5\frac{1}{2}} = \frac{2\frac{1}{2}+\frac{1}{2}(x-1)}{2\frac{1}{2}}$$

or $\frac{9x-2-2x}{12} + \frac{2(5-x)}{55} = \frac{60+x-1}{55}$

or $55(7x-2)+24(5-x) = 12(59+x)$

or $385x-110+120-24x = 708+12x$

or $385x-24x-12x = 708+110-120$

or $349x = 698, \quad x=2$

$$72 \quad \frac{1}{3}(x-a) - \frac{1}{2}(2x-3b) - \frac{1}{2}(a-x) = 30(10a+11b)$$

or $10(x-a) - 6(2x-3b) + 15(x-a) = 30(10a+11b)$

or $10x-12x+15x = 300a+330b+10a-18b+15a$

or $13x = 325a+312b = 13(25a+24b)$

$x = 25a+24b.$

$$73 \quad \frac{2x+a}{b} - \frac{x-b}{a} = \frac{3ax+(a-b)^2}{ab}$$

or $a(2x+a)-b(x-b) = 3ax+(a-b)^2$

or $x(2a-b-3a) = (a-b)^2 - a^2 - b^2$

or $-x(a+b) = -2ab, \quad x = \frac{2ab}{a+b}$

$$74 \quad \frac{2x+1}{29} - \frac{402-3x}{12} = 9 - \frac{471-6x}{2}$$

or $12(2x+1) - 29(402-3x) = 29 \times 108 - 174(471-6x)$

or $24x+12-1044x = 3132-81954-12+11658x$

or $-933x = -67176, \quad x=72$

$$75 \quad \frac{x+5}{4x-9} = \frac{x+10\frac{1}{2}}{4x-7}$$

or $(4x-7)(x+5) = \frac{1}{2}(4x-9)(3x+31)$

$$\text{or } 3(4x^2 + 13x - 35) = 12x^2 + 97x - 297$$

$$\text{or } 39x - 97x = -279 + 105$$

$$\text{or } -58x = -174, \quad x=3$$

76

$$\frac{7x+2}{17x+14} = \frac{7x+6}{17x+26}$$

$$\text{or } \frac{7x+2}{17x+14} = \frac{7x+2}{17x+26} + \frac{4}{17x+26}$$

$$\text{or } (7x+2) \left(\frac{1}{17x+14} - \frac{1}{17x+26} \right) = \frac{4}{17x+26}$$

$$\text{or } (7x+2) \left\{ \frac{17x+26-17x-14}{(17x+14)(17x+26)} \right\} = \frac{4}{17x+26}$$

$$\text{or } \frac{12(7x+2)}{17x+14} = 4$$

$$\text{or } 3(7x+2) = 17x+14$$

$$\text{or } 21x - 17x = 14 - 6$$

$$\text{or } 4x = 8, \quad x=2$$

77

$$\frac{2x-7}{3x-8} = \frac{3(2x-1)}{9x-2}$$

$$\text{or } (2x-7)(9x-2) = (3x-8)(6x-3)$$

$$\text{or } 18x^2 - 67x + 14 = 18x^2 - 57x + 24$$

$$\text{or } -67 + 57x = 24 - 14$$

$$\text{or } -10x = 10; \quad x = -1$$

78

$$\frac{13-6x}{3+4x} = \frac{7-9x}{7+6x}$$

$$\text{or } (13-6x)(7+6x) = (7-9x)(3+4x)$$

$$\text{or } 91 + 36x - 36x^2 = 21 + x - 36x^2$$

$$\text{or } 36x - x = 21 - 91$$

$$\text{or } 35x = -70, \quad x = -2$$

79

$$\frac{10(7+4x)}{7+15x} = \frac{27+8x}{4+3x}$$

$$\text{or } 10(7+4x)(4+3x) = (7+15x)(27+8x)$$

$$\text{or } 280 + 370x + 120x^2 = 189 + 461x + 120x^2$$

$$\text{or } 370x - 461x = 189 - 280$$

$$\text{or } -91x = -91, \quad x=1$$

80

$$\frac{15 - \frac{2}{5}x}{5} - \frac{2x + 5}{2\frac{1}{2}} = \frac{17 - \frac{1}{4}x}{3}$$

or $3 - \frac{2}{15}x - \frac{4}{5}x - 2 = \frac{17}{3} - \frac{1}{6}x$

or $-\frac{2}{15}x - \frac{4}{5}x + \frac{1}{6}x = \frac{17}{3} - 3 + 2$

or $\frac{-6x - 36x + 70x}{45} = \frac{14}{3}$

or $\frac{28x}{45} = \frac{14}{3}$ or $\frac{2x}{15} = 1$, $x = \frac{15}{2} = 7\frac{1}{2}$

Exercise 63

1

$$5x - 21 = 3x - 15$$

or $5x - x(2 + 3) = -15$

or $5x - 5x = -15$

or $-051 = -15$, $x = \frac{15}{05} = \frac{15}{5} = \frac{15+9}{5} = 27$

2

$$375x + 5 = 225x + 8$$

or $x(375 - 225) = 8 - 5$

or $15x = 75$, $x = \frac{75}{15} = 5$

3

$$12x - \frac{18x - 05}{5} = 4x + 89$$

or $6x - 18x - 05 = 2x + 445$

or $6x - 18x - 2x = 445 - 05$

or $22x = 440$, $x = \frac{440}{22} = 20$

4

$$\frac{x + 75}{125} - \frac{x - 25}{25} = 15$$

or $2x + 15 - x + 25 = 375$ (multiplying both sides by 25)

$x = 375 - 15 - 25 = 2$

5

$$\frac{x}{5} - \frac{1}{05} + \frac{x}{005} - \frac{1}{0005} = 0$$

or $x - 10 + 100x - 1000 = 0$

or $101x = 1010$, $x = 10$

6

$$5x + \frac{45x-75}{6} = \frac{12}{2} - \frac{3x-6}{9}$$

$$\text{or } 5x + \frac{15x-25}{2} = 6 - \frac{x-2}{3}$$

$$\text{or } 3x + 15x - 75 = 36 - 2x + 4$$

$$\text{or } 3x + 15x + 2x = 36 + 4 + 75$$

$$\text{or } 95x = 115; \quad \therefore x = 5$$

7.

$$\frac{x+2}{x-2} + 68 = 538$$

$$\text{or } \frac{x+2}{x-2} = 538 - 68$$

$$\text{or } \frac{x+2}{x-2} = -15$$

$$\text{or } x+2 = -15x+3$$

$$\text{or } 25x = 1; \quad \therefore x = \frac{1}{25} = .04$$

8

$$7x - 1 = 67x - 5$$

$$\text{or } 7x - 67x = 5 - 1$$

$$\text{or } 60x = -1 \quad \therefore x = -\frac{1}{60} = -.016\bar{6}$$

9

$$\frac{2x-3}{3x-4} = \frac{1x-6}{6x-7}$$

$$\text{or } \frac{2x-3}{3x-4} = -\frac{4x-6}{6x-7} = \frac{4x-6}{6x-7}$$

$$\text{or } (2x-3)(6x-7) = (4x-6)(3x-4)$$

$$\text{or } 12x^2 - 32x - 21 = 12x^2 - 34x + 24$$

$$\text{or } -32x - 34x = 24 - 21 = 3$$

$$\text{or } 2x = 3, \quad \therefore x = 1\frac{1}{2}$$

10

$$15x + \frac{135x-225}{6} = \frac{36}{2} - \frac{99x-18}{9}$$

$$\text{or } 15x + \frac{45x-75}{2} = 18 - 11x + 2$$

$$\text{or } 30x - 45x - 75 = 36 - 22x + 4$$

$$\text{or } 30x - 45x - 75 = 360 - 22x + 40$$

$$\text{or } 30x - 45x + 22x = 360 - 40 - 75$$

$$\text{or } 95x = 245 \quad \therefore x = 5$$

$$\begin{aligned}
 11 \quad & 5x + \frac{02x + 07}{03} - \frac{x+2}{9} = 95 \\
 \text{or} \quad & 45x + 6x + 21 - x - 2 = 855 \\
 \text{or} \quad & 45x + 6x - 1 = 855 - 21 + 2 \\
 \text{or} \quad & 95x = 665, \quad x = 7
 \end{aligned}$$

$$\begin{aligned}
 12 \quad & \frac{405}{9x} - \frac{3}{8-2x} = \frac{18}{x} - \frac{36}{24-6x} \\
 \text{or} \quad & \frac{135}{9x} - \frac{1}{2(4-x)} = \frac{6}{x} - \frac{12}{6(4-x)} \\
 \text{or} \quad & \frac{15}{x} - \frac{1}{2(4-x)} = \frac{6}{x} - \frac{2}{(4-x)} \\
 \text{or} \quad & -\frac{1}{2(4-x)} + \frac{2}{(4-x)} = \frac{6}{x} - \frac{15}{x} \\
 \text{or} \quad & \frac{3\left(\frac{1}{4-x}\right)}{2} = \frac{45}{x} \\
 \text{or} \quad & \frac{1}{2(4-x)} = \frac{15}{x} \\
 \text{or} \quad & x = 12 - 3x \\
 \text{or} \quad & 4x = 12, \quad x = 3
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & 011x + \frac{001x - 125}{6} = \frac{5-1}{03} - 145 \\
 \text{or} \quad & 11x + \frac{x - 125}{6} = \frac{5000 - 10001}{03} - 145 \\
 \text{or} \quad & 33x + 5x - 625 = 500000 - 100000x - 435 \\
 \text{or} \quad & 38x + 100000x = 500000 - 435 + 625 \\
 \text{or} \quad & 100038x = 500190, \quad x = 5
 \end{aligned}$$

Exercise (64)

$$\begin{aligned}
 1 \quad & \frac{5x+6}{4} + \frac{64x-35}{15} = \frac{20x+23}{16} + \frac{13x-7}{3} \\
 \text{or} \quad & \frac{5x+6}{4} - \frac{20x+23}{16} = \frac{13x-7}{3} - \frac{64x-35}{15} \\
 \text{or} \quad & \frac{20x+24-20x-23}{16} = \frac{65x-35-64x+35}{15} \\
 \text{or} \quad & \frac{1}{16} = \frac{x}{15}, \quad x = \frac{15}{16}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & \frac{17x-13}{9} + \frac{108x-75}{32} = \frac{271+19}{8} + \frac{(50\frac{1}{18})x-39}{27} \\
 \text{or} \quad & \frac{17x-13}{9} - \frac{(50\frac{1}{18})x-39}{27} = \frac{271+19}{8} - \frac{108x+75}{32} \\
 \text{or} \quad & \frac{51x-39-50\frac{1}{18}x+39}{27} = \frac{1081+76-108x-75}{32} \\
 \text{or} \quad & \frac{\frac{9}{18}x}{27} = -\frac{1}{2} \quad \text{or} \quad \frac{9}{18}x \times 32 = 27, \quad x = \frac{2}{18} = \frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & \frac{29x-18}{8} + \frac{189x-93}{49} = \frac{(86\frac{1}{2})x-54}{24} + \frac{271-13}{7} \\
 \text{or} \quad & \frac{29x-18}{8} - \frac{86\frac{1}{2}x-54}{24} = \frac{271-13}{7} - \frac{189x-93}{49} \\
 \text{or} \quad & \frac{87x-54-86\frac{1}{2}x+54}{24} = \frac{189x-91-189x+93}{49} \\
 \text{or} \quad & \frac{-\frac{1}{2}}{24} = \frac{2}{49} \quad \text{or} \quad \frac{8x}{21 \times 24} = \frac{2}{49}, \\
 & x = \frac{63 \times 2}{49} = \frac{18}{7} = 2\frac{4}{7}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & \frac{16x-17}{9} - \frac{23x-15}{16} = \frac{142\frac{1}{18}x-153}{81} - \frac{92x-65}{64} \\
 \text{or} \quad & \frac{16x-17}{9} - \frac{142\frac{1}{18}x-153}{81} = -\frac{92x-65}{64} + \frac{23x-15}{16} \\
 \text{or} \quad & \frac{144x-153-142\frac{1}{18}x+153}{81} = \frac{-92x+65+92x-60}{64} \\
 \text{or} \quad & \frac{\frac{9}{18}x}{81} = \frac{5}{64}, \quad x = \frac{81 \times 5 \times 16}{64 \times 25} = \frac{81}{20} = 4\frac{1}{20}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \frac{18x-19}{7} + \frac{135x+62\frac{1}{2}}{65} = \frac{271+14}{13} + \frac{106\frac{5}{18}x-114}{42} \\
 \text{or} \quad & \frac{18x-19}{7} - \frac{106\frac{5}{18}x-114}{42} = \frac{271+14}{13} - \frac{135x+62\frac{1}{2}}{65} \\
 \text{or} \quad & \frac{108x-114-106\frac{5}{18}x+114}{42} = \frac{135x+70-135x-62\frac{1}{2}}{65} \\
 \text{or} \quad & \frac{\frac{9}{18}x}{42} = \frac{7\frac{1}{2}}{65} \quad \text{or} \quad \frac{31x}{13 \times 42} = \frac{15}{2 \times 65} \\
 \text{or} \quad & \frac{x}{2 \times 13} = \frac{3}{2 \times 13}, \quad x = 3
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \frac{33-19x}{15} - \frac{41+27x}{28} + \frac{164+(107\frac{1}{11})x}{112} - \frac{164\frac{1}{11}-95x}{75} = 0 \\
 \text{or} \quad & \frac{33-19x}{15} - \frac{164\frac{1}{11}-95x}{75} = \frac{41+27x}{28} - \frac{164+(107\frac{1}{11})x}{112} \\
 \text{or} \quad & \frac{165-95x-164\frac{1}{11}+95x}{75} = \frac{164+108x-164-107\frac{1}{11}x}{112} \\
 \text{or} \quad & \frac{1}{75} = \frac{1}{112} \quad \text{or} \quad \frac{1}{5 \times 28} = \frac{1}{15 \times 28}, \quad x=3
 \end{aligned}$$

$$\begin{aligned}
 7 \quad & \frac{18-41x}{9} - \frac{17-16x}{8} + \frac{91\frac{1}{5}-10x}{5} - \frac{14-32x}{7} = 0 \\
 \text{or} \quad & \frac{18-41x}{9} - \frac{14-32x}{7} = \frac{17-16x}{8} - \frac{91\frac{1}{5}-10x}{5} \\
 \text{or} \quad & \frac{126-287x-126+288x}{63} = \frac{85-80x-78\frac{1}{5}x+80x}{40} \\
 \text{or} \quad & \frac{1}{63} = \frac{61\frac{1}{5}}{40}, \quad x = \frac{145 \times 63}{21 \times 40} = \frac{87}{8} = 10\frac{7}{8}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad & \frac{98x-73}{21} = \frac{14x-9}{3} - \frac{13x-16}{5x-9} \\
 \text{or} \quad & \frac{98x-73}{7} = 14x-9 - \frac{13x-16}{5x-3} \\
 \text{or} \quad & \frac{98x-73-98x+63}{7} = -\frac{13x-16}{5x-3} \\
 \text{or} \quad & \frac{10}{7} = \frac{13x-16}{5x-3} \\
 \text{or} \quad & 50x-30=91x-112 \\
 \text{or} \quad & 50x-91x=-112+30 \\
 \text{or} \quad & 41x=82 \quad x=2
 \end{aligned}$$

$$\begin{aligned}
 9 \quad & \frac{95x-159}{35} = \frac{19x-29}{7} - \frac{17x-47}{23x-59} \\
 \text{or} \quad & \frac{95x-159}{35} - \frac{19x-29}{7} = -\frac{17x-47}{23x-59} \\
 \text{or} \quad & \frac{95x-159-95x+145}{35} = -\frac{17x-47}{23x-59} \\
 \text{or} \quad & -\frac{14}{35} = -\frac{17x-47}{23x-59}
 \end{aligned}$$

$$\begin{array}{l} \text{or} \quad 461 - 118 = 85x - 235 \\ \text{or} \quad 39x = 117, \quad x = 3 \end{array}$$

10

$$\begin{array}{l} \frac{91x-21}{56} + \frac{24x-93}{35x-138} = \frac{131+9}{8} \\ \text{or} \quad \frac{911-21}{56} - \frac{131+9}{8} = -\frac{24x-93}{35x-138} \\ \text{or} \quad \frac{91x-21-91x-63}{56} = -\frac{24x-93}{35x-138} \\ \text{or} \quad \frac{-84}{56} = -\frac{24x-93}{35x-138} \\ \text{or} \quad \frac{3}{2} = \frac{24x-93}{35x-138} \\ \text{or} \quad 105x-414 = 48x-186 \\ \text{or} \quad 1051-48x = -186+414 \\ \text{or} \quad 57x = 228; \quad x = 4 \end{array}$$

11

$$\begin{array}{l} \frac{117x-26}{135} + \frac{161-77}{23x-110} = \frac{13x+4}{15} + \frac{3\frac{1}{2}}{27} \\ \text{or} \quad \frac{16x-77}{23x-110} = \frac{13x+4}{15} + \frac{1\frac{1}{2}}{27} - \frac{117x-26}{135} \\ \text{or} \quad \frac{16x-77}{23x-110} = \frac{117x+36+19-117x+26}{135} \\ \text{or} \quad \frac{16x-77}{23x-110} = \frac{81}{135} = \frac{3}{5} \\ \text{or} \quad 80x-385 = 69x-330 \\ \text{or} \quad 80x-69x = -330+385 = 55 \\ \text{or} \quad 11x = 55, \quad x = 5 \end{array}$$

12

$$\begin{array}{l} \frac{61-7\frac{1}{2}}{13-2x} + 2x + \frac{1+16x}{24} = 4\frac{5}{12} - \frac{12\frac{5}{8}-8x}{3} \\ \text{or} \quad \frac{6x-7\frac{1}{2}}{13-2x} = \frac{53}{12} - \frac{101-8x}{3} - 2x - \frac{1+16x}{24} \\ \text{or} \quad \frac{61-7\frac{1}{2}}{13-2x} = \frac{106-101+64x-48x-1-16x}{24} = \frac{4}{24} = \frac{1}{6} \\ \text{or} \quad 36x-44 = 13-21 \\ \text{or} \quad 38x = 57, \quad x = \frac{57}{38} = 1\frac{1}{2} \end{array}$$

13

$$\frac{2x+8\frac{1}{2}}{9} - \frac{13x-2}{17x-32} + \frac{1}{3} = \frac{7x}{12} - \frac{x+16}{36}$$

$$\begin{aligned} \text{or } \frac{15x-2}{17x-32} &= \frac{2x+1\frac{1}{2}}{9} + \frac{1}{3} - \frac{7x}{12} + \frac{x+16}{36} \\ &= \frac{8x+34+12x-21x+1+16}{36} = \frac{50}{36} = \frac{25}{18} \end{aligned}$$

$$\text{or } 234x - 36 = 425x - 800$$

$$\text{or } 234x - 425x = -800 + 36$$

$$\text{or } -191x = -764, \quad x=4$$

14

$$\frac{41-35x}{105} - \frac{7-2x^2}{14(x-1)} = \frac{1+3x}{21} - \frac{2x-2\frac{1}{2}}{6}$$

$$\begin{aligned} \text{or } \frac{7-2x^2}{14(x-1)} &= \frac{41-35x}{105} - \frac{1+3x}{21} + \frac{2x-1\frac{1}{2}}{6} \\ &= \frac{82-70x-10-30x+70x-77}{210} \end{aligned}$$

$$= \frac{-5-30x}{210} = -\frac{1+6x}{14 \times 3}$$

$$\text{or } \frac{7-2x^2}{x-1} = -\frac{1+6x}{3}$$

$$\text{or } 21-6x^2 = -1+1-6x^2+6x$$

$$\text{or } 5x=20, \quad x=4$$

15

$$\frac{1}{x-1} - \frac{2}{x+7} = \frac{1}{7(x-1)}$$

$$\text{or } \frac{1}{x-1} - \frac{1}{7(x-1)} = \frac{2}{x+7}$$

$$\text{or } \frac{6}{7(x-1)} = \frac{2}{x+7} \text{ or } \frac{3}{7(x-1)} = \frac{1}{x+7}$$

$$\text{or } 3x+21=7x-7$$

$$\text{or } 3x-7x=-7-21=-28$$

$$\text{or } -4x=-28, \quad x=7$$

16

$$\frac{2}{5(3x+4)} + \frac{4}{2x+3} = \frac{6}{3x+4}$$

$$\text{or } \frac{2}{5(3x+4)} - \frac{6}{3x+4} = -\frac{4}{2x+3}$$

$$\text{or } \frac{2-30}{5(3x+4)} = -\frac{4}{2x+3}$$

$$\text{or } \frac{28}{5(3x+4)} = \frac{4}{2x+3} \quad \text{or } \frac{7}{5(3x+4)} = \frac{1}{2x+3}$$

$$\text{or } 14x+21=15x+20, \quad x=1$$

17

$$\frac{3}{3x-5} - \frac{6}{7(4x-7)} = \frac{7}{9(3x-5)} + \frac{2}{4x-7}$$

$$\text{or } \frac{3}{3x-5} - \frac{7}{9(3x-5)} = \frac{2}{4x-7} + \frac{6}{7(4x-7)}$$

$$\text{or } \frac{27-7}{9(3x-5)} = \frac{14+6}{7(4x-7)}$$

$$\text{or } \frac{20}{9(3x-5)} = \frac{20}{7(4x-7)}$$

$$\text{or } 27x-45=28x-49, \quad x=4$$

18

$$\frac{11}{12(14x-19)} + \frac{7}{9(13x-14)} = \frac{3}{14x-19} - \frac{2}{13x-14}$$

$$\text{or } \frac{11}{12(14x-19)} - \frac{3}{14x-19} = -\frac{2}{13x-14} - \frac{7}{9(13x-14)}$$

$$\text{or } \frac{11-36}{12(14x-19)} = \frac{-18-7}{9(13x-14)}$$

$$\text{or } \frac{25}{4(14x-19)} = \frac{25}{3(13x-14)}$$

$$\text{or } 56x-76=39x-42$$

$$\text{or } 56x-39x=-42+76$$

$$\text{or } 17x=34, \quad x=2$$

19

$$\frac{50}{3x-1} + \frac{37-\frac{1}{2}x}{12x-1} = \frac{35}{12x-1} + \frac{49-\frac{1}{2}x}{3x-1}$$

$$\text{or } \frac{50-49+\frac{1}{2}x}{3x-1} = \frac{35-37+\frac{1}{2}x}{12x-1}$$

$$\text{or } \frac{\frac{1}{2}x+1}{3x-1} = \frac{\frac{1}{2}x-2}{12x-1}$$

$$\text{or } x^2+11\frac{1}{2}x-1=x^2-6\frac{1}{2}x+2$$

$$\text{or } 18\frac{1}{2}x=3, \quad x=\frac{3 \times 4}{73} = \frac{12}{73}$$

20

$$\frac{(1\frac{1}{2})x + 19\frac{1}{2}}{2x+5} - \frac{7x+8}{x+8} = \frac{20\frac{1}{2} - (1\frac{1}{2})x}{2x+5} + \frac{(1\frac{1}{2})x - 9}{2(x+8)}$$

$$\text{or } \frac{(1\frac{1}{2})x + 19\frac{1}{2} - 20\frac{1}{2} + (1\frac{1}{2})x}{(2x+5)} = \frac{(1\frac{1}{2})x - 9 + (1\frac{1}{2})x + 16}{2(x+8)}$$

$$\text{or } \frac{3x-1}{2x+5} = \frac{3x+7}{2(x+8)}$$

$$\text{or } 6x^2 + 46x - 16 = 6x^2 + 29x + 35$$

$$\text{or } 17x = 51, \quad x = 3$$

21

$$\frac{(9\frac{1}{2})x - 32}{4x+7} + \frac{65x + 4\frac{1}{2}}{8x+29} = \frac{75x + 5\frac{1}{2}}{8x+29} + \frac{(4\frac{1}{2})x - 29}{4x+7}$$

$$\text{or } \frac{(9\frac{1}{2})x - 32 - (4\frac{1}{2})x + 29}{4x+7} = \frac{75x + 5\frac{1}{2} - 65x - 4\frac{1}{2}}{8x+29}$$

$$\text{or } \frac{5x-3}{4x+7} = \frac{10x+1}{8x+29}$$

$$\text{or } 40x^2 + 121x - 87 = 40x^2 + 74x + 7$$

$$\text{or } 121x - 74x = 7 + 87$$

$$\text{or } 47x = 94, \quad x = 2$$

22

$$\frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$$

$$\text{or } \frac{1}{x-1} - \frac{1}{x-3} = \frac{2}{x-3} - \frac{2}{x-2}$$

$$\text{or } \frac{x-3-x+1}{(x-1)(x-3)} = 2 \left\{ \frac{x-2-x+3}{(x-3)(x-2)} \right\}$$

$$\text{or } \frac{-2}{x-1} = \frac{2}{x-2}$$

$$\text{or } -x+2 = x-1 \quad \text{or } 2x = 3,$$

$$x = \frac{3}{2}$$

23

$$\frac{3}{4x+1} + \frac{4}{4x+5} = \frac{7}{4x+3}$$

$$\text{or } \frac{7}{4x+1} - \frac{3}{4x+3} = \frac{4}{4x+3} - \frac{4}{4x+5}$$

$$\text{or } 3 \left\{ \frac{4x+3-4x-1}{(4x+1)(4x+3)} \right\} = 4 \left\{ \frac{4x+5-4x-3}{(4x+3)(4x+5)} \right\}$$

$$\begin{aligned} \text{or} \quad & \frac{6}{4x+1} = \frac{8}{4x+5} \quad \text{or} \quad \frac{3}{4x+1} = \frac{4}{4x+5} \\ \text{or} \quad & 12r+15=16r+4 \\ \text{or} \quad & 4x=11, \quad r=2\frac{3}{4} \end{aligned}$$

24

$$\begin{aligned} & \frac{15}{3x+11} - \frac{8}{3x+17} = \frac{7}{3x+5} \\ \text{or} \quad & \frac{8}{3x+11} - \frac{8}{3x+17} = \frac{7}{3x+5} - \frac{7}{3x+11} \\ \text{or} \quad & 8 \left\{ \frac{3x+17-3x-11}{(3x+11)(3x+17)} \right\} = 7 \left\{ \frac{3x+11-3x-5}{(3x+5)(3x+11)} \right\} \\ \text{or} \quad & \frac{8 \times 6}{3x+17} = \frac{7 \times 6}{3x+5} \\ \text{or} \quad & 24x+40=21x+119 \\ \text{or} \quad & 3x=79, \quad x=26\frac{1}{3} \end{aligned}$$

25

$$\begin{aligned} & \frac{6}{5x+7} - \frac{4}{5x+13} = \frac{9}{5x+13} - \frac{7}{5x+19} \\ \text{or} \quad & \frac{6}{5x+7} - \frac{6}{5x+13} = \frac{7}{5x+13} - \frac{7}{5x+19} \\ \text{or} \quad & 6 \left\{ \frac{5x+13-5x-7}{(5x+7)(5x+13)} \right\} = 7 \left\{ \frac{5x+19-5x-13}{(5x+13)(5x+19)} \right\} \\ \text{or} \quad & \frac{6 \times 6}{5x+7} = \frac{7 \times 6}{5x+19} \\ \text{or} \quad & 30x+114=35x+49 \\ \text{or} \quad & 5x=65, \quad x=13 \end{aligned}$$

26

$$\begin{aligned} & \frac{8}{2x+17} - \frac{12}{2x+25} = \frac{5}{2x+25} - \frac{9}{2x+33} \\ \text{or} \quad & \frac{8}{2x+17} - \frac{8}{2x+25} = \frac{9}{2x+25} - \frac{9}{2x+33} \\ \text{or} \quad & 8 \left\{ \frac{2x+25-2x-17}{(2x+17)(2x+25)} \right\} = 9 \left\{ \frac{2x+33-2x-25}{(2x+25)(2x+33)} \right\} \\ \text{or} \quad & \frac{8 \times 8}{2x+17} = \frac{9 \times 8}{2x+33} \\ \text{or} \quad & 16x+264=18x+153 \\ \text{or} \quad & 2x=111, \quad x=55\frac{1}{2} \end{aligned}$$

27.

$$\frac{5}{3-4x} + \frac{9}{4x+13} - \frac{4}{4x+5} = 0$$

$$\text{or } \frac{5}{3-4x} + \frac{5}{4x+13} = \frac{4}{4x+5} - \frac{4}{4x+13}$$

$$\text{or } 5 \left\{ \frac{4x+13+3-4x}{(3-4x)(4x+13)} \right\} = 4 \left\{ \frac{4x+13-4x-5}{(4x+5)(4x+13)} \right\}$$

$$\text{or } \frac{5 \times 16}{3-4x} = \frac{4 \times 8}{4x+5}$$

$$\text{or } 20x + 25 = 6 - 8x$$

$$\text{or } 28x = -19, \quad x = -\frac{19}{28}$$

28

$$\frac{6}{5-6x} + \frac{13}{6x+19} = \frac{7}{6x+7}$$

$$\text{or } \frac{6}{5-6x} + \frac{6}{6x+19} = \frac{7}{6x+7} - \frac{7}{6x+19}$$

$$\text{or } 6 \left\{ \frac{6x+19+5-6x}{(5-6x)(6x+19)} \right\} = 7 \left\{ \frac{6x+19-6x-7}{(6x+7)(6x+19)} \right\}$$

$$\text{or } \frac{6 \times 24}{5-6x} = \frac{7 \times 12}{6x+7}$$

$$\text{or } 72x + 84 = 35 - 42x$$

$$\text{or } 114x = -49, \quad x = -\frac{49}{114}$$

29

$$\frac{7}{4-5x} - \frac{8}{17-5x} = \frac{1}{5x+9}$$

$$\text{or } \frac{7}{4-5x} - \frac{7}{17-5x} = \frac{1}{5x+9} + \frac{1}{17-5x}$$

$$\text{or } 7 \left\{ \frac{17-5x-4+5x}{(4-5x)(17-5x)} \right\} = \frac{17-5x+5x+9}{(5x+9)(17-5x)}$$

$$\text{or } \frac{7 \times 13}{4-5x} = \frac{26}{5x+9}$$

$$\text{or } 35x + 63 = 8 - 10x$$

$$\text{or } 45x = -55; \quad x = -\frac{11}{9}$$

30

$$\frac{9}{3-7x} + \frac{1}{7x+15} = \frac{8}{12-7x}$$

$$\text{or } \frac{1}{3-7x} + \frac{1}{7x+15} = \frac{8}{12-7x} - \frac{8}{3-7x}$$

$$\text{or } \frac{7x+15+3-7x}{(3-7x)(7x+15)} = 8 \left\{ \frac{3-7x-12+7x}{(12-7x)(3-7x)} \right\}$$

$$\text{or } \frac{18}{7x+15} = \frac{8 \times (-9)}{12-7x}$$

$$\text{or } \frac{1}{7x+15} = \frac{-4}{12-7x}$$

$$\text{or } 12-7x = -28x-60$$

$$\text{or } 21x = -72, \quad x = -\frac{24}{7}.$$

31

$$\frac{2}{2x-5} + \frac{1}{x-3} = \frac{6}{3x-1}$$

$$\text{or } \frac{2}{2x-5} - \frac{3}{3x-1} = \frac{3}{3x-1} - \frac{1}{x-3}$$

$$\text{or } \frac{6x-2-6x+15}{(2x-5)(3x-1)} = \frac{3x-9-3x+1}{(3x-1)(x-3)}$$

$$\text{or } \frac{13}{2x-5} = \frac{-8}{x-3}$$

$$\text{or } 13x-39 = -16x+40$$

$$\text{or } 29x = 79, \quad x = \frac{79}{29} = 2\frac{21}{29}$$

32

$$\frac{10}{2x-5} + \frac{1}{x+5} = \frac{18}{3x-5}$$

$$\text{or } \frac{10}{2x-5} - \frac{15}{3x-5} = \frac{3}{3x-5} - \frac{1}{x+5}$$

$$\text{or } \frac{30x-50-30x+75}{(2x-5)(3x-5)} = \frac{3x+15-3x+5}{(3x-5)(x+5)}$$

$$\text{or } \frac{25}{2x-5} = \frac{20}{x+5}$$

$$\text{or } 5x+25 = 8x-20$$

$$\text{or } 3x = 45, \quad x = 15$$

33

$$\frac{9}{3x-4} + \frac{20}{4x+1} = \frac{8}{x+7}$$

$$\text{or } \frac{9}{3x-4} - \frac{3}{x+7} = \frac{5}{x+7} - \frac{20}{4x+1}$$

$$\text{or } \frac{9x+63-9x+12}{(3x-4)(x+7)} = \frac{20x+5-20x-140}{(x+7)(4x+1)}$$

$$\begin{aligned}
 \text{or} \quad & \frac{75}{3x-4} = \frac{-135}{4x+1} \\
 \text{or} \quad & 5(4x+1) = -9(3x-4) \\
 \text{or} \quad & 20x+5 = -27x+36 \\
 \text{or} \quad & 47x = 31, \quad x = \frac{31}{47}
 \end{aligned}$$

34

$$\begin{aligned}
 & \frac{10}{5x-9} + \frac{14}{2x+9} = \frac{9}{x+8} \\
 \text{or} \quad & \frac{10}{5x-9} - \frac{2}{x+8} = \frac{7}{x+8} - \frac{14}{2x+9} \\
 \text{or} \quad & \frac{10x+80-10x+18}{(5x-9)(x+8)} = \frac{14x+63-14x-112}{(x+8)(2x+9)} \\
 \text{or} \quad & \frac{98}{5x-9} = \frac{-49}{2x+9} \\
 \text{or} \quad & 4x+18 = -5x+9 \\
 \text{or} \quad & 9x = -9, \quad x = -1
 \end{aligned}$$

35

$$\begin{aligned}
 & \frac{12}{3x-8} = \frac{20}{4x-13} - \frac{1}{x+9} \\
 \text{or} \quad & \frac{12}{3x-8} - \frac{16}{4x-13} = \frac{4}{4x-13} - \frac{1}{x+9} \\
 \text{or} \quad & \frac{48x-156-48x+128}{(3x-8)(4x-13)} = \frac{4x+36-4x+13}{(4x-13)(x+9)} \\
 \text{or} \quad & \frac{-28}{3x-8} = \frac{49}{x+9} \\
 \text{or} \quad & \frac{-4}{3x-8} = \frac{7}{x+9} \\
 \text{or} \quad & -4x-36 = 21x-56 \\
 \text{or} \quad & 25x = 20, \quad x = \frac{4}{5}
 \end{aligned}$$

36

$$\begin{aligned}
 & \frac{a+b}{x-c} = \frac{a}{x-a} + \frac{b}{x-b} \\
 \text{or} \quad & \frac{a}{x-c} - \frac{a}{x-a} = \frac{b}{x-b} - \frac{b}{x-c} \\
 \text{or} \quad & a \left\{ \frac{x-a-x+c}{(x-c)(x-a)} \right\} = b \left\{ \frac{x-c-x+b}{(x-b)(x-c)} \right\}
 \end{aligned}$$

$$\begin{aligned}
 \text{or} \quad & \frac{a(c-a)}{r-a} = \frac{b(b-c)}{(x-b)} \\
 \text{or} \quad & a(c-a)(x-b) = b(b-c)(x-a) \\
 \text{or} \quad & (ac-a^2)x - b(ac-a^2) = (b^2-bc)x - a(b^2-bc) \\
 \text{or} \quad & x(ac-a^2-b^2+bc) = -ab^2+abc+abc-a^2b \\
 & = ab(2c-a-b) \\
 & x = \frac{ab(2c-a-b)}{ac-a^2-b^2+bc} = \frac{ab(a+b-2c)}{a^2+b^2-ac-bc}
 \end{aligned}$$

37

$$\begin{aligned}
 & \frac{a^2}{ax-b} + \frac{b^2}{bx-a} = \frac{a+b}{x+c} \\
 \text{or} \quad & \frac{a^2}{ax-b} - \frac{a}{x+c} = \frac{b}{x+c} - \frac{b^2}{bx-a} \\
 \text{or} \quad & \frac{a^2x+a^2c-a^2x+ab}{(ax-b)(x+c)} = \frac{b^2x-ab-b^2x-b^2c}{(x+c)(bx-a)} \\
 \text{or} \quad & \frac{a(ac+b)}{ax-b} = \frac{-b(a+bc)}{bx-a} \\
 \text{or} \quad & abx(ac+b) - a^2(ac+b) = -abx(a+bc) + b^2(a+bc) \\
 \text{or} \quad & abx(ac+b+a+bc) = ab^2+b^2c+a^2c+a^2b \\
 \text{or} \quad & abx\{c(a+b)+(a+b)\} = ab(a+b)+c(a+b)(a^2-ab+b^2) \\
 \text{or} \quad & abx(c+1) = ab + c(a^2-ab+b^2), \\
 & x = \frac{c(a^2-ab+b^2)+ab}{ab(c+1)}
 \end{aligned}$$

38

$$\begin{aligned}
 & \frac{m(x+a)}{x+b} + \frac{n(x+b)}{x+a} = m+n \\
 \text{or} \quad & \frac{m(x+a)}{x+b} - m = n - \frac{n(x+b)}{x+a} \\
 \text{or} \quad & m\left(\frac{x+a-x-b}{x+b}\right) = n\left(\frac{x+a-x-b}{x+a}\right) \\
 \text{or} \quad & \frac{m(a-b)}{x+b} = \frac{n(a-b)}{x+a} \\
 \text{or} \quad & m(x+a) = n(x+b) \\
 \text{or} \quad & x(m-n) = nb-am, \quad x = \frac{nb-am}{m-n}
 \end{aligned}$$

39

$$\frac{b-c}{r+a} + \frac{a-b}{x+b} = \frac{a-c}{r+c}$$

$$\text{or } \frac{b-c}{r+a} + \frac{a-b}{r+b} = \frac{a-b+b-c}{r+c}$$

$$\text{or } b-c \left\{ \frac{1}{x+a} - \frac{1}{r+c} \right\} = a-b \left\{ \frac{1}{r+c} - \frac{1}{x+b} \right\}$$

$$\text{or } b-c \left\{ \frac{r+c-x-a}{(r+a)(x+c)} \right\} = a-b \left\{ \frac{x+b-r-c}{(r+c)(r+b)} \right\}$$

$$\text{or } \frac{(b-c)(c-a)}{x+a} = \frac{(a-b)(b-c)}{r+b}$$

$$\text{or } (c-a)(r+b) = (a-b)(x+a)$$

$$\text{or } r\{(c-a)-(a-b)\} = a(a-b) - b(c-a)$$

$$= a^2 - ab - bc + ab$$

$$\text{or } r(b+c-2a) = a^2 - bc, \quad r = \frac{a^2 - bc}{b+c-2a}$$

40

$$\frac{2a-3b}{r-a+b} - \frac{2b-3a}{x+a-b} = \frac{5(a-b)}{r+a+b}$$

$$\text{or } \frac{2a-3b}{r-a+b} - \frac{2b-3a}{r+a-b} = \frac{2a-3b+3a-2b}{r+a+b}$$

$$\text{or } (2a-3b) \left\{ \frac{1}{(r-a+b)} - \frac{1}{(r+a+b)} \right\}$$

$$= (3a-2b) \left\{ \frac{1}{(r+a+b)} - \frac{1}{(x+a-b)} \right\}$$

$$\text{or } (2a-3b) \left\{ \frac{x+a+b-r+a-b}{(r-a+b)(x+a+b)} \right\}$$

$$= (3a-2b) \left\{ \frac{r+a-b-r-a-b}{(x+a+b)(r+a-b)} \right\}$$

$$\text{or } \frac{2a(2a-3b)}{r-a+b} = \frac{-2b(3a-2b)}{r+a-b}$$

$$\text{or } a(2b-3b)(r+a-b) = b(2b-3a)(r-a+b)$$

$$\text{or } r(2a^2 - 3ab - 2b^2 + 3ab)$$

$$= -b(2b-3a)(a-b) - a(a-b)(2a-3b)$$

$$= (a-b)(-2b^2 + 3ab - 2a^2 + 3ab)$$

$$\text{or } 2x(a+b)(a-b) = 2(a-b)(3ab - a^2 - b^2),$$

$$x = \frac{3ab - a^2 - b^2}{a+b}$$

41

$$\frac{1}{1-6a} + \frac{2}{1+3a} + \frac{3}{1-2a} = \frac{6}{1-a}$$

$$\text{or } \frac{1}{1-6a} - \frac{1}{1-a} + \frac{2}{1+3a} - \frac{2}{1-a} + \frac{3}{1-2a} - \frac{3}{1-a} = 0$$

$$\text{or } \frac{1-a-1+6a}{(1-6a)(1-a)} + \frac{21-2a-21-6a}{(1-a)(1+3a)} + \frac{31-3a-31-6a}{(1-2a)(1-a)} = 0$$

$$\text{or } \frac{5a}{1-6a} + \frac{3a}{1-2a} = \frac{8a}{1+3a} = \frac{5a+3a}{1+3a}$$

$$\text{or } 5a \left\{ \frac{1}{1-6a} - \frac{1}{1+3a} \right\} = 3a \left\{ \frac{1}{1+3a} - \frac{1}{1-2a} \right\}$$

$$\text{or } \frac{5a \cdot 9a}{1-6a} = \frac{3a \times (-5a)}{1-2a}$$

$$\text{or } \frac{3}{1-6a} = -\frac{1}{1-2a}$$

$$\text{or } 31-6a = -1+6a$$

$$\text{or } 41 = 12a, \quad 1 = 3a$$

42

$$\frac{1}{x+1} + \frac{4}{21-1} + \frac{9}{31-1} = \frac{36}{61-1}$$

$$\text{or } \frac{1}{1+1} - \frac{6}{61-1} + \frac{4}{21-1} - \frac{12}{61-1} + \frac{9}{31-1} - \frac{18}{61-1} = 0$$

$$\text{or } \frac{61-1-61-6}{(1+1)(61-1)} + \frac{241-4-241+12}{(21-1)(61-1)} + \frac{541-9-541+18}{(31-1)(61-1)} = 0$$

$$\text{or } \frac{-7}{1+1} + \frac{8}{21-1} + \frac{9}{31-1} = 0$$

$$\text{or } \frac{8}{21-1} - \frac{4}{1+1} + \frac{9}{31-1} - \frac{3}{1+1} = 0$$

$$\text{or } \frac{81+8-81+4}{(21-1)(1+1)} + \frac{91+9-91+3}{(31-1)(1+1)} = 0$$

$$\text{or } \frac{12}{21-1} + \frac{12}{31-1} = 0$$

$$\text{or } 31-1+21-1=0$$

$$\text{or } 5x=2, \quad x=\frac{2}{5}$$

Exercise (65)

$$\begin{aligned}
 1 \quad & \frac{2x-1}{x-1} + \frac{3x-4}{x-2} = \frac{5x-12}{x-3} \\
 \text{or} \quad & 2 + \frac{1}{x-1} + 3 + \frac{2}{x-2} = 5 + \frac{3}{x-3} \\
 \text{or} \quad & \frac{1}{x-1} - \frac{1}{x-3} = \frac{2}{x-3} - \frac{2}{x-2} \\
 \text{or} \quad & \frac{1-3-x+1}{(x-1)(x-3)} = \frac{2x-4-2x+6}{(x-3)(x-2)} \\
 \text{or} \quad & \frac{-2}{x-1} = \frac{2}{x-2} \\
 \text{or} \quad & -x+2 = x-1 \\
 \text{or} \quad & 2x = 3, \quad x = 1\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & \frac{2x+7}{x+2} + \frac{4x+29}{x+6} - \frac{6x-10}{x-3} = 0 \\
 \text{or} \quad & 2 + \frac{3}{x+2} + 4 + \frac{5}{x+6} - 6 - \frac{8}{x-3} = 0 \\
 \text{or} \quad & \frac{3}{x+2} - \frac{3}{x-3} + \frac{5}{x+6} - \frac{5}{x-3} = 0 \\
 \text{or} \quad & \frac{3x-9-3x-6}{(x+2)(x-3)} + \frac{5x-15-5x-30}{(x+6)(x-3)} = 0 \\
 \text{or} \quad & \frac{15}{x+2} + \frac{45}{x+6} = 0 \\
 \text{or} \quad & \frac{1}{x+2} + \frac{3}{x+6} = 0 \\
 \text{or} \quad & x+6+3x+6=0 \\
 \text{or} \quad & 4x = -12, \quad x = -3
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & \frac{25x-40}{5x-6} - \frac{7x+9}{x+2} + \frac{6x-1}{3x+4} = 0 \\
 \text{or} \quad & 5 - \frac{10}{5x-6} - 7 + \frac{5}{x+2} + 2 - \frac{9}{3x+4} = 0 \\
 \text{or} \quad & -\frac{10}{5x-6} + \frac{2}{x+2} + \frac{3}{x+2} - \frac{9}{3x+4} = 0 \\
 \text{or} \quad & \frac{-10x-20+10x-12}{(5x-6)(x+2)} + \frac{9x+12-9x-18}{(x+2)(3x+4)} = 0
 \end{aligned}$$

$$\text{or } -\frac{32}{5x-6} - \frac{6}{3x+4} = 0$$

$$\text{or } \frac{16}{5x-6} + \frac{3}{3x+4} = 0$$

$$\text{or } 48x + 64 + 15x - 18 = 0$$

$$\text{or } 63x = -46, \quad x = -\frac{46}{63}$$

4

$$\frac{12x+5}{4x+3} + \frac{15x+11}{3x+4} = \frac{8x+44}{x+6}$$

$$\text{or } 3 - \frac{4}{4x+3} + 5 - \frac{9}{3x+4} = 8 - \frac{4}{x+6}$$

$$\text{or } \frac{4}{4x+3} - \frac{1}{x+6} = \frac{3}{x+6} - \frac{9}{3x+4}$$

$$\text{or } \frac{4x+24-4x-3}{(4x+3)(x+6)} = \frac{9x+12-9x-54}{(x+6)(3x+4)}$$

$$\text{or } \frac{21}{4x+3} = \frac{-42}{3x+4}$$

$$\text{or } \frac{1}{4x+3} = \frac{-2}{3x+4}$$

$$\text{or } 3x+4 = -8x-6$$

$$\text{or } 11x = -10, \quad x = -\frac{10}{11}$$

5

$$\frac{15x-13}{3x-5} + \frac{12x+59}{3x+5} = \frac{9x+80}{x+7}$$

$$\text{or } 5 + \frac{12}{3x-5} + 4 + \frac{39}{3x+5} = 9 + \frac{17}{x+7}$$

$$\text{or } \frac{12}{3x-5} - \frac{4}{x+7} = \frac{13}{x+7} - \frac{39}{3x+5}$$

$$\text{or } \frac{12x+84-12x+20}{(3x-5)(x+7)} = \frac{39x+65-39x-273}{(x+7)(3x+5)}$$

$$\text{or } \frac{104}{3x-5} = \frac{-208}{3x+5}$$

$$\text{or } \frac{1}{3x-5} = \frac{-2}{3x+5}$$

$$\text{or } 3x+5 = -6x+10$$

$$\text{or } 9x = 5, \quad x = \frac{5}{9}$$

$$6 \quad \frac{42x-37}{6x-1} + \frac{20x+13}{5x+12} = \frac{11x+76}{x+8}$$

$$\text{or } 7 - \frac{30}{6x-1} + 4 - \frac{35}{5x+12} = 11 - \frac{12}{x+8}$$

$$\text{or } \frac{30}{6x-1} + \frac{35}{5x+12} = \frac{12}{x+8}$$

$$\text{or } \frac{30}{6x-1} - \frac{5}{x+8} = \frac{7}{x+8} - \frac{35}{5x+12}$$

$$\text{or } \frac{30x+240-30x+5}{(6x-1)(x+8)} = \frac{35x+84-35x-280}{(x+8)(5x+12)}$$

$$\text{or } \frac{245}{6x-1} = \frac{-196}{5x+12}$$

$$\text{or } \frac{5}{6x-1} = \frac{-4}{5x+12}$$

$$\text{or } 25x+60 = -24x+4$$

$$\text{or } 49x = -56, \quad x = -\frac{56}{49} = -\frac{8}{7}$$

$$7 \quad \frac{15x-7}{5x-4} + \frac{4x+3}{4x-3} = \frac{8x+1}{2x-1}$$

$$\text{or } 3 + \frac{5}{5x-4} + 1 + \frac{6}{4x-3} = 4 + \frac{5}{2x-1}$$

$$\text{or } \frac{5}{5x-4} - \frac{2}{2x-1} = \frac{3}{2x-1} - \frac{6}{4x-3}$$

$$\text{or } \frac{10x-5-10x+8}{(5x-4)(2x-1)} = \frac{12x-9-12x+6}{(2x-1)(4x-3)}$$

$$\text{or } \frac{3}{5x-4} = \frac{-3}{4x-3}$$

$$\text{or } 5x-4 = -4x+3$$

$$\text{or } 9x=7, \quad x=\frac{7}{9}$$

$$8 \quad \frac{4x-7}{4x+5} + \frac{15x+11}{5x+7} = \frac{12x+1}{3x+4}$$

$$\text{or } 1 - \frac{12}{4x+5} + 3 - \frac{10}{5x+7} = 4 - \frac{15}{3x+4}$$

$$\text{or } \frac{12}{4x+5} + \frac{10}{5x+7} = \frac{15}{3x+4}$$

$$\text{or } \frac{12}{4x+5} - \frac{9}{3x+4} = \frac{6}{3x+4} - \frac{10}{5x+7}$$

$$\text{or } \frac{36x+48-36x-45}{(4x+5)(3x+4)} = \frac{30x+42-30x-40}{(3x+4)(5x+7)}$$

$$\text{or } \frac{3}{4x+5} = \frac{2}{5x+7}$$

$$\text{or } 15x+21=8x+10$$

$$\text{or } 7x=-11, \quad x=-1\frac{1}{7}$$

9

$$\frac{4x^2+4x^2+8x+1}{2x^2+2x+3} = \frac{2x^2+2x+1}{x+1}$$

$$\text{or } \frac{2x(2x^2+2x+3)+2x+1}{2x^2+2x+3} = \frac{2x(x+1)+1}{x+1}$$

$$\text{or } 2x + \frac{2x+1}{2x^2+2x+3} = 2x + \frac{1}{x+1}$$

$$\text{or } (2x+1)(x+1) = 2x^2+2x+3$$

$$\text{or } 2x^2+3x+1-2x^2-2x-3, \quad x=2$$

10

$$\frac{12x^3+16x^2+29x-1}{3x^2+4x+8} = \frac{4x^2+20x-1}{x+5}$$

$$\text{or } \frac{4x(3x^2+4x+8)-(3x+1)}{3x^2+4x+8} = \frac{4x(x+5)-1}{x+5}$$

$$\text{or } 4x - \frac{3x+1}{3x^2+4x+8} = 4x - \frac{1}{x+5}$$

$$\text{or } 3x^2+4x+8 = (3x+1)(x+5) = 3x^2+16x+5$$

$$\text{or } -12x = -3, \quad x = \frac{1}{4}$$

11

$$\frac{x^2-x+1}{x-1} + \frac{x^2-2x+1}{x-2} = 2x + \frac{2}{x-3}$$

$$\text{or } x + \frac{1}{x-1} + x + \frac{1}{x-2} = 2x + \frac{2}{x-3}$$

$$\text{or } \frac{1}{x-1} - \frac{1}{x-3} = \frac{1}{x-3} - \frac{1}{x-2}$$

$$\text{or } \frac{x-3-x+1}{(x-1)(x-3)} = \frac{x-2-x+3}{(x-3)(x-2)}$$

$$\text{or } \frac{-2}{x-1} = \frac{1}{x-2}$$

$$\text{or } -2x+4 = x-1$$

$$\text{or } -3x = -5, \quad x = \frac{5}{3}$$

12

$$\frac{x^2+3}{x-1} + \frac{x^2-x+1}{x-2} = \frac{2x^2-4x+1}{x-3}$$

$$\text{or } \frac{(x^2-1)+4}{x-1} + \frac{(x-2)(x+1)+3}{x-2} = \frac{(2x+2)(x-3)+7}{x-3}$$

$$\text{or } (x+1) + \frac{4}{x-1} + (x+1) + \frac{3}{x-2} = 2x+2 + \frac{7}{x-3}$$

$$\text{or } \frac{4}{x-1} - \frac{4}{x-3} = \frac{3}{x-3} - \frac{3}{x-2}$$

$$\text{or } \frac{4(x-3-x+1)}{(x-1)(x-3)} = \frac{3(x-2-x+3)}{(x-3)(x-2)}$$

$$\text{or } \frac{-8}{x-1} = \frac{3}{x-2}$$

$$\text{or } -8x+16=3x-3$$

$$\text{or } -11x = -19 \quad x = \frac{19}{11} = 1\frac{8}{11}$$

13

$$\frac{2x^2-3x+7}{2x-1} + \frac{6x^2+2x+21}{3x+1} = \frac{3x^2+8x+7}{x+3}$$

$$\text{or } x - \frac{2x-7}{2x-1} + 2x + \frac{21}{3x+1} = 3x - \frac{2-7}{x+3}$$

$$\text{or } -1 + \frac{6}{2x-1} + \frac{21}{3x+1} = -1 + \frac{10}{x+3}$$

$$\text{or } \frac{6}{2x-1} - \frac{3}{x+3} = \frac{7}{x+3} - \frac{21}{3x+1}$$

$$\text{or } \frac{6x+18-6x+3}{(2x-1)(x+3)} = \frac{21x+7-21x-63}{(x+3)(3x+1)}$$

$$\text{or } \frac{21}{2x-1} = \frac{-56}{3x+1}$$

$$\text{or } \frac{3}{2x-1} = \frac{-8}{3x+1}$$

$$\text{or } 9x+3 = -16x+8$$

$$\text{or } 25x = 5, \quad x = \frac{1}{5}$$

14

$$\frac{3+2x}{1+2x} - \frac{5+2x}{7+2x} = 1 - \frac{4x^2-2}{7+16x+4x^2}$$

$$\text{or } 1 + \frac{2}{1+2x} - 1 + \frac{2}{7+2x} = 1 - 1 + \frac{16x+9}{7+16x+4x^2}$$

$$\text{or } \frac{2(7+2x+1+2x)}{(1+2x)(7+2x)} = \frac{16x+9}{(7+2x)(1+2x)}$$

$$\begin{aligned} \text{or} \quad & 16+8x=16x+9 \\ \text{or} \quad & -8x=-7, \quad x=\frac{7}{8} \end{aligned}$$

15

$$\begin{aligned} & \frac{2x-3}{x-2} + \frac{3x-20}{x-7} = \frac{x-3}{x-4} + \frac{4x-19}{x-5} \\ \text{or} \quad & 2 + \frac{1}{x-2} + 3 + \frac{1}{x-7} = 1 + \frac{1}{x-4} + 4 + \frac{1}{x-5} \\ \text{or} \quad & \frac{1}{x-2} - \frac{1}{x-4} = \frac{1}{x-5} - \frac{1}{x-7} \\ \text{or} \quad & \frac{x-4-x+2}{(x-2)(x-4)} = \frac{x-7-x+5}{(x-5)(x-7)} \\ \text{or} \quad & \frac{-2}{(x-2)(x-4)} = \frac{-2}{(x-5)(x-7)} \\ & (x-2)(x-4) = (x-5)(x-7) \\ \text{or} \quad & x^2 - 6x + 8 = x^2 - 12x + 35 \\ \text{or} \quad & 6x = 27, \quad x = \frac{27}{6} = 4\frac{1}{2} \end{aligned}$$

16

$$\begin{aligned} & \frac{3x-8}{x-3} + \frac{4x-35}{x-9} = \frac{2x-9}{x-5} + \frac{5x-34}{x-7} \\ \text{or} \quad & 3 + \frac{1}{x-3} + 4 + \frac{1}{x-9} = 2 + \frac{1}{x-5} + 5 + \frac{1}{x-7} \\ \text{or} \quad & \frac{1}{x-3} - \frac{1}{x-5} = \frac{1}{x-7} - \frac{1}{x-9} \\ \text{or} \quad & \frac{x-5-x+3}{(x-3)(x-5)} = \frac{x-9-x+7}{(x-7)(x-9)} \\ \text{or} \quad & \frac{-2}{(x-3)(x-5)} = \frac{-2}{(x-7)(x-9)} \\ \text{or} \quad & (x-3)(x-5) = (x-7)(x-9) \\ \text{or} \quad & x^2 - 8x + 15 = x^2 - 16x + 63 \\ \text{or} \quad & 8x = 48, \quad x = 6 \end{aligned}$$

17

$$\begin{aligned} & \frac{3x-13}{x-4} + \frac{4x-41}{x-10} = \frac{2x-13}{x-6} + \frac{5x-41}{x-8} \\ \text{or} \quad & 3 - \frac{1}{x-4} + 4 - \frac{1}{x-10} = 2 - \frac{1}{x-6} + 5 - \frac{1}{x-8} \\ \text{or} \quad & \frac{1}{x-4} - \frac{1}{x-6} = \frac{1}{x-8} - \frac{1}{x-10} \\ \text{or} \quad & \frac{x-6-x+4}{(x-4)(x-6)} = \frac{x-10-x+8}{(x-8)(x-10)} \end{aligned}$$

$$\begin{aligned}
 \text{or} \quad & \frac{-2}{(x-4)(x-6)} = \frac{-2}{(x-8)(x-10)} \\
 \text{or} \quad & (x-4)(x-6) = (x-8)(x-10) \\
 \text{or} \quad & x^2 - 10x + 24 = x^2 - 18x + 80 \\
 \text{or} \quad & 8x = 56, \quad x = 7
 \end{aligned}$$

18

$$\begin{aligned}
 & \frac{4x+21}{x+5} + \frac{5x-69}{x-14} = \frac{3x-5}{x-2} + \frac{6x-41}{x-7} \\
 \text{or} \quad & 4 + \frac{1}{x-5} + 5 + \frac{1}{x-14} = 3 + \frac{1}{x-2} + 6 + \frac{1}{x-7} \\
 \text{or} \quad & \frac{1}{x+5} + \frac{1}{x-14} = \frac{1}{x-2} + \frac{1}{x-7} \\
 \text{or} \quad & \frac{1}{x+5} - \frac{1}{x-7} = \frac{1}{x-2} - \frac{1}{x-14} \\
 \text{or} \quad & \frac{x-7-x-5}{(x+5)(x-7)} = \frac{x-14-x+2}{(x-2)(x-14)} \\
 \text{or} \quad & \frac{-12}{(x+5)(x-7)} = \frac{-12}{(x-2)(x-14)} \\
 \text{or} \quad & (x+5)(x-7) = (x-2)(x-14) \\
 \text{or} \quad & x^2 - 2x - 35 = x^2 - 16x + 28 \\
 \text{or} \quad & -2x + 16x = 28 + 35 \\
 \text{or} \quad & 14x = 63, \quad x = 4\frac{1}{2}
 \end{aligned}$$

19

$$\begin{aligned}
 & \frac{5-6x}{3x-1} - \frac{2x+7}{x+3} = \frac{31-12x}{3x-7} + \frac{4x+21}{x+5} \\
 \text{or} \quad & -2 + \frac{3}{3x-1} + 2 + \frac{1}{x+3} = -4 + \frac{3}{3x-7} + 4 + \frac{1}{x+5} \\
 \text{or} \quad & \frac{3}{3x-1} - \frac{1}{x+5} = \frac{3}{3x-7} - \frac{1}{x+3} \\
 \text{or} \quad & \frac{3x+15-3x+1}{(3x-1)(x+5)} = \frac{3x+9-3x+7}{(3x-7)(x+3)} \\
 \text{or} \quad & \frac{16}{(3x-1)(x+5)} = \frac{16}{(3x-7)(x+3)} \\
 \text{or} \quad & (3x-1)(x+5) = (3x-7)(x+3) \\
 \text{or} \quad & 3x^2 + 14x - 5 = 3x^2 + 2x - 21 \\
 \text{or} \quad & 12x = -16, \quad x = -\frac{4}{3}
 \end{aligned}$$

20

$$\frac{x^2+3x+3}{x+2} + \frac{x^2-15}{x-4} = \frac{x^2+7x+11}{x+5} + \frac{x^2-4x-20}{x-7}$$

$$\text{or } \frac{(x+2)(x+1)+1}{x+2} + \frac{(x^2-16)+1}{x-4} = \frac{(x+5)(x+2)+1}{x+5} + \frac{(x-7)(x+3)+1}{x-7}$$

$$\text{or } (x+1) + \frac{1}{x+2} + (x+4) + \frac{1}{x-4} = (x+2) + \frac{1}{x+5} + (x+3) + \frac{1}{x-7}$$

$$\text{or } \frac{1}{x+2} - \frac{1}{x-7} = \frac{1}{x+5} - \frac{1}{x-4}$$

$$\text{or } \frac{x-7-x-2}{(x+2)(x-7)} = \frac{x-4-x-5}{(x+5)(x-4)}$$

$$\text{or } \frac{-9}{(x+2)(x-7)} = \frac{-9}{(x+5)(x-4)}$$

$$\text{or } x^2 - 5x - 14 = x^2 + x - 20$$

$$\text{or } 6x = 6, \quad x = 1$$

21

$$\frac{2x+11}{x+5} - \frac{9x-9}{3x-4} = \frac{4x+13}{x+3} - \frac{15x-47}{3x-10}$$

$$\text{or } 2 + \frac{1}{x+5} - 3 - \frac{3}{3x-4} = 4 + \frac{1}{x+3} - 5 - \frac{3}{3x-10}$$

$$\text{or } \frac{3x-4-3x-15}{(x+5)(3x-4)} = \frac{3x-10-3x-9}{(x+3)(3x-10)}$$

$$\text{or } \frac{-19}{(x+5)(3x-4)} = \frac{-19}{(x+3)(3x-10)}$$

$$\text{or } (x+5)(3x-4) = (x+3)(3x-10)$$

$$\text{or } 3x^2 + 11x - 20 = 3x^2 - x - 30$$

$$\text{or } 12x = -10, \quad x = -\frac{5}{6}$$

22

$$\frac{x-2}{x-3} + \frac{x-3}{x-4} = \frac{x-1}{x-2} + \frac{x-4}{x-5}$$

$$\text{or } 1 + \frac{1}{x-3} + 1 + \frac{1}{x-4} = 1 + \frac{1}{x-2} + 1 + \frac{1}{x-5}$$

$$\text{or } \frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{x-5} - \frac{1}{x-4}$$

$$\text{or } \frac{x-2-x+3}{(x-3)(x-2)} = \frac{x-4-x+5}{(x-5)(x-4)}$$

$$\text{or } \frac{1}{(x-3)(x-2)} = \frac{1}{(x-5)(x-4)}$$

$$\text{or } (x-3)(r-2) = (x-5)(r-4)$$

$$\text{or } x^2 - 5x + 6 = x^2 - 9r + 20$$

$$\text{or } 4r = 14, \quad x = 3$$

Exercise 66.

$$1 \quad \frac{2r}{r-4} + \frac{7r-3}{x+1} = 9$$

$$\text{or } 2 + \frac{8}{x-4} + 7 - \frac{10}{1+1} = 9$$

$$\text{or } \frac{4}{r-4} - \frac{5}{1+1} = 0$$

$$\text{or } \frac{1}{r-4} = \frac{5}{x+1}$$

$$\text{or } 4x+4 = 51-20, \quad x = 24$$

$$2 \quad \frac{x+4a+b}{x+a+b} + \frac{4r+a+2b}{x+a-b} = 5$$

$$\text{or } 1 + \frac{3a}{x+a+b} + 4 - \frac{3a-6b}{x+a-b} = 5$$

$$\text{or } \frac{a}{x+a+b} - \frac{a-2b}{x+a-b} = 0$$

$$\text{or } a(x+a-b) = a(x+a+b) - 2b(x+a+b)$$

$$\text{or } -ab = ab - 2bx - 2ab - 2b^2$$

$$\text{or } br = -b^2, \quad x = -b$$

$$3 \quad \frac{3r+5}{x+1} = \frac{4r+8}{3x+3} + \frac{10r+1}{6r+3}$$

$$\text{or } \frac{3x+5}{x+1} - \frac{4x+8}{3(r+1)} = \frac{10r+1}{3(2r+1)}$$

$$\text{or } \frac{9x+15-4x-8}{3(x+1)} = \frac{10x+1}{3(2x+1)}$$

$$\text{or } \frac{5x+7}{x+1} = \frac{10x+1}{2x+1}$$

$$\text{or } 5 + \frac{2}{x+1} = 5 - \frac{4}{2x+1}$$

$$\text{or } 4x+2 = -4x-4$$

$$\text{or } -8x = -6, \quad x = -\frac{3}{4}$$

4

$$\frac{6x+8}{2x+1} - \frac{2x+38}{x+12} = 1$$

$$\text{or } 3 + \frac{5}{2x+1} - 2 - \frac{14}{x+12} = 1$$

$$\text{or } \frac{5}{2x+1} = \frac{14}{x+12}$$

$$\text{or } 5x+60 = 28x+14$$

$$\text{or } -23x = -46, \quad x=2$$

5

$$\frac{1+18}{x-2} - \frac{27-31}{3x-19} = 2$$

$$\text{or } 1 + \frac{20}{x-2} - 1 - \frac{8}{3x-19} = 2$$

$$\text{or } \frac{5}{x-2} = \frac{2}{3x-19}$$

$$\text{or } 15x-95 = 2x-4$$

$$\text{or } 13x = 91 \quad x=7$$

6

$$\frac{1-b}{x-a} - \frac{x-a}{(x-b)} = \frac{2(a-b)}{x-(a+b)}$$

$$\text{or } 1 + \frac{a-b}{x-a} - 1 + \frac{a-b}{x-b} = \frac{2(a-b)}{x-(a+b)}$$

$$\text{or } \frac{1}{x-a} - \frac{1}{x-b} = \frac{2}{x-(a+b)}$$

$$\text{or } \frac{1}{x-a} - \frac{1}{x-(a+b)} = \frac{1}{x-(a+b)} - \frac{1}{x-b}$$

$$\text{or } \frac{x-a-b-x+a}{(x-a)\{x-(a+b)\}} = \frac{x-b-x+a+b}{\{x-(a+b)\}(x-b)}$$

$$\text{or } \frac{-b}{x-a} = \frac{a}{x-b}$$

$$\text{or } -bx + b^2 = ax - a^2$$

$$\text{or } x(a+b) = a^2 + b^2, \quad x = \frac{a^2 + b^2}{a+b}$$

7

$$\frac{1+2a}{2b-x} + \frac{x-2a}{2b+x} - \frac{4ab}{x^2-4b^2} = 0$$

$$\text{or } -1 + \frac{2(a+b)}{2b-x} + 1 - \frac{2(a+b)}{2b+x} - \frac{4ab}{4b^2-x^2} = 0$$

$$\text{or } \frac{2(a+b)(2b+x-2b+1)}{4b^2-x^2} = \frac{4ab}{4b^2-x^2}$$

$$\text{or } 4x(a+b) = 4ab, \quad r = \frac{ab}{a+b}$$

8

$$\frac{(x-a)(r-b)}{x-a-b} = \frac{(x-c)(x-d)}{x-c-d}$$

$$\text{or } \frac{x^2-(a+b)x+ab}{r-(a+b)} = \frac{x^2-(c+d)x+cd}{r-(c+d)}$$

$$\text{or } x + \frac{ab}{x-(a+b)} = 1 + \frac{cd}{x-(c+d)}$$

$$\text{or } 2ab - ab(c+d) = xcd - cd(a+b)$$

$$\text{or } x(ab+cd) = ab(c+d) - cd(a+b),$$

$$x = \frac{ab(c+d) - cd(a+b)}{ab - cd}$$

9

$$\frac{1}{r^2+3r+2} + \frac{2x}{x^2+4x+3} + \frac{1}{x^2+5x+6} = 14 - \frac{60+4x}{x+3}$$

$$\text{or } \frac{1}{(x+1)(x+2)} + \frac{2x}{(x+1)(x+3)} - \frac{2}{x+3}$$

$$+ \frac{1}{(r+2)(x+3)} = 14 - \frac{62+4x}{x+3}$$

$$\text{or } \frac{1}{(x+1)(x+2)} + \frac{2r-2x-2}{(x+1)(r+3)} + \frac{1}{(r+2)(x+3)}$$

$$= \frac{14x+42-62-4x}{r+3}$$

$$\text{or } \left(\frac{1}{(r+1)(x+2)} - \frac{2}{(x+1)(x+3)} + \frac{1}{(r+2)(x+3)} \right)$$

$$= \frac{10x-20}{x+3}$$

$$\text{or } \frac{x+3-2x-4+x+1}{(x+1)(x+2)(x+3)} = \frac{10x-20}{x+3}$$

$$\text{or } 10x-20=0, \quad r=2$$

10

$$\frac{a+r}{a^2+ax+x^2} + \frac{a-r}{a^2-ax+x^2} = \frac{3a}{x(a^4+a^2r^2+x^4)}$$

$$\text{or } \frac{a^3+x^3+a^3-x^3}{a^4+a^2x^2+x^4} = \frac{3a}{x(a^4+a^2x^2+x^4)}$$

$$\text{or } 2a^3 = \frac{3a}{x}, \quad x = \frac{3a}{2a^3} = \frac{3}{2a^2}$$

$$11 \quad \frac{x}{x-2} + \frac{x-9}{x-7} = \frac{x+1}{x-1} + \frac{x-8}{x-6}$$

$$\text{or} \quad 1 + \frac{2}{x-2} + 1 - \frac{2}{x-7} = 1 + \frac{2}{x-1} + 1 - \frac{2}{x-6}$$

$$\text{or} \quad \frac{1}{x-2} - \frac{1}{x-7} = \frac{1}{x-1} - \frac{1}{x-6}$$

$$\text{or} \quad \frac{x-7-x+2}{(x-2)(x-7)} = \frac{x-6-x+1}{(x-1)(x-6)}$$

$$\text{or} \quad \frac{-5}{(x-2)(x-7)} = \frac{-5}{(x-1)(x-6)}$$

$$\text{or} \quad (x-2)(x-7) = (x-1)(x-6)$$

$$\text{or} \quad x^2 - 9x + 14 = x^2 - 7x + 6$$

$$\text{or} \quad 2x = 8, \quad x = 4$$

$$12 \quad \frac{1}{(x+a)^2 - b^2} + \frac{1}{(x+b)^2 - a^2} = \frac{1}{x^2 - (a+b)^2} + \frac{1}{x^2 - (a-b)^2}$$

$$\text{or} \quad \frac{1}{(x+a+b)(x+a-b)} + \frac{1}{(x+a+b)(x-a+b)} = \frac{1}{(x+a+b)(x-a-b)} + \frac{1}{(x-a+b)(x+a-b)}$$

$$\text{or} \quad \frac{1}{(x+a+b)(x+a-b)} - \frac{1}{(x+a+b)(x-a-b)} = \frac{1}{(x-a+b)(x+a-b)} - \frac{1}{(x-a+b)(x+a-b)}$$

$$\text{or} \quad \frac{x-a-b-x-a+b}{(x+a+b)(x+a-b)(x-a-b)} = \frac{x+a+b-x-a-b}{(x-a+b)(x+a-b)(x+a-b)}$$

$$\text{or} \quad \frac{-a}{x-a-b} = \frac{b}{x-a+b}$$

$$\text{or} \quad -ax + a^2 - ab = bx - ab - b^2$$

$$\text{or} \quad 2(a+b) = a^2 + b^2 \quad x = \frac{a^2 + b^2}{a+b}$$

$$13 \quad \frac{3x^2 + 5x + 8}{5x^2 + 6x + 12} = \frac{3x + 5}{5x + 6}$$

$$\text{or} \quad \frac{x(3x+5)+8}{x(5x+6)+12} = \frac{3x+5}{5x+6}$$

Now putting m for $3x+5$ and n for $5x+6$, we get

$$\frac{1}{x} \frac{m+8}{n+12} = \frac{m}{n}$$

or $xmn+8 = xmn+12m$

or $8n=12m$

or $2n=3m$

or (substituting) $2(5x+6)=3(3x+5)$

or $10x+12=9x+15$; $x=3$

14

$$\frac{58x^2+87x+7}{87x^2+145x+11} = \frac{2x+3}{3x+5}$$

or $\frac{29x(2x+3)+7}{29x(3x+5)+11} = \frac{2x+3}{3x+5}$

Putting a for $2x+3$ and b for $3x+5$, we get

$$\frac{29xa+7}{29xb+11} = \frac{a}{b}$$

or $29xab+7b=29abx+11a$

or $7b=11a$

or (substituting) $7(3x+5)=11(2x+3)$

or $21x-35=22x+33$, $x=2$

15

$$\frac{a^2(a-b)}{b(a-b)^2}x + \frac{2abc}{a-b} - \frac{ax}{b} = 2cx - \frac{a^2b^2}{(a-b)^2}$$

or $x \left\{ \frac{a^2(a-b)}{b(a-b)^2} - \frac{a}{b} - 2c \right\} = -\frac{ab}{a-b} \left\{ \frac{ab}{(a-b)^2} + 2c \right\}$

or $x \left\{ \frac{a^3-2a^2b-a^2+2a^2b-ab^2}{b(a-b)^2} - 2c \right\} = -\frac{ab}{a-b} \left\{ \frac{ab}{(a-b)^2} + 2c \right\}$

or $x \left\{ \frac{-ab^2}{b(a-b)^2} - 2c \right\} = -\frac{ab}{a-b} \left\{ \frac{ab}{(a-b)^2} + 2c \right\}$

or $x \left\{ -\frac{ab}{(a-b)^2} - 2c \right\} = \frac{ab}{a-b} \left\{ -\frac{ab}{(a-b)^2} - 2c \right\}$

$$x = \frac{ab}{a-b}$$

16

$$(x-23)^2 + (x-27)^2 = 2(x-25)^2$$

Putting a for $x-25$, we get

$$(a+2)^2 + (a-2)^2 = 2a^2$$

$$\text{or } a^3 + 6a^2 + 12a + 8 + a^3 - 6a^2 + 12a - 8 = 2a^3$$

$$\text{or } 2a^3 + 24a = 2a^3$$

$$\text{or } 24a = 0$$

$$\text{or } a = 0$$

$$\text{or } r - 25 = 0, \quad r = 25$$

17

$$\frac{4x-17}{9} - \frac{3^2-22x}{33} = x - \frac{6}{x} \left(1 - \frac{x^2}{54} \right)$$

$$\text{or } \frac{4x-17}{9} - \frac{11-66x}{99} = r - \frac{6}{x} \left(\frac{54-x^2}{54} \right)$$

$$\text{or } \frac{4x-17}{9} - \frac{1-6x}{9} = r - \frac{54-x^2}{9x}$$

$$\text{or } \frac{4x-17-1+6x}{9} = \frac{9x^2-54+x^2}{9x}$$

$$\text{or } r(101-18) = 10x^2 - 54$$

$$\text{or } -18r = -54, \quad r = 3$$

18

$$\left(\frac{r-2a}{x+2b} \right)^2 = \frac{r-2a-2b}{r+2a+2b}$$

$$\text{or } \left\{ 1 - \frac{2(a+b)}{x+2b} \right\}^2 = 1 - \frac{4(a+b)}{x+2a+2b}$$

$$\text{or } 1 - \frac{4(a+b)}{x+2b} + \frac{4(a+b)^2}{(x+2b)^2} = 1 - \frac{4(a+b)}{x+2a+2b}$$

$$\text{or } \frac{1}{x+2b} - \frac{1}{x+2a+2b} = \frac{a+b}{(x+2b)^2}$$

$$\text{or } \frac{x+2a+2b-x-2b}{(x+2b)(x+2a+2b)} = \frac{a+b}{(x+2b)^2}$$

$$\text{or } \frac{2a}{x+2a+2b} = \frac{a+b}{x+2b}$$

$$\text{or } 2ax + 4ab = ax + bx + 2(a+b)^2$$

$$\text{or } x(2a-a-b) = 2\{(a+b)^2 - 2ab\}$$

$$\text{or } x(a-b) = 2(a^2+b^2), \quad x = \frac{2(a^2+b^2)}{a-b}$$

19

$$\frac{x+19}{x+10} = \left(\frac{2x+33}{2x+24} \right)^2$$

$$\text{or } 1 + \frac{9}{x+10} = \left(1 + \frac{9}{2x+24} \right)^2$$

$$= 1 + \frac{9}{x+12} + \frac{81}{4(x+12)^2}$$

$$\text{or } \frac{1}{1+10} - \frac{1}{1+12} = \frac{9}{4(r+12)^2}$$

$$\text{or } \frac{x+12-x-10}{(x+10)(x+12)} = \frac{9}{4(x+12)^2}$$

$$\text{or } \frac{2}{x+10} = \frac{9}{4(1+12)}$$

$$\text{or } 8(x+12) = 9(x+10)$$

$$\text{or } 81+96=91+90, \quad x=6$$

20

$$\left(\frac{1-a}{x+b}\right)^3 = \frac{x-2a-b}{x+a+2b}$$

$$\text{or } \left(1 - \frac{a+b}{1+b}\right)^3 = 1 - \frac{3(a+b)}{x+a+2b}$$

$$\begin{aligned} \text{or } 1 - \frac{3(a+b)}{(1+b)} + \frac{3(a+b)^2}{(1+b)^2} - \frac{(a+b)^3}{(x+b)^3} \\ = 1 - \frac{3(a+b)}{x+a+2b} \end{aligned}$$

$$\begin{aligned} \text{or } \frac{3(a+b)}{(1+b)^3} - \frac{(a+b)^3}{(x+b)^3} &= 3\left(\frac{1}{x+b} - \frac{1}{x+a+2b}\right) \\ &= 3\left\{\frac{x+a+2b-x-b}{(x+b)(1+a+2b)}\right\} \\ &= \frac{3(a+b)}{(x+b)(1+a+2b)} \end{aligned}$$

$$\text{or } \frac{3}{1+b} - \frac{3}{(x+a+2b)} = \frac{a+b}{(1+b)^2}$$

$$\text{or } 3\left\{\frac{x+a+2b-x-b}{(1+b)(x+a+2b)}\right\} = \frac{a+b}{(1+b)^2}$$

$$\text{or } \frac{3(a+b)}{(1+a+2b)} = \frac{a+b}{(x+b)}$$

$$\text{or } 3x+3b=1+a+2b$$

$$\text{or } 2x=a-b, \quad 1=\frac{1}{2}(a-b)$$

21

$$\frac{3x}{2} - \frac{81x^2-9}{(3x-1)(x+3)} = 3x - \frac{3}{2} \frac{2x^2-1}{1+3} - \frac{57-31}{2}$$

$$\text{or } -\frac{9(3x+1)}{1+3} = \frac{3x}{2} - \frac{3}{2} \frac{2x^2-1}{1+3} - \frac{57-31}{2}$$

$$\text{or } -\frac{3(3x+1)}{1+3} = \frac{1}{2} - \frac{1}{2} \frac{2x^2-1}{1+3} - \frac{19-x}{2}$$

$$\text{or } \frac{2x^2-1}{2(x+3)} - \frac{3(3x+1)}{x+3} = \frac{2}{2} - \frac{19-x}{2} = \frac{2x-19}{2}$$

$$\text{or } \frac{2x^2-1-18x-6}{2(x+3)} = \frac{2x-19}{2}$$

$$\text{or } 2x^2-18x-7=2x^2-13x-57$$

$$\text{or } -5x = -50, \quad x = 10$$

Exercise (67)

- 1 Let the other number be y , 2 Let y be the other number,
then we have $x+y=15$, then we have $x-y=20$,
 $y=15-x$ $y=x-20$
- 3 Let y be the other number 4 Let x be the required excess,
then we have $y-x=25$, then $y-x=25$,
 $y=x+25$ $y=25+x$
- 5 Let z be the required defect, 6 Let y be the other factor,
then we have $y-z=21$ then we have $xy=21$,
 $z=y-21$ $y=\frac{21}{x}$
- 7 Let y be the required number, 8 Let z be the required number,
then we have $y+3x=100$, then we have $4x-z=3y$,
 $y=100-3x$ $z=4x-3y$
- 9 Since in one hour y mile is travelled,
in x hours $(x \times y)$ or (xy) mile is travelled
- 10 Since y mile is travelled in 1 hour,
1 mile $\frac{1}{y}$ th of an hour,
 x mile $\frac{x}{y}$ hour
- 11 Let y years be the age 20 years later,
and z years 3 years ago,
then $y-20=x$, $y=x+20$,
and $x-z=3$, $z=x-3$
- 12 Let z miles be the required rate,
 $xz=60$, $z=\frac{60}{x}$

- 13 Since in x hours 30 miles is travelled
 or in $(1 \times 60 \times 60)$ seconds $(30 \times 1760 \times 3)$ feet is travelled ,
 in 1 second $\frac{30 \times 1760 \times 3}{60 \times 60 \times x}$ feet or $\frac{44}{x}$ feet is travelled

- 14 In one week x annas is spent
 or $\frac{x}{16}$ Rupees is spent ,
 in one year $\left(\frac{x}{16} \times 52\right)$ Rupees is spent
 or $\frac{13x}{4}$ Rupees is spent

Now therefore the saving $= \left(5x - \frac{13x}{4}\right) \text{Rs} = \frac{7x}{4} \text{Rs}$

- 15 As the preceding number is evidently greater than the succeeding one, the two numbers just preceding x are $(x-1)$ and $(x-2)$ and those just succeeding x are $(x+1)$ and $(x+2)$. Thus the required numbers are $x-2, x-1, x, x+1$ and $x+2$

- 16 The three numbers are $x-1, x$ and $x+1$,
 the required sum $= (x-1) + x + (x+1) = 3x$

- 17 The next number after $2m+1$ is
 $\{(2m+1)+1\}$, i.e., $2(m+1)$ which is even ,
 the required odd number is $2(m+1)+1$, i.e., $2m+3$

- 18 The even number next before $2x$ is evidently $2x-2$

- 19 x men take 10 days to do the work
 1 man takes 10x days
 y men will take $\frac{10x}{y}$ days to do the work

- 20 $a \text{ yds} = 3a \text{ ft}$,
 the required measure $= (3a \times b) \text{sq ft} = 3ab \text{ sq ft}$

- 21 The length and breadth evidently become

$$\frac{3a}{4} + \frac{b}{4} \text{ respectively ,}$$

the number of sq units required is $\frac{3ab}{16}$

- 22 In y hours he walks x miles
 in 1 hour $\frac{x}{y}$ miles
 in 1 minute $\frac{x}{60y}$ miles
 in 20 minutes $\frac{x}{3y}$ miles
- 23 He walks r miles in a hours
 1 miles in $\frac{a}{r}$ hours
 16 miles in $\frac{16a}{r}$ hours
- 24 His present age $= \{(x-5)+20\}$ years $= x+15$ years
 and his age 30 years hence $= \{(1+15)+30\}$ years
 $= (1+45)$ years
- 25 The number is evidently equal to $10y+r$
- 26 The number is evidently equal to $100x+10y+z$
- 27 The number in this case evidently becomes $100z+10y+x$

Exercise (68)

- 1 Let r be the greater number
 Then the other number $= 50-x$,
 also that number $= x-30$,
 $50-x = x-30$
 or $2x=80$, $x=40$,
 the other number $= 50-40=10$
- 2 Let x be the number
 Then $x=5(96-x)$
 or $6x=5 \times 96$,
 $x=5 \times 16=80$
- 3 Let x be the number
 Then $8x=\frac{1}{2}r+90$
 or $16x=x+180$
 or $15x=180$, $x=12$

- 4 Let x be the required number
 Then $x - 40 = \frac{1}{3}x$
 or $3x - 120 = x$
 or $2x = 120$
 $x = 60$
- 5 Let x be the number
 Then $x - 35 = (67 - x) - 22$
 or $2x = 67 - 22 + 35 = 80$
 $x = 40$
- 6 Let x be the number
 Then $4(x - 16) = 416 - x$
 or $5x = 416 + 64 = 480$,
 $x = 96$
- 7 Let x be the middle number
 Then $3x = 129$,
 $x = 43$,
 the numbers are 42, 43 and 44
- 8 Let x be the number
 Then $7x - 132 = 132 - x$
 or $8x = 264$,
 $x = 33$
- 9 Let x be one part
 Then the other part $= 90 - x$
 $3x + 4(90 - x) = 335$
 or $x = 335 - 360$
 $= -25$,
 $x = 25$,
 and the other part is $(90 - 25) = 65$
- 10 Let x be one of the numbers
 the other number $= 39 - x$,
 $\frac{x}{5} = \frac{39 - x}{8}$
 or $8x = 195 - 5x$
 or $13x = 195$,

$r = 15$,

the other number is $(39 - 15) = 24$

11 Let x be the number

$$\frac{1}{4}x - \frac{1}{8}x = 5$$

or $9x - 4x = 180$

or $5x = 180$,

$$x = 36$$

12 Let x be the number -

Then $\frac{1}{2}x - \frac{1}{8}x = 3$

or $8x - 6x = 144$

or $2x = 144$,

$$x = 72$$

13 Let x be one of the parts

Then the other part $= 21 - x$,

$$10x = 9(21 - x) + 1$$

or $19x = 189 + 1 = 190$,

$$x = 10$$
 ,

the other part $= 21 - 10 = 11$

14 Let x be the price of the house

Then the price of the garden $= \frac{5x}{12}$,

$$x + \frac{5x}{12} = £850$$

or $12x + 5x = £10200$

or $17x = £10200$

$$x = £600$$
 ,

the price of the garden $= \frac{5}{12} \times £600 = £250$

15 Let one of the persons get x shillings or $£\frac{x}{20}$ and the other gets x half-crown or $£\frac{x}{8}$

We have $\frac{x}{20} + \frac{x}{8} = 420$

or $2x + 5x = 420 \times 40$,

or $7x = 420 \times 40$,

$$r = 60 \times 40 = 2400,$$

they will receive 2400 shillings and 2400 half-crowns respectively i.e., £120 and £300

- 16 Let x be the value of a sheep,

$$72x + £35 = 92x - £35$$

or

$$20x = £70$$

$$x = £3 \text{ } 10s$$

- 17 Let x be the age of the younger person,

the age of the elder man $= (x + 10)$ years,

$$(x + 10) - 15 = 2(1 - 15)$$

or

$$x - 5 = 2x - 30,$$

$$x = 25,$$

the age of the elder man $= 25 + 10 = 35$

- 18 Let x be the breadth of the smaller field,

$2x$ is the length of the smaller field,

$$2x^2 + 6800 = (x + 10)(2x + 50)$$

$$= 2x^2 + 70x + 500,$$

$$70x = 6300,$$

$$x = 90$$

The size of the smaller field is 90 yds by 180 yds,

and the size of the larger field is 100 yds by 230 yds

- 19 Let the speed of the luggage train be r miles per hour

Now the luggage train travels $(15 + 10)$ mts before it is overtaken by the train

$$\frac{35 \times 10}{60} = \frac{25x}{60}$$

$$25x = 35 \times 10$$

or

$$x = \frac{35 \times 10}{25} = 14$$

The required speed is 14 miles per hour

- 20 Let x miles be the required distance. Then he must take $\frac{x}{8}$ hrs to ride on and $\frac{x}{2}$ hours to come back

$$\frac{x}{8} + \frac{x}{2} = 5$$

or $x + 4x = 40,$

$x = 8$ miles

21 Let x years be the son's age Then the father's age is $3x$ years Ten years afterwards the father's age will be $(3x+10)$ and the son's age $(x+10)$

$$\text{We get } 3x + 10 = 2(x + 10)$$

$$\text{or } x = 10,$$

the father's age is 30 years,
and the son's age is 10 years

22 Let x ft be the breadth of the room Then the length is $(x+3)$ ft Also the increased length becomes $(x+6)$ ft and the diminished breadth becomes $(x-2)$ ft

$$\text{We get } x(x+3) = (x+6)(x-2)$$

$$\text{or } x^2 + 3x = x^2 + 4x - 12$$

$$\text{or } x = 12,$$

the breadth is 12 ft
and the length is 15 ft

23 Let x pounds be the required sum After losing $\frac{5}{11} B$ had $\pounds \frac{6x}{11}$ with him

$$A \text{ gained } \pounds \left\{ \frac{1}{2} \left(\frac{6x}{11} \right) + 6 \right\}$$

$$\text{or } \pounds \left(\frac{3x}{11} + 6 \right),$$

$$\text{we get } \frac{5x}{11} = \frac{3x}{11} + 6$$

$$\text{or } \frac{2x}{11} = 6, \quad \text{or } x = 33$$

They had at first $\pounds 33$

24 Let x years be the son's age, then the father's age is $(80-x)$ years

$$\text{We get } 2x = (80-x) + 10$$

$$\text{or } 3x = 90, \quad x = 30, \text{ and}$$

$$80 - x = 50$$

The father's age is 50 years,
and the son's age is 30 years

- 25 Let there were x men, therefore there were $(36-x)$ women

$$\text{We get } \frac{3}{20}x + \frac{5}{2 \times 20}(36-x) = 5$$

$$\text{or } 6x + 5(36-x) = 200$$

$$\text{or } x = 200 - 180 = 20 ,$$

$$\text{and } 36 - x = 16 ,$$

there were 20 men and 16 women

- 26 Let them have travelled for x hours, then A travelled $\frac{3x}{2}$ miles and B travelled $\frac{5x}{4}$ miles

$$\text{We get } \frac{3x}{2} + \frac{5x}{4} = 154$$

$$\text{or } 6x + 5x = 616 ,$$

$$11x = 616 ,$$

$$x = 56 ,$$

$$\text{and } \frac{3x}{2} = 84 \text{ and } \frac{5x}{4} = 70$$

A travelled 84 miles and B 70 miles in 56 hours

- 27 Let him have worked for x days, then he was absent for $(36-x)$ days

$$\text{We get } x(21) - (36-x)(11) = 58$$

$$\text{or } 5x - (36-x)11 = 58$$

$$\text{or } 8x = 116 + 106 = 224 ,$$

$$x = 28 ,$$

he worked for 28 days

- 28 Let x shillings be the cost of the picture or that of the frame,

$$\text{we have } x + 15 = 2(x - 20)$$

$$\text{or } x = 55 ,$$

$$\text{the cost of the picture} = 55s = \text{£}2 \ 15s$$

- 29 Let x feet be the required length

$$\text{We get } \frac{x}{4} + \frac{x}{3} + 10 = x$$

$$\text{or } 3x + 4x + 120 = 12x$$

$$\text{or } 5x = 120$$

$$\text{or } x = 24 ,$$

the required length = 24 ft

30. Let x be the number of working days, he remained idle for $(30-x)$ days. For x days he received $x(2\frac{1}{2})$ and $(30-x)$ days he lost $(30-x)$.

We get $x(2\frac{1}{2}) - (30-x) = £27s$

$$\text{or } \frac{5x}{2} - 30 + x = 47$$

$$\text{or } 5x - 60 + 2x = 94$$

$$\text{or } 7x = 154$$

$$\text{or } x = 22,$$

he worked for 22 days

31. Let x be the required number of days. Then in one day they can together do $\frac{1}{x}$ of the work.

A can do $\frac{1}{9}$ of the work in one day, B can do $\frac{1}{18}$ and C can do $(\frac{1}{9} \times \frac{2}{3})$ or $\frac{1}{12}$ of the work in the same time.

$$\text{We get } \frac{1}{9} + \frac{1}{18} + \frac{1}{12} = \frac{1}{x}$$

$$\text{or } \frac{4+2+3}{36} = \frac{1}{x}$$

$$\text{or } \frac{9}{36} = \frac{1}{x}, \text{ or, } \frac{1}{4} = \frac{1}{x}, \text{ or, } x = 4$$

They would require 4 days to do the work.

32. Let x pounds and x shillings be the required sums.

$$\text{We get } x + \frac{x}{20} = 54\frac{3}{5} = \frac{273}{5}$$

$$\text{or } 20x + x = 1092$$

$$\text{or } 21x = 1092$$

$$\text{or } x = 52,$$

the required sums are £52 and 52s.

33. Let x pounds be the sum to be divided.

A is to get $(\frac{x}{2} - 30)$ pounds,

B is to get $(\frac{x}{3} - 10)$ pounds,

and C is to get $(\frac{x}{4} + 8)$ pounds,

$$\text{we get } \frac{2}{2} - 30 + \frac{2}{3} - 10 + \frac{2}{4} + 8 = 2$$

$$\text{or } \frac{2}{2} + \frac{2}{3} + \frac{2}{4} - r = 32$$

$$\text{or } \frac{62 + 42 + 32 - 122}{12} = 32$$

$$\text{or } \frac{1}{12} = 32, \quad r = £384$$

$$A \text{ receives } £\left(\frac{384}{2} - 30\right) \text{ or } £162,$$

$$B \quad ,, \quad £\left(\frac{384}{3} - 10\right) \text{ or } £118,$$

$$\text{and } C \quad , \quad £\left(\frac{384}{4} + 8\right) \text{ or } £104$$

34 Let x be the number of sheep

$$\text{Then we get } x(£2 - 2s) - (£1 - 8s) = x(£2) + £2$$

$$\text{or } x(2s) = £3 - 8s = 68s$$

$$\text{or } x = 34, \quad \text{there were 34 sheep,}$$

$$\text{and he had with him } £(34 \times 2 + 2), \text{ or } £70$$

35 Let them meet x hours after starting

Then we get

$$9\frac{1}{2}x + 9\frac{1}{2}x = 200$$

$$\text{or } 18\frac{1}{2}x = 200$$

$$\text{or } 75x = 800$$

$$\text{or } x = 10\frac{2}{3}$$

They met at a distance of $(9\frac{1}{2} \times 10\frac{2}{3})$,

or $98\frac{2}{3}$ miles from London, also the required time is $10\frac{2}{3}$ hours

36 Let the number of apples at 3 a penny be x , the number of the kind at 4 a penny is therefore $\frac{5x}{6}$

The sum spent in buying them is $\left(\frac{x}{3} + \frac{5x}{6 \times 4}\right)d$ and

the sum obtained by selling them is $\left\{\left(x + \frac{5x}{6}\right)\frac{6}{16}\right\}d$ As the selling price is greater than the buying price by $3\frac{1}{2}d$,

we get $\frac{x}{3} + \frac{5}{24}x + \frac{7}{2} = \frac{11}{8}x$

or $16x + 10x + 168 = 33x$

or $33x - 26x = 168$

or $7x = 168,$ or $x = 24,$

the total number of apples bought

$$= 24 + \frac{5}{6} \cdot 24 = 24 + 20 = 44$$

37 Let the number representing the left digit be x The number representing the other digit is $(5-x)$, the required number is $10x + (5-x)$, or $9x + 5$

Also we get $x + 1 = \frac{1}{9}(9x + 5)$

or $8x + 8 = 9x + 5$

or $x = 3$

The required number = 32

38 Let the digit in the unit's place be x

Then the ten's place is $(x+5)$,

the number is $10(x+5) + x$ or $11x + 50$

Also we get $(11x + 50) - 5\{x + (x+5)\} = 10x + (x+5)$

or $11x + 50 - 10x - 25 = 11x + 5$

or $10x = 20,$ or $x = 2$

The required number = 72

39 Let the digit in the ten's place be x ,

the other digit is $(5-x)$,

the number is $10x + (5-x)$ or $9x + 5$

Also $10x + 4(5-x) = 10(5-x) + x$

or $6x + 20 = 50 - 9x$

or $15x = 30,$ or $x = 2$

The required number is 23

40 Let the common result be x

The 1st part is $x-1$,

„ 2nd $x+2$,

„ 3rd $\frac{x}{3}$

„ 4th $4x$

We get $(1-1) + (1+2) + \frac{x}{3} + 41 = 39$

or $6x + \frac{x}{3} = 38$

or $18x + x = 114$

or $19x = 114$

or $x = 6$

The parts are $(6-1)$, $(6+2)$, $(\frac{x}{3})$ and (6×4)

or 5, 8, 2 and 24

41

Let the common result be x

The 1st part $= x + 3$,

„ 2nd „ $= x - 11$,

„ 3rd „ $= \frac{x}{4}$,

„ 4th „ $= 2x$

We get $(x+3) + (x-11) + \frac{x}{4} + 2x = 60$

or $4x + \frac{x}{4} = 60 + 11 - 3 = 68$

or $16x + x = 68 \cdot 4$

or $17x = 68 \cdot 4$

or $x = 16$

The parts are $(16+3)$, $(16-11)$, $(\frac{x}{4})$ and (16×2)

or 19, 5, 4 and 32

42

Let the common result be x

The 1st part $= x - 5$,

„ 2nd „ $= x + 4$,

„ 3rd „ $= \frac{x}{3}$,

„ 4th „ $= 2x$

We get $(x-5) + (x+4) + \frac{x}{3} + 2x = 116$

or $4x + \frac{x}{3} = 117$

or $12x + x = 117 \cdot 3$

or $13x = 117 \cdot 3$,

$x = 27$

The parts are $(27-5\frac{1}{2})$ $(27-4)$ $(\frac{2}{3})$ and (27×2)
 or $22, 31, 9, 54$

Exercise 69

1 Let x minutes past 3 o'clock be the required time. Now the minute hand travels 12 times faster than the hour-hand so that

$$x = 12(\tau - 15)$$

$$\text{or } 11x = 180 \text{ or } x = 16\frac{4}{11}$$

the required time is $16\frac{4}{11}$ minutes past 3

2 Let τ mts past 5 be the required time.

$$\text{We get } x = 12(\tau - 25)$$

$$\text{or } 11x = 300$$

$$\tau = 27\frac{2}{11}$$

The required time is $27\frac{2}{11}$ minutes past 5

3 At 7 the hands are 35 minute-divisions apart.

(i) They will be opposite to each other when they will be 30 minute-divisions apart.

Let x minutes past 7 be the required time.

$$\text{We get } x = 12\{x - (35 - 30)\} = 12(x - 5)$$

$$\text{or } 11x = 60 \quad x = 5\frac{5}{11}$$

The hands will be opposite to each other at $5\frac{5}{11}$ minutes past 7

(ii) The hands will be at right angles when they will be 15 minute-divisions apart.

Let x minutes past 7 be the required time

$$\text{We get } x = 12\{x - (35 \pm 15)\}$$

$$= 12\{x - 50\}$$

$$\text{or } 12(x - 20)$$

$$\text{or } 11x = 600 \text{ or } 240$$

$$\text{or } x = 54\frac{6}{11} \text{ or } 21\frac{6}{11}$$

The hands will be at right angles at $54\frac{6}{11}$ minutes and $21\frac{6}{11}$ minutes past 7

(iii) Let the required time be τ minutes past 7

We get $\tau = 12(2 - 35)$

$$\text{or} \quad 11\tau = 420,$$

$$\tau = \frac{420}{11} = 38\frac{2}{11}$$

The required time is $38\frac{2}{11}$ minutes past 7

4 At 6 the hands are 30 minute-divisions apart

(i) they will be again opposite when the minute hand will pass 60 minute-divisions more than the hour-hand

Let x minutes past 6 be the required time

We get $x = 12(x - 60)$

$$\text{or} \quad 11x = 720$$

$$\text{or} \quad x = \frac{720}{11} = 65\frac{5}{11}$$

The required time is $65\frac{5}{11}$ minutes past 6 i.e. $5\frac{5}{11}$ minutes past 7

(ii) The hands will be at right angles to each other when they will be 15 minute divisions apart

Let x be the required time.

We get $x = 12\{x - (30 - 15)\}$

$$= 12(x - 15)$$

$$\text{or} \quad 11x = 180, \quad x = 16\frac{4}{11}$$

The required time is $16\frac{4}{11}$ minutes past 6

5 Let x miles be the required distance. Then the man from A walked x miles in $\frac{x}{p}$ hours and the man from B walked $(a-x)$ miles in $\frac{a-x}{q}$ hours

We get $\frac{x}{p} = \frac{a-x}{q}$

$$\text{or} \quad xq = ap - xp$$

$$\text{or} \quad x(p+q) = ap, \quad x = \frac{ap}{p+q}$$

The required distance is $\frac{ap}{p+q}$ miles

6 Let the second meeting place be x miles from the place whence the man walking 6 miles an hour started. Then he

walked $\{22 + (22 - x)\}$ miles or $(44 - x)$ miles in $\frac{44 - x}{6}$ hours and the other walked $(22 + x)$ miles in $\frac{22 + x}{5}$ hours

$$\frac{44 - x}{6} = \frac{22 + x}{5}$$

$$\text{or} \quad 220 - 5x = 132 + 6x$$

$$\text{or} \quad 11x = 88$$

$$x = 8$$

Thus the required place is 8 miles distant from the starting place of the faster walker and the required time $= \frac{44 - x}{6}$ hours

$$= \frac{36}{6} \text{ hours} = 6 \text{ hours}$$

7 Let x miles be the distance from A to B . Then the man travelled $\frac{x}{3}$ miles in $\frac{x}{3a}$ hours and $\frac{2x}{3}$ miles in $\frac{2x}{3b}$ hours or $\frac{x}{3b}$ hours and at the rate of $3c$ miles per hour he would have ridden $2x$ miles in $\frac{2x}{3c}$ hours

$$\text{We get } \frac{2x}{3c} = \frac{x}{3a} + \frac{x}{3b}$$

$$\text{or} \quad \frac{2}{3c} = \frac{1}{a} + \frac{1}{b}$$

8 Let x minutes be the time for which B ran

In 5 minutes B ran $(900 - 75)$ yds $= 825$ yds, in x minutes he ran $\frac{825}{5}x$ yds or $165x$ yds

Again A ran before he fell $\frac{900}{5}$ yds or 180 yds per minute and after the accident he ran 160 yds per minute, in the remaining $(x - 5 + \frac{1}{2})$ minutes he ran $(x - \frac{9}{2}) \times 160$ yds or $(160x - 720)$ yds

$$\text{We get } 165x = 160x - 720 + 900$$

$$\text{or} \quad 5x = 180$$

$$\text{or} \quad x = 36$$

The time required is 36 minutes

9 Let x miles per hour be his rate when he first started
Then $\frac{1}{x}$ ($\frac{5}{6}$ of the journey in mile) is the fixed time

Also $\left\{ \frac{1}{x} \left(\frac{1}{4} \text{ of the journey in miles} \right) + \frac{1}{x+1} \left(\frac{3}{4} \text{ of the journey in miles} \right) \right\}$ is the fixed time

$$\text{We get } \frac{1}{x} \frac{5}{6} = \frac{1}{x} \frac{1}{4} + \frac{1}{x+1} \frac{3}{4}$$

$$\text{or } \frac{1}{x} \left(\frac{5}{6} - \frac{1}{4} \right) = \frac{1}{x+1} \frac{3}{4}$$

$$\text{or } \frac{1}{x} \left(\frac{10-3}{12} \right) = \frac{1}{x+1} \frac{9}{12}$$

$$\text{or } \frac{7}{x} = \frac{9}{x+1}, \text{ or } 7x+7=9x, \text{ or } x=\frac{7}{2}=3\frac{1}{2}$$

The required rate is $3\frac{1}{2}$ miles and $4\frac{1}{2}$ miles per hour

10 Let x be the required number of bushels

Then $(\pounds 1 \ 15s) - \{ \pounds 80 + (\pounds 1 \ 5s)x \}$ is the number of acres in his farm

$$\text{Also } (\pounds 2) - \{ \pounds 80 + (\pounds 1 \ 10s)x \}$$

$$\text{We get } 1\frac{1}{4} - (80 + 1\frac{1}{4}x) = 2 - (80 + 1\frac{1}{2}x)$$

$$\text{or } \frac{7}{4(80 + \frac{1}{2}x)} = \frac{2}{80 + \frac{1}{2}x}$$

$$\text{or } 560 + 2\frac{1}{2}x = 640 + 10x$$

$$\text{or } \frac{3}{2}x = 80, \text{ or } x = 160$$

The required number of bushels = 160

11 Let x pounds be the value of the livery $(8+x)$ pounds is his yearly pay
 $\frac{7}{12}(8+x)$ pounds is his 7 months' pay

Again $(\pounds 2 \ 3s \ 4d + \pounds 1)$ is his 7 months' pay

$$\text{We get } \frac{7}{12}(8+x) = 2\frac{1}{3} + 1$$

$$\text{or } 56 + 7x = 26 + 12x$$

$$\text{or } 5x = 30$$

$$x = 6,$$

the required value = $\pounds 6$

12 Let x be the required number of leaps of the greyhound

But in that time the hare took $\frac{4x}{3}$ leaps Also x leaps of the hound are as much as $\frac{3x}{2}$ leaps of the hare

$$\text{We get } 50 + \frac{4x}{3} = \frac{3x}{2}$$

$$\text{or } 300 + 8x = 9x$$

$$\text{or } x = 300$$

The required number of leaps of the hound = 300

13 Let τ be the number of leaps made by the hound Then the number of leaps made by the hare is $\frac{5\tau}{4}$ Again τ leaps of the hound are as much as $\frac{4\tau}{3}$ leaps of the hare

$$\text{We get } \frac{4\tau}{3} = \frac{5\tau}{4} + \frac{4}{3}(60)$$

$$\text{or } 16\tau = 15\tau + 960$$

$$\text{or } \tau = 960$$

$$\text{and } \frac{5}{4}\tau = \frac{5}{4}(960) = 1200$$

The number of leaps made by the hound = 960 and that made by the hare = 1200-

14 Let x be the required number of strokes pulled by the Caius The number of strokes pulled by the Johnious is $\frac{4x}{3}$ But the τ strokes of the Caius are equivalent to $\frac{3\tau}{2}$ of the Johnious

$$\text{We get } \frac{3\tau}{2} = 30 + \frac{4x}{3}$$

$$\text{or } 9x = 180 + 8x$$

$$\text{or } x = 180$$

The required number of strokes = 180

15 Let x be the number of shillings in the purse Then A took $\{2 + \frac{1}{6}(\tau - 2)\}$ shillings or $(\frac{\tau}{6} + \frac{5}{3})$ shillings and B took $\{3 + \frac{1}{6}(x - \frac{x}{6} - \frac{5}{3} - 3)\}$ shillings or $(\frac{5x}{36} + \frac{20}{9})$ shillings.

$$\text{We get } \frac{x}{6} + \frac{5}{3} = \frac{5x}{36} + \frac{20}{9} -$$

$$\text{or } 6x + 60 = 5x + 80$$

$$\text{or } x = 20,$$

$$\text{and } \frac{x}{6} + \frac{5}{3} = \frac{20}{6} + \frac{5}{3} = \frac{30}{6} = 5$$

There were 20 shillings in the purse and 5 shillings were taken by each of A and B

16 Let x be the required number of crew. Then there were 602 pounds of biscuit. Evidently after 20 days there remained 402 pounds of biscuits. Again for the remaining $(40+21)$ days or 64 days the remaining $(x-5)$ men were supplied with $\{21-5, 64\}$ pounds of biscuit.

$$\text{We get } 402 = \frac{5 \times 64(-5)}{7}$$

$$\text{or } 280x = 320x - 1600$$

$$\text{or } 40x = 1600$$

$$\text{or } x = 40$$

The required number of crew is 40

17 Let x lbs be the weight of gold in 106 lbs of the mass, then the weight of silver there is $(106-x)$ lbs. Again x lbs of gold will weigh $\frac{18}{19}x$ lbs in water and $(106-x)$ lbs of silver will weigh $\frac{9(106-x)}{10}$ lbs in water

$$\text{We get } \frac{18x}{19} + \frac{9(106-x)}{10} = 99$$

$$\text{or } 180x + 171(106-x) = 99 \times 190$$

$$\text{or } 180x + 18126 - 171x = 18810$$

$$\text{or } 9x = 684, \text{ or } x = 76,$$

$$\text{and } 106-x = 106-76 = 30$$

There were 76 lbs of gold and 30 lbs of silver

18 Let x hours be the time of his going with the current, then $(10-x)$ hours is the time of his returning. The rates of his going with and against the current are $\frac{20}{x}$ miles and $\frac{20}{10-x}$ miles per hour respectively.

We get $\frac{20}{x} = \frac{3}{2} \frac{20}{10-x}$

or $\frac{1}{x} = \frac{3}{2(10-x)}$

or $20-2x=3x$

or $5x=20$, or $x=4$, and $10-x=6$

The time of going with the current is 4 hours and the time of returning is 6 hours

19 Let x years be the required age of the son Then the father's age is $2x$ years Also the father's age is

$$\frac{2x}{6} + \frac{2x}{12} + \frac{2x}{7} + 5 + 1 + 1$$

We get $\frac{2x}{6} + \frac{2x}{12} + \frac{2x}{7} + 1 + 9 = 2x$

or $14x + 7x + 12x + 42x + 378 = 84x$

or $9x = 378$, $x = 42$

The required age is 42 years

20 Let x oz. be taken from the first bar and then $(20-x)$ oz. is taken from the second bar Now x oz. of the first bar contains $(x \times \frac{1}{10})$ oz. of silver and $(20-x)$ oz. of the second bar contains $\{(20-x) \times \frac{2}{30}\}$ oz. of silver

We get $\frac{1}{10}x + \frac{2}{30}(20-x) = 10$

or $\frac{1}{10}x + \frac{2}{3}(20-x) = 10$

or $7x + 80 - 4x = 100$

or $3x = 20$

or $x = \frac{20}{3} = 6\frac{2}{3}$ and $20-x = 13\frac{1}{3}$

$6\frac{2}{3}$ oz. from the first bar and $13\frac{1}{3}$ oz. from the second bar should be taken

21 Let the greater sum be $\pounds x$, then the smaller is $(\pounds 607$ is $8d - \pounds x)$

The interest on the greater sum for 2 years at $3\frac{1}{2}\%$ p.c. is $3\frac{1}{2} \times 2 \times \frac{\pounds x}{100}$ or $\pounds \frac{7x}{100}$ and that on the smaller for $2\frac{1}{2}$ years at $3\frac{1}{2}\%$ p.c. is $\frac{5}{2} \times \frac{1}{2} \left(\frac{\pounds 607 \text{ is } 8d - \pounds x}{100} \right)$ or $\frac{5}{8} \left(\frac{\pounds 607 \text{ is } 8d - \pounds x}{100} \right)$

$$\text{We get } \frac{7x}{100} = \frac{65}{800} \left(607 \frac{20}{240} - x \right) + 18 \frac{16}{20}$$

$$\text{or } \frac{7x}{100} = \frac{65}{800} \left(607 \frac{1}{12} - x \right) - 18 \frac{4}{5}$$

$$\text{or } \frac{7x}{100} = \frac{65}{800} \left(7285 - 12x \right) + \frac{94}{5}$$

$$\text{or } 56 \times 12x = 65 \times 7285 - 65 \times 12x + 94 \times 12 \times 8 \times 20$$

$$\text{or } 12x(56+65) = 5 \times 47(13 \times 155 + 2 \times 96 \times 4)$$

$$\text{or } 12x \times 121 = 235(2015 + 768)$$

$$= 235 \times 2783$$

$$\text{or } 12x = 235 \times 23 = 5405,$$

$$x = £450 \frac{5}{12} = £450 \text{ } 8s \text{ } 4d$$

$$\text{The greater sum} = £450 \text{ } 8s \text{ } 4d,$$

$$\text{and the other sum} = £156 \text{ } 13s \text{ } 4d$$

22 Let there be x pice in each group

Each man of the 3rd group obtained $\frac{x+5}{4}$ pice and

each man of the 4th group obtained $\frac{x+9}{5}$ pice,

$$\text{we have } \frac{x+5}{4} = \frac{x+9}{5}$$

$$\text{or } 5x+25=4x+36$$

$$\text{or } x=11,$$

there were 11 pice in each group

Each man of the 1st set obtained

$$\left(\frac{11+1}{2} = \right) 6 \text{ pice,}$$

each of the 2nd set $\left(\frac{11+4}{3} = \right) 5$ pice,

and each of the 3rd and 4th sets $\left(\frac{11+5}{4} = \right) 4$ pice

23 Let there be x guineas in the parcel

The weight of the parcel $\frac{4}{15}(x-9)$ oz,

The weight of each guinea $= \frac{4(x-9)}{15 \times 2}$ oz

Again the weight of $\left(\frac{x}{2} - 10\frac{1}{2}\right)$ guineas

$$\text{was } \left\{ \frac{2}{15} \times \left(\frac{x-21}{2} \right) - \frac{1}{3} \right\} \text{ oz. or } \frac{2}{3} \left(\frac{x-21}{10} - 1 \right) \text{ oz.}$$

∴ The weight of each guinea

$$= \left\{ \frac{(x+11)}{30} - \frac{x-21}{2} \right\} \text{ oz.}$$

$$= \frac{(x-11)}{15(x-21)} \text{ oz.}$$

$$\therefore \text{ We get } \frac{(x+11)}{15(x-21)} = \frac{(x-9)}{15x}$$

$$\text{or } \frac{x+11}{x-21} = \frac{x-9}{x}$$

$$\text{or } x^2 - 11x = x^2 - 12x - 189$$

$$\text{or } x = 189$$

There were 189 guineas

24. Let x oz. be the weight of the 1st wrought plate sold.

Then price of x oz. of unwrought plate = £10 - £3. 15s

$$= £6. 5s = £\frac{25}{4}$$

∴ the price of 8 oz. of that plate

$$= £\frac{25}{4} \times \frac{8}{x} = £\frac{50}{x}$$

Again the price of 12 oz. of wrought plate = £ $\frac{12 \times 10}{x}$

∴ The price of 8 oz. of unwrought plate

$$= £ \left(\frac{120}{x} - 2\frac{2}{5} \right)$$

$$\therefore \text{ We get } \frac{50}{x} = \frac{120}{x} - \frac{14}{5}$$

$$\text{or } -\frac{70}{x} = -\frac{14}{5}$$

$$\text{or } 14x = 350$$

$$\text{or } x = 25$$

∴ The weight of the first plate sold is 25 oz. and the price of wrought plate per oz. = £ $\frac{10}{25}$ = 8s

25 Let $2s$ be the charge for the luggage allowed free. Then if all the luggage belonged to one man, he would have been charged $(xs + 2s \text{ 10d} + 7s \text{ 6d})$

$$\text{or } (xs + 10s \text{ 4d}) \quad \text{or } \left(x + \frac{31}{3}\right)s,$$

$$\text{we get } \left(x + \frac{31}{3}\right)s = 14s \text{ 6d} = \frac{29}{2}s$$

$$\text{or } x + \frac{31}{3} = \frac{29}{2}$$

$$\text{or } x = \frac{29}{2} - \frac{31}{3} = \frac{87 - 62}{6} = \frac{25}{6} = 4\frac{1}{6}$$

The charge for the luggage allowed free = $4\frac{1}{6}s = 4s \text{ 2d}$

If none had been allowed free the passengers would have been charged $\{(2s \text{ 10d}) + (4s \text{ 2d})\}$ or $7s$, and $\{(7s \text{ 6d}) + (4s \text{ 2d})\}$ or $11s \text{ 8d}$ respectively

26 Let x be the required number of bundles. Then the buying price of the two kinds of hay = $\left(\frac{5x}{1000} + \frac{5600 \times 6}{1000}\right) \text{Rs}$

$$= \left(\frac{x}{200} + \frac{168}{5}\right) \text{Rs}$$

The selling price must be

$$\frac{120}{100} \left(\frac{x}{200} + \frac{168}{5}\right) \text{Rs to have a profit of } 20\% \text{ c}$$

$$\text{We get } \frac{11}{10} \times \frac{x + 5600}{100} = \frac{120}{100} \left(\frac{x}{200} + \frac{168}{5}\right)$$

$$\text{or } \frac{11x + 61600}{1600} = \frac{120}{100} \times \frac{x + 6720}{200}$$

$$\text{or } \frac{11x + 61600}{8} = \frac{6(x + 6720)}{5}$$

$$\text{or } 55x + 308000 = 48x + 322560$$

$$\text{or } 7x = 14560,$$

$$x = 2080,$$

the required number of bundles = 2080

27 Let xd be the required selling price of each orange. The buying price of 3 of the 1st kind and one of the 2nd kind

is $(2 + \frac{1}{2}) = \frac{5}{2}d$ the selling price of those four must be $\frac{120 \times \frac{5}{2}}{100 \times \frac{5}{2}}d$ or $3d$

We get $4x = 3$, $x = \frac{3}{4}$,
the required price is $\frac{3}{4}d$ each

Let x be the number bought,
the selling price of x oranges is

$$\frac{120}{20}(64)d \text{ or } (6 \times 64)d$$

$$x = \frac{6 \times 64}{\frac{3}{4}} = 6 \times 64 \times \frac{4}{3} = 512,$$

the required number is 512

28 Let x be the required number of coins. The new coins were equivalent to $(1 - \frac{1}{5}) = \frac{4}{5}$ of the original coins. With x original coins he made $\frac{5}{4}x$ light coins. If he could pass all these coins his gain would have been equivalent to $(\frac{5x}{4} - x) = \frac{x}{4}$ original coins or $\frac{1}{4}(\frac{5x}{4})$ light coins. $\{\frac{1}{3}(\frac{5x}{4}) - 1\}$ light coins were seized, and his original gain was equivalent to $\frac{2}{3}\{\frac{1}{4}(\frac{5x}{4})\}$ light coins, and by passing the one light coin with which he decamped his gain will be $\frac{1}{4}$ of a light coin more.

$$\text{We get } \left\{ \frac{1}{3} \left(\frac{5x}{4} \right) - 1 \right\} - \frac{2}{3} \left\{ \frac{1}{4} \left(\frac{5x}{4} \right) \right\} - \frac{1}{4}$$

$$= \frac{1}{3} \left\{ \frac{1}{4} \left(\frac{5x}{4} \right) \right\}$$

$$\text{or } \frac{5x}{12} - 1 - \frac{5x}{24} - \frac{1}{4} = \frac{5x}{48}$$

$$\text{or } 20x - 48 - 10x - 12 = 5x$$

$$\text{or } 5x = 60$$

$$x = 12,$$

there were at first 12 coins

29 Let x be the digit in the unit's place. Then the required number $= 100(r+2) + 10(r+1) + x$ and the number formed by inverting the digits $= 100x + 10(x+1) + (r+2)$, we get

$$\{100(r+2) + 10(r+1) + x\} - \frac{1}{4}\{100x + 10(x+1) + (r+2)\}$$

$$= 36\{(r+2) + (x+1) + x\}$$

$$\begin{aligned} \text{or } 100x + 200 + 10x + 10 + 7 - \frac{1}{4}(100x + 10x + 10 + x + 2) \\ = 108(x + 1) \end{aligned}$$

$$\begin{aligned} \text{or } 400x + 800 + 40x + 40 + 4x - 100x - 10x - 10 - x - 2 \\ = 432x + 432 \end{aligned}$$

$$\text{or } 444x - 111x - 432x = 432 - 840 + 12$$

$$\text{or } -99x = -396, \quad x = 4$$

The required number = 654

30 Let x be the number of men in front of the square, the required number of men = $x^2 + 60$

Again the number of men in the new complete column will be $(x + 5)(x - 3)$, we get

$$\begin{aligned} x^2 + 60 &= (x + 5)(x - 3) - 1 \\ &= x^2 + 2x - 15 - 1 \end{aligned}$$

$$\text{or } 60 = 2x - 16$$

$$\text{or } 2x = 76 \quad x = 38,$$

$$\text{and } x^2 + 60 = 1444 + 60 = 1504$$

The required number of men = 1504

31 Let x be the number of men in front of the hollow square

Then the total number of men in the regiment

$$= x^2 - \{x - 2(10)\}^2 = 40x - 400,$$

$$\text{we get } 40x - 400 = 2800$$

$$\text{or } 40x = 3200, \quad x = 80,$$

the number of men in front of the hollow square
= 80

32 Let x be the number of men in front of the 1st square

Then the number of men in front of the 2nd square = $(x - 19)$

From the 1st square we get the number of men = $x^2 - (x - 8)^2$
and from the 2nd square it becomes = $(x - 19)^2 - (x - 19 - 16)^2$,

$$\text{we get } x^2 - (x - 8)^2 = (x - 19)^2 - (x - 35)^2$$

$$\text{or } (2x - 8)8 = 16(2x - 54)$$

$$\text{or } x - 4 = 2x - 54$$

$$\text{or } -x = -50, \quad x = 50,$$

the required number of men

$$= (50)^2 - (42)^2 = 92 \times 8 = 736$$

33 Let x be the number of men in front of the 1st column

The number of men in the depth of the column $= (x + 5)$,
the total number of men $= x(x + 5)$

Again from the 2nd column we get the number to be equal to $5(x + 845)$,

$$x(x + 5) = 5(x + 845)$$

$$\text{or} \quad x^2 + 5x = 5x + 845 \times 5$$

$$\text{or} \quad x^2 = 845 \times 5$$

$$\text{or} \quad x = 65$$

The required number of men $65(65 + 5) = 65 \times 70 = 4550$

Exercise 70.

1 From the 1st equation $x = 14 - 4y$, and substituting this value of x in the 2nd equation we get $7(14 - 4y) - 3y = 5$

$$\text{or} \quad 98 - 28y - 3y = 5$$

$$\text{or} \quad 31y = 93, \quad y = 3,$$

$$\text{and} \quad x = 14 - 4y = 14 - 12 = 2,$$

$$\left. \begin{array}{l} x = 2 \\ y = 3 \end{array} \right\}$$

2 From the 1st equation $x = \frac{9 \times 8y}{5}$, and substituting this value of x in the 2nd equation we get $13\left(\frac{9 + 8y}{5}\right) + 7y = 79$

$$\text{or} \quad 117 + 104y + 35y = 395$$

$$\text{or} \quad 139y = 278, \quad y = 2,$$

$$\text{and} \quad x = \frac{9 + 8y}{5} = \frac{9 + 16}{5} = 5,$$

$$\left. \begin{array}{l} x = 5 \\ y = 2 \end{array} \right\}$$

3 From the 1st equation $x = \frac{32 - 3y}{2}$, and substituting this value of x in the 2nd equation we get $11y - 9\left(\frac{32 - 3y}{2}\right) = 3$

$$\text{or} \quad 22y - 288 + 27y = 6$$

$$\text{or} \quad 49y = 294, \quad y = 6,$$

$$\text{and} \quad x = \frac{32 - 3y}{2} = \frac{32 - 18}{2} = 7,$$

$$\left. \begin{array}{l} x = 7 \\ y = 6 \end{array} \right\}$$

4 From the 1st equation $x = \frac{8+4y}{9}$, and substituting this value of x in the 2nd equation we get $13\left(\frac{8+4y}{9}\right) + 7y = 101$

$$\text{or} \quad 104 + 52y + 63y = 909$$

$$\text{or} \quad 115y = 805, \quad y = 7,$$

$$\text{and } x = \frac{8+4y}{9} = \frac{8+28}{9} = 4,$$

$$\left. \begin{array}{l} x = 4 \\ y = 7 \end{array} \right\}$$

5 From the 1st equation $r = b - ay$, and substituting this value of x in the 2nd equation we get

$$a(b - ay) - by = c$$

$$\text{or} \quad ab - a^2y - by = c$$

$$\text{or} \quad y(a^2 + b) = ab - c, \quad y = \frac{ab - c}{a^2 + b},$$

$$\text{and } x = b - a\left(\frac{ab - c}{a^2 + b}\right) = \frac{a^2b + b^2 - a^2b + ac}{a^2 + b} = \frac{b^2 + ac}{a^2 + b},$$

$$x = \frac{b^2 + ac}{a^2 + b} \text{ and } y = \frac{ab - c}{a^2 + b}$$

6 The equations become

$$\left. \begin{array}{l} 10x - (y - 3) = 20 \\ 9y + (x - 2) = 27 \end{array} \right\} \text{ From the 1st equation}$$

$$x = \frac{20 + y - 3}{10} = \frac{17 + y}{10},$$

and substituting this value of x in the 2nd we get

$$9y + \left(\frac{17 + y}{10} - 2\right) = 27$$

$$\text{or} \quad 90y + 17 + y - 20 = 270$$

$$\text{or} \quad 91y = 270 + 20 - 17 = 273, \quad y = 3,$$

$$\text{and } x = \frac{17 + y}{10} = \frac{20}{10} = 2,$$

$$\left. \begin{array}{l} x = 2 \\ y = 3 \end{array} \right\}$$

7 The equations become

$$\left. \begin{array}{l} 3(x + y) = 2(2x + 4) \\ 2(x - y) = 3(x - 24) \end{array} \right\}$$

$$\text{or } \begin{cases} 3x+3y-4x-8=0 \\ 2x-2y-3x+72=0 \end{cases}$$

$$\text{or } \begin{cases} 3y-x-8=0 \\ x+2y-72=0 \end{cases} \quad \text{From the 1st equation}$$

$x=3y-8$, and substituting this value of x in the 2nd equation we get $3y-8+2y-72=0$

$$\text{or } \begin{aligned} 5y &= 80, & y &= 16, \\ & \text{and } x = 3y - 8 = 48 - 8 = 40, \\ & & x &= 40 \\ & & y &= 16 \end{aligned}$$

8 The equations become

$$\begin{cases} 4(x-y)=3(y-1) \\ 4x-5y=7(x-7) \end{cases}$$

$$\text{or } \begin{cases} 4x-4y-3y+3=0 \\ 4x-5y-7x+49=0 \end{cases}$$

$$\text{or } \begin{cases} 4x-7y+3=0 \\ 3x+5y-49=0 \end{cases} \quad \text{From the 1st equation}$$

$x=\frac{7y-3}{4}$, and substituting this value of x in the 2nd equation

$$\text{we get } 3\left(\frac{7y-3}{4}\right)+5y-49=0$$

$$\text{or } 21y-9+20y-196=0$$

$$\text{or } 41y=205, \quad y=5,$$

$$\text{and } x = \frac{7y-3}{4} = \frac{35-3}{4} = 8, \quad \begin{cases} x=8 \\ y=5 \end{cases}$$

9 The equations become

$$\begin{cases} 2(3x-2y)-12=(2x-y) \\ 3(5x-4y)-18=2(4x-3y) \end{cases}$$

$$\text{or } \begin{cases} 6x-4y-12-2x+y=0 \\ 15x-12y-18-8x+6y=0 \end{cases}$$

$$\text{or } \begin{cases} 4x-3y-12=0 \\ 7x-6y-18=0 \end{cases} \quad \text{From the 1st equation}$$

$x=\frac{3y+12}{4}$, and substituting this value of x in the 2nd equation

$$\text{we get } 7\left(\frac{3y+12}{4}\right)-6y-18=0$$

$$\text{or } 21xy+84-24y-72=0$$

$$\text{or } -3y=-12, \quad y=4,$$

$$' \text{ and } r = \frac{3j + 12}{4} = \frac{12 + 12}{4} = 6,$$

$$\left. \begin{array}{l} x=6 \\ j'=4 \end{array} \right\}$$

10 The equations become

$$\left. \begin{array}{l} 2x + 3j' + 2x = 48 \\ 7j' - 3r - 2y = 22 \end{array} \right\}$$

$$\text{or } \left. \begin{array}{l} 4x + 3j' = 48 \\ -3x + 5j' = 22 \end{array} \right\} \text{ From the 1st equation}$$

$$x = \frac{48 - 3j'}{4}, \text{ and substituting this value of } x \text{ in the}$$

$$\text{2nd equation we get } -3\left(\frac{48 - 3j'}{4}\right) + 5j' = 22$$

$$\text{or } -144 + 9j' + 20j' = 88$$

$$\text{or } 29j' = 232, \quad j' = 8,$$

$$\text{and } r = \frac{48 - 3j'}{4} = \frac{48 - 24}{4} = 6,$$

$$\left. \begin{array}{l} x=6 \\ j'=8 \end{array} \right\}$$

Exercise 71

1 From the 1st equation, we have $5x = 9 + 3y$

$$r = \frac{9 + 3y}{5} \quad (1)$$

Again from the 2nd equation, we get $2x = 16 - 5y$,

$$r = \frac{16 - 5y}{2} \quad (2)$$

Hence, from (1) and (2) we have

$$\frac{9 + 3y}{5} = \frac{16 - 5y}{2}$$

$$\text{or } 18 + 6y = 80 - 25y$$

$$\text{or } 31y = 62$$

$$y = 2$$

$$\text{Hence, } r = \frac{9 + 3y}{5} = \frac{9 + 6}{5} = 3$$

$$\left. \begin{array}{l} r=3 \\ y=2 \end{array} \right\}$$

- 2 From the 1st equation, we have $3y = 1 + 4x$,

$$y = \frac{1+4x}{3} \quad (1)$$

From the 2nd equation, we have $4y = 18 - 3x$,

$$y = \frac{18-3x}{4} \quad (2)$$

Hence, from (1) and (2), we have

$$\frac{1+4x}{3} = \frac{18-3x}{4}$$

$$\text{or} \quad 4 + 16x = 54 - 9x$$

$$\text{or} \quad 25x = 50, \quad x = 2$$

$$\text{Hence,} \quad y = \frac{1+4x}{3} = \frac{1+8}{3} = 3$$

$$\left. \begin{array}{l} x=2 \\ y=3 \end{array} \right\}$$

- 3 From the 1st equation, we have $3x = 7 + 7y$,

$$x = \frac{7+7y}{3} \quad (1)$$

From the 2nd equation, we have $11x = 87 - 5y$,

$$x = \frac{87-5y}{11} \quad (2)$$

Hence, from (1) and (2), we have

$$\frac{7+7y}{3} = \frac{87-5y}{11}$$

$$\text{or} \quad 77 + 77y = 261 - 15y$$

$$\text{or} \quad 92y = 184, \quad y = 2$$

$$\text{Hence,} \quad x = \frac{7+7y}{3} = \frac{7+14}{3} = 7$$

$$\left. \begin{array}{l} x=7 \\ y=2 \end{array} \right\}$$

- 4 From the 1st equation, we have $3y + xy = 7x + 14$,

$$y = \frac{7x}{3} \quad (1)$$

From the 2nd equation, we have $5y = 4x + 9 + 14$,

$$y = \frac{4x+23}{5} \quad (2)$$

Hence, from (1) and (2), we have

$$\frac{7x}{3} = \frac{4x+23}{5}$$

$$\text{or} \quad 35x = 12x + 69$$

$$\text{or} \quad 23x = 69$$

$$x = 3$$

$$\text{Hence,} \quad y = \frac{7x}{3} = \frac{21}{3} = 7$$

$$\begin{array}{l} x=3 \\ y=7 \end{array}$$

5 From the 1st equation, we have $32x = 28 + 25y$,

$$x = \frac{28+25y}{32} \quad (1)$$

From the 2nd equation, we have $14x = 116 - 15y$,

$$x = \frac{116-15y}{14} \quad (2)$$

Hence, from (1) and (2), we have

$$\frac{28+25y}{32} = \frac{116-15y}{14}$$

$$\text{or} \quad 196 + 175y = 1856 - 240y$$

$$\text{or} \quad 415y = 1660,$$

$$y = 4$$

$$\text{Hence, } x = \frac{28+25y}{32} = \frac{28+100}{32} = 4.$$

$$x=4$$

$$y=4$$

6 From the 1st equation, we have

$$15x + 5y = 14x + 7y + 7 \quad [\text{Multiplying both sides by } 35]$$

$$\text{or} \quad x = 2y + 7 \quad (1)$$

From the 2nd equation, we have

$$40 - (x - y) = 30$$

$$\text{or} \quad x - y = 10$$

$$\text{or} \quad x = y + 10 \quad (2)$$

Hence, from (1) and (2), we have

$$2y + 7 = y + 10$$

$$y = 3,$$

from (1) we get $x=6+7=13$

$$\left. \begin{array}{l} x=13 \\ y=3 \end{array} \right\}$$

7 Multiplying both sides of the 1st equation by 12,

$$55x - 66y + 39x = 52y - 26$$

$$\text{or} \quad 94x = 118y - 26$$

$$\text{or} \quad 47x = 59y - 13,$$

$$x = \frac{59y - 13}{47} \quad (1)$$

Multiplying both sides of the 2nd equation by 12,

$$10x + 12y - 9x + 6y = 24y - 24$$

$$\text{or} \quad x + 18y = 24y - 24,$$

$$x = 6y - 24 \quad (2)$$

Hence, from (1) and (2) we have

$$\frac{59y - 13}{47} = 6y - 24$$

$$\text{or} \quad 59y - 13 = 282y - 1128$$

$$\text{or} \quad 223y = 1115,$$

$$y = 5,$$

$$\text{from (2) } x = 30 - 24 = 6$$

$$\left. \begin{array}{l} x=6 \\ y=5 \end{array} \right\}$$

8 Multiplying both sides of the first equation by 20,

$$40x - 5y - 15 = 140 + 12y - 81$$

$$\text{or} \quad 48x = 17y + 155$$

$$x = \frac{17y + 155}{48} \quad (1)$$

Multiplying both sides of the second equation by 6,

$$24y + 2x - 4 = 159 - 6y - 3$$

$$\text{or} \quad 2x = 160 - 30y$$

$$x = 80 - 15y \quad (2)$$

Hence, from (1) and (2), we get

$$\frac{17y + 155}{48} = 80 - 15y$$

$$\text{or} \quad 17y + 155 = 3840 - 720y$$

$$\begin{aligned} \text{or } 737y' &= 3685, & y' &= 5, \\ \text{from (2) } z &= 80 - 75 = 5, \\ & \begin{array}{l} z = 5 \\ y' = 5 \end{array} \end{aligned}$$

9 Multiplying both sides of the first equation by 24,

$$48z - 16y' + 8 = 77 + 18z - 12y'$$

$$\text{or } 30z = 4y' + 69$$

$$z = \frac{4y' + 69}{30} \quad (1)$$

Multiplying both sides of the second equation by 20,

$$80y' - 25 + 10z = 120 - 12z + 8y'$$

$$\text{or } 10z = 133 - 72y'$$

$$z = \frac{133 - 72y'}{10} \quad (2)$$

Hence, from (1) and (2), we get

$$\frac{4y' + 69}{30} = \frac{133 - 72y'}{10}$$

$$\begin{aligned} \text{or } 4y' + 69 &= 3(133 - 72y') \\ &= 399 - 216y' \end{aligned}$$

$$\text{or } 220y' = 330, \quad y' = \frac{3}{2} = 1\frac{1}{2},$$

$$\text{from (1) } z = \frac{6 + 69}{30} = \frac{75}{30} = 2\frac{1}{2},$$

$$\begin{array}{l} x = 2\frac{1}{2} \\ y' = 1\frac{1}{2} \end{array}$$

10 From the first equation, we have

$$12z(23 - z) - 2(2y' - x) = 40(23 - x) - (59 - 12z)(23 - x)$$

$$\text{or } 276x - 12x^2 - 4y' + 2z = 920 - 40z - (1357 - 335x + 12x^2)$$

$$\text{or } 278z - 4y' = 295z - 437$$

$$\text{or } 4y' = -17z + 437$$

$$y' = \frac{-17x + 437}{4} \quad (1)$$

From the second equation, we have

$$9y(x - 18) + 3(y - 3) = 60(1 - 18) - (1 - 18)(43 - 9y)$$

$$\text{or } 9zy' - 162y + 3y - 9 = 60x - 1080 - (431 - 9xy - 774 + 162y)$$

$$\text{or } 3y - 9 = 60x - 1080 - 43z + 774$$

$$\text{or } 3y = 17z - 297,$$

$$y' = \frac{172 - 297}{3} \quad (2)$$

From (1) and (2), we get

$$\frac{-17x + 437}{4} = \frac{172 - 297}{3}$$

$$\text{or } -51x + 1311 = 68x - 1188$$

$$\text{or } 119x = 2499$$

$$\text{or } 17x = 357, \quad x = 21,$$

$$\text{from (2) } y' = \frac{357 - 297}{3} = 20,$$

$$\left. \begin{array}{l} x = 21 \\ y' = 20 \end{array} \right\}$$

Exercise 72

1 Multiplying the 1st equation by 3, and the 2nd by 7, we have

$$21x - 15y = 33$$

$$\text{and } 21x + 14y = 91$$

Hence, by subtraction,

$$-29y = -58,$$

$$y = 2$$

Substituting this value of y in the 1st equation, we have

$$7x = 11 + 10 = 21, \quad x = 3$$

$$\text{Thus we have } \left. \begin{array}{l} x = 3 \\ y = 2 \end{array} \right\}$$

2 Multiplying the 1st equation by 11, and the 2nd by 6, we have

$$143x + 66y = 638$$

$$\text{and } 30x - 66y = 54$$

Hence, by addition, $173x = 692, \quad x = 4$

Substituting this value of x in the 2nd equation, we have

$$11y = 20 - 9 = 11$$

$$y = 1$$

$$\text{Thus we have } \left. \begin{array}{l} x = 4 \\ y = 1 \end{array} \right\}$$

3 Multiplying the first equation by 10, and the second by 9, we have

$$80x - 90y = 200$$

$$\text{and } 63x - 90y = 81$$

Hence, by subtraction, $17x = 119$, $x = 7$

Substituting this value of x in the second equation, we have

$$10y = 49 - 9 = 40, \quad y = 4$$

Thus we have $x = 7$, $y = 4$

4 Keeping the first equation as it is, and multiplying the second by 2, we have

$$25x - 14y = 8$$

$$\text{and } 24x + 14y = 90$$

Hence, by addition, $49x = 98$, $x = 2$

Substituting this value of x in the second equation, we have

$$7y = 45 - 24 = 21, \quad y = 3$$

Thus we have $x = 2$, $y = 3$

5 Multiplying the 1st equation by 2, and the 2nd by 3, we have $24x + 22y = 140$,

$$\text{and } 24x - 31y = 54$$

Hence, by subtraction, $43y = 86$, $y = 2$

Substituting this value of y in the 2nd equation,

$$\text{we have } 8x = 18 + 14 = 32, \quad x = 4$$

Thus we have $x = 4$, $y = 2$

6 Multiplying the 1st equation by 3, and the 2nd by 2, we have $39x - 42y = 66$

$$\text{and } 34x - 42y = 36$$

Hence, by subtraction, $5x = 30$, $x = 6$

Substituting this value of x in the 1st equation,

$$\text{we have } 14y = 78 - 22 = 56, \quad y = 4$$

Thus we have $x = 6$, $y = 4$

7 Multiplying the first equation by 3, and second by 4, we have

$$84x - 45y = 123$$

$$\text{and } 84x + 52y = 220$$

Hence, by subtraction, $-97y = -97$, $y = 1$

Substituting this value of y in the second equation, we have

$$21x = 55 - 13 = 42, \quad x = 2$$

Thus we have $x = 2$, $y = 1$

8 Multiplying the first equation by 3, and the second by 2, we have

$$57x + 72y = 102$$

$$\text{and } 46x + 72y = 124$$

Hence, by subtraction, $11x = -22$, $x = -2$

Substituting this value of x in the first equation, we have

$$24y = 34 - (-38) = 72, \quad y = 3$$

Thus we have $x = -2$, $y = 3$

9 Multiplying the 1st equation by 3, and the 2nd by 2, we have

$$141x - 168y = 369$$

$$\text{and } 50x + 168y = 586$$

Hence, by addition, $191x = 955$,

$$x = 5$$

Substituting this value of x in the 1st equation, we have

$$56y = 235 - 123 = 112, \quad y = 2$$

Thus we have $x = 5$, $y = 2$

10 Multiplying the 1st equation by 4, and the 2nd by 3, we have

$$204x - 4y = 12$$

$$\text{and } 204x + 69y = 411$$

Hence, by subtraction,

$$-133y = -399, \quad y = 3$$

Substituting this value of y in the 1st equation, we have

$$51x = 3 + 48 = 51, \quad x = 1$$

Thus we have $x = 1$, $y = 3$

11 Multiplying the 1st equation by 3, and the 2nd by 4, we have

$$156x - 27y = 102$$

$$\text{and } 156x + 56y = 268$$

Hence, by subtraction, $-83y = -166$,

$$y = 2$$

Substituting this value of y in the 1st equation, we have

$$52x = 34 + 18 = 52, \quad x = 1$$

Thus we have $x = 1, y = 2$

12 Multiplying the 1st equation by 2, and the 2nd by 5, we have

$$24x + 170y = -98$$

$$\text{and } 95x - 170y = 455$$

Hence, by addition $119x = 357, \quad x = 3$

Substituting this value of x in the 2nd equation, we have

$$34y = 57 - 91 = -34, \quad y = -1$$

Thus we have $x = 3, y = -1$

13 Multiplying the 1st equation by 7, and the 2nd by 5, we have

$$455x - 98y = 63$$

$$\text{and } 455x - 75y = 155$$

Hence, by subtraction, $-23y = -92, \quad y = 4$

Substituting this value of y in the 2nd equation, we have

$$91x = 31 + 60 = 91, \quad x = 1$$

Thus $x = 1, y = 4$

14 Multiplying the 1st equation by 3, and the 2nd equation by 2, we have

$$45x + 138y = 51$$

$$\text{and } 26x + 138y = 146$$

Hence, by subtraction, $19x = -95, \quad x = -5$

Substituting this value of x in the 1st equation, we have

$$46y = 17 - (-75) = 92, \quad y = 2$$

Thus we have $x = -5, y = 2$

15 Multiplying the 1st equation by 5, and the 2nd by 3, we have

$$70x + 405y = 265$$

$$\text{and } 51x + 405y = 303$$

Hence, by subtraction, $19x = -38, \quad x = -2$

Substituting this value of x in the 1st equation, we have

$$81y = 53 - (-28) = 81, \quad y = 1$$

Thus we have $x = -2, y = 1$

16 Multiplying the 1st equation by 5, and the 2nd by 11, we have

$$25x + 55y = 730$$

$$\text{and } 121x + 55y = 1210$$

Hence, by subtraction, $-96x = -480$, $x = 5$

Substituting this value of x in the 1st equation, we have

$$11y = 146 - 25 = 121, \quad y = 11$$

Thus we have $x = 5$, $y = 11$

17 Multiplying the 1st equation by b , and keeping the 2nd as it is, we have

$$abx + b^2y = bc$$

$$\text{and } a^2x + b^2y = c^2$$

Hence, by subtraction, $x(ab - a^2) = bc - c^2$,

$$x = \frac{bc - c^2}{ab - a^2}$$

Substituting this value of x in the 1st equation, we have

$$\frac{a(bc - c^2)}{ab - a^2} + by = c$$

or

$$by = c - \frac{a(bc - c^2)}{ab - a^2}$$

$$= \frac{abc - a^2c - abc + ac^2}{ab - a^2}$$

$$= \frac{c^2 - ac}{b - a}$$

$$y = \frac{ac - c^2}{ab - b^2}$$

Thus we have

$$x = \frac{bc - c^2}{ab - a^2}, \quad y = \frac{ac - c^2}{ab - b^2}$$

18 Multiplying the 1st equation by 4, and the second by 126, we have

$$2x + 2y + 3x - 5y = 8$$

$$\text{or } 5x - 3y = 8 \quad (1)$$

$$\text{and } 9x + 7y = 126 \quad (2)$$

Now by multiplying (1) by 7, and (2) by 3, we have

$$35x - 21y = 56$$

$$\text{and } 27x + 21y = 378$$

Hence, by addition, we have $62x = 434$,

$$x = 7$$

Substituting this value of x in the 2nd equation,

$$\text{we have } \frac{y}{18} = 1 - \frac{1}{2} = \frac{1}{2},$$

$$y = 9$$

Thus we have

$$x = 7, y = 9$$

19 From the 1st equation, we have

$$4x + 5y = 40 \text{ or } -40y$$

$$\text{or } -36x + 45y = 0$$

$$\text{or } 4x - 5y = 0. \quad (1)$$

From the 2nd equation, we have

$$\frac{2x - y + 6y}{3} = \frac{1}{2}$$

$$\text{or } 4x + 10y = 3 \quad (2)$$

Hence, from (1) and (2), we have

$$15y = 3,$$

$$y = \frac{1}{5}$$

Substituting this value of y in (1), we have

$$4x = 1, \quad x = \frac{1}{4}$$

Thus we have $x = \frac{1}{4}, y = \frac{1}{5}$

20 From the 1st equation, we have

$$\frac{4x - 3y - 7}{5} = \frac{9x - 4y - 25}{30}$$

$$\text{or } 4x - 3y - 7 = \frac{9x - 4y - 25}{6}$$

$$\text{or } 24x - 18y - 42 = 9x - 4y - 25$$

$$\text{or } 15x - 14y = 17 \quad (1)$$

From the 2nd equation, we have

$$\frac{20y - 20 + 30x - 9y}{60} = \frac{2y - 2x + 5x + 33}{30}$$

$$\text{or } \frac{11y + 30x - 20}{2} = 2y + 3x + 33$$

$$\text{or } 11y + 30x - 20 = 4y + 6x + 66$$

$$\text{or } 24x + 7y = 86 \quad (2)$$

Multiplying (2) by 2, we have

$$\text{also, } \begin{array}{l} 48x + 14y = 172 \\ 15x - 14y = 17 \end{array} \}$$

Hence, by addition, $63x = 189$,

$$x = 3$$

Substituting this value of x in (1), we have

$$14y = 45 - 17 = 28,$$

$$y = 2$$

Thus we have $x = 3, y = 2$

21 From the 1st equation, we have

$$\frac{(2x-3)(6x+9)+2(3x+5y)}{4(2x-3)} = \frac{13+2(3x+4)}{4}$$

$$\text{or } (2x-3)(6x+9)+2(3x+5y)=13(2x-3) \\ + (2x-3)(6x+8)$$

$$\text{or } 12x^2 - 27 + 6x + 10y = 26x - 39 + 12x^2 - 2x - 24$$

$$\text{or } 18x - 10y = 36 \quad . \quad (1)$$

From the 2nd equation, we have

$$\frac{(8y+7)(2y-8)+10(6x-3y)}{10(2y-8)} = \frac{20+4y-9}{5}$$

$$\text{or } 16y^2 - 50y - 56 + 60x - 30y = (4y-16)(4y+11) \\ = 16y^2 - 20y - 176$$

$$\text{or } 60x - 60y = -120 \quad \text{or } 10x - 10y = -20 \quad (2)$$

Hence, from (1) and (2), we have

$$8x = 65, \quad x = 7$$

Substituting this value of x in (1), we have

$$10y = 126 - 36 = 90, \quad y = 9$$

Thus we have $x = 7, y = 9$

22 From the 1st equation, we have

$$\frac{12x - 20y - 2x + 8y + 33}{12} = \frac{6y + 4x + 3}{12}$$

$$\text{or } 12x - 20y - 2x + 8y + 33 = 6y + 4x + 3$$

$$\text{or } 6x - 18y = -30$$

$$\text{or } x - 3y = -5 \quad (1)$$

From the 2nd equation, we have

$$\frac{x}{2} + \frac{7y}{8} + \frac{14}{3} = \frac{40x}{3} - \frac{5y}{12} - 80$$

$$\text{or } \frac{12x + 21y + 112}{24} = \frac{160x - 5y - 960}{12}$$

$$\text{or } 12x + 21y + 112 = 320x - 10y - 1920$$

$$\text{or } 308x - 31y = 2032 \quad (2)$$

Multiplying (1) by 31 and (2) by 3, we have

$$31x - 93y = -155, \quad \text{and } 924x - 93y = 6096$$

Hence, by subtraction, $893x = 6251$,

$$x = 7$$

Substituting this value of x in (1), we have

$$3y = 7 + 5 = 12, \quad y = 4$$

$$\text{Thus we have } x = 7, \quad y = 4$$

23 From the 1st equation, we have

$$\frac{6x + 08y - 18x - 025}{25} = \frac{4x + 52 + 01y}{5}$$

$$\text{or } 42x + 08y - 025 = 2x + 26 + 005y$$

$$\text{or } 22x + 075y = 2625$$

$$\text{or } 88x + 3y = 103$$

$$\text{or } 88x + 30y = 1030 \quad (1)$$

From the 2nd equation, we have

$$\frac{2y + 5}{15} = \frac{7x - 1}{6}$$

$$\text{or } 12y + 30 = 105x - 15$$

$$\text{or } 24y + 60 = 21x - 30$$

$$\text{or } 21x - 24y = 90 \quad (2)$$

Multiplying (1) by 4, and (2) by 5, we have

$$352x + 120y = 4120$$

$$\text{and } 105x - 120y = 450$$

Hence, by addition, $457x = 4570$,

$$x = 10$$

Substituting this value of x in (2) we have

$$24y = 210 - 90 = 120,$$

$$y = 5$$

Thus we have $x = 10, \quad y = 5$

24 Multiplying the 1st equation by 3, and the 2nd by 4, we have

$$\frac{12}{x} + \frac{30}{y} = 6$$

and
$$\frac{12}{x} + \frac{8}{y} = \frac{19}{5}$$

Hence, by subtraction,
$$\frac{22}{y} = \frac{11}{5}$$

or
$$11y = 110, \quad y = 10$$

Substituting this value of y in the 1st equation, we have

$$\frac{4}{x} = 2 - \frac{10}{10} = 1,$$

$$x = 4$$

Thus we have $x = 4, \quad y = 10$

25 Multiplying the 1st equation by 5, and the 2nd by 2, we have

$$\frac{10}{x} + \frac{15}{y} = 10$$

and
$$\frac{10}{x} + \frac{20}{y} = 11\frac{2}{3}$$

Hence, by subtraction,
$$\frac{5}{y} = \frac{5}{3}, \quad y = 3$$

Substituting this value of y in the 1st equation, we have

$$\frac{2}{x} = 2 - \frac{3}{3} = 1,$$

$$x = 2$$

Thus we have $x = 2, \quad y = 3$

26 Multiplying the 1st equation by a , and the 2nd equation by b , we have

$$\frac{a^2}{x} + \frac{ab}{y} = am$$

and
$$\frac{b^2}{x} + \frac{ab}{y} = bn$$

Hence, by subtraction,
$$\frac{1}{x}(a^2 - b^2) = am - bn,$$

$$x = \frac{a^2 - b^2}{am - bn}$$

Substituting this value of r in the 1st equation, we have

$$\begin{aligned}\frac{b}{j} &= m - \frac{a'cr - br}{a^2 - t^2} \\ &= \frac{a^2n - b^2n - a^2m - abn}{a^2 - t^2} \\ &= \frac{t ar - cm}{a^2 - t^2}\end{aligned}$$

Or

$$\begin{aligned}\frac{1}{j} &= \frac{an - bn}{a^2 - t^2} \\ &= \frac{a^2 - b^2}{an - bn}\end{aligned}$$

Thus we have $x = \frac{a^2 - t^2}{an - bt}$ $y = \frac{a^2 - b^2}{an - bm}$

27 Multiplying the 1st equation by 3, and the 2nd. by 5, we have

$$\frac{1}{x} + \frac{3}{y} = 3$$

and

$$\frac{1}{x} + \frac{5}{y} = \frac{17}{3}$$

Hence, by subtraction,

$$\frac{1}{j} \left(\frac{3}{5} - \frac{5}{3} \right) = 3 - \frac{17}{3}$$

or $\frac{1}{j} \left(\frac{-16}{15} \right) = \frac{9-17}{3} = -\frac{8}{3}$

or $\frac{1}{j} \cdot \frac{2}{5} = 1, \quad y = \frac{2}{5}$

Substituting this value of y in the 1st equation, we have

$$\frac{1}{3x} = 1 - \frac{1}{2} = \frac{1}{2}$$

or $3x = 2, \quad x = \frac{2}{3}$

Thus we have $x = \frac{2}{3}, \quad y = \frac{2}{5}$

28 Multiplying the 1st equation by $\frac{2}{5}$ we have

$$\frac{6}{5} - \frac{2}{5x} = \frac{2}{5},$$

also $\frac{5}{21} - \frac{2}{5x} = 7$ (2nd equation)

Hence, by addition,

$$\frac{1}{y}\left(\frac{6}{5} + \frac{5}{2}\right) = 7\frac{2}{5} = \frac{37}{5}$$

or $\frac{1}{y}\left(\frac{37}{10}\right) = \frac{37}{5}$

or $\frac{1}{2y} = 1, \quad y = \frac{1}{2}$

Substituting this value of y in the 1st equation, we have

$$\frac{1}{x} = \frac{3}{\frac{1}{2}} - 1 = 5, \quad x = \frac{1}{5}$$

Thus we have $x = \frac{1}{5}, y = \frac{1}{2}$

29 Multiplying the 1st equation by $\frac{3}{4}$, we have

$$\frac{3x}{16} + \frac{3}{2y} = \frac{3}{2},$$

also, $\frac{2x}{5} + \frac{3}{2y} = \frac{47}{20}$

Hence, by subtraction,

$$x\left(\frac{3}{16} - \frac{2}{5}\right) = \left(\frac{3}{2} - \frac{47}{20}\right)$$

or $x\left(\frac{15-32}{80}\right) = \frac{60-94}{40}$

or $\frac{17x}{2} = 34, \quad x = 4$

Substituting this value of x in the 1st equation, we have

$$\frac{2}{y} = 2 - \frac{4}{4} = 1, \quad y = 2$$

Thus we have $x = 4, y = 2$

30 Multiplying the 1st equation by $\frac{1}{3}$, and the 2nd by $\frac{1}{5}$, we have

$$\frac{1}{15x} + \frac{y'}{27} = \frac{5}{3}$$

and $\frac{1}{15x} + \frac{y'}{10} = \frac{14}{5}$

Hence, by subtraction

$$j\left(\frac{1}{27} - \frac{1}{10}\right) = \frac{5}{3} - \frac{14}{5}$$

$$\text{or } j\left(\frac{10-27}{270}\right) = \frac{25-42}{15}$$

$$\text{or } j\frac{17}{270} = \frac{17}{15}$$

$$\text{or } 15j = 270, \quad j = 18$$

Substituting this value of j in the 1st equation, we have

$$\frac{1}{5i} = 5 - \frac{18}{9} = 3, \quad i = \frac{1}{15}$$

$$\text{Thus we have } i = \frac{1}{15}, j = 18$$

Exercise 73

$$\begin{array}{l} 1 \quad 2x+3j-8=0 \\ \quad 3x-4j+5=0 \end{array}$$

$$\text{Hence, } \frac{x}{3 \times 5 - (-8)(-4)} = \frac{j}{(-8)5 - 2 \times 5} = \frac{1}{2(-4) - 3 \times 3}$$

$$\text{or } \frac{x}{15-32} = \frac{j}{-24-10} = \frac{1}{-8-9}$$

$$\text{or } \frac{x}{-17} = \frac{j}{-34} = \frac{1}{-17}$$

$$x = \frac{-17}{-17} = 1, \text{ and } j = \frac{-34}{-17} = 2$$

Thus we have $x=1$, and $j=2$

$$\begin{array}{l} 2 \quad 3x-5j+9=0 \\ \quad 5x+2j-16=0 \end{array}$$

$$\text{Hence } \frac{x}{(-5)(-16)-9 \times 2} = \frac{j}{9 \times 5 - 3(-16)} = \frac{1}{3 \times 2 - 5(-5)}$$

$$\text{or } \frac{x}{80-18} = \frac{j}{45+48} = \frac{1}{6+25}$$

$$\text{or } \frac{x}{62} = \frac{j}{93} = \frac{1}{31}, \quad x = \frac{62}{31} = 2, \text{ and}$$

$$j = \frac{93}{31} = 3 \quad \text{Thus we have } x=2, \text{ and } j=3$$

$$\begin{cases} 4x - 5y + 8 = 0 \\ 2x - 3y + 6 = 0 \end{cases}$$

$$\text{Hence, } \frac{x}{(-5)6 - 8(-3)} = \frac{y}{8 \times 2 - 4 \times 6} \\ = \frac{1}{4(-3) - (-5)2}$$

$$\text{or } \frac{x}{-30 + 24} = \frac{y}{16 - 24} = \frac{1}{-12 + 10}$$

$$\text{or } \frac{x}{-6} = \frac{y}{-8} = \frac{1}{-2}$$

$$\text{Hence, } x = 3, \text{ and } y = 4$$

$$\begin{cases} -3x + 2y + 2 = 0 \\ 5x - 3y - 5 = 0 \end{cases}$$

$$\text{Hence, } \frac{x}{2(-5) - 2(-3)} = \frac{y}{2 \times 5 - (-3)(-5)} \\ = \frac{1}{(-3)(-3) - 2 \times 5}$$

$$\text{or } \frac{x}{-10 + 6} = \frac{y}{10 - 15} = \frac{1}{9 - 10}$$

$$\text{or } \frac{x}{-4} = \frac{y}{-5} = \frac{1}{-1}$$

$$\text{Hence, } x = 4, \text{ and } y = 5$$

$$\begin{cases} 6x - 7y + 12 = 0 \\ -7x + 4y + 11 = 0 \end{cases}$$

$$\text{Hence, } \frac{x}{(-7)11 - 12 \times 4} = \frac{y}{12(-7) - 6 \times 11} = \frac{1}{6 \times 4 - (-7)(-7)}$$

$$\text{or } \frac{x}{-77 - 48} = \frac{y}{-84 - 66} = \frac{1}{24 - 49}$$

$$\text{or } \frac{x}{-125} = \frac{y}{-150} = \frac{1}{-25}$$

$$\text{Hence, } x = 5, \text{ and } y = 6$$

$$\begin{cases} \text{From the 1st equation, } 7x - 8y + 14 = 0 \\ \text{and from the 2nd equation, } 5x - 3y - 9 = 0 \end{cases}$$

$$\text{Hence, } \frac{x}{(-8)(-9) - 14(-3)} = \frac{y}{14 \times 5 - 7(-9)} \\ = \frac{1}{7(-3) - 5(-8)}$$

$$\text{or} \quad \frac{x}{72+42} = \frac{y}{70+63} = \frac{1}{-21+40}$$

$$\text{or} \quad \frac{x}{114} = \frac{y}{133} = \frac{1}{19}$$

Hence, $x=6$ and $y=7$

$$\begin{array}{l} 7 \quad -6x+5y+2=0 \\ \quad 13x-9y-19=0 \end{array} \quad \text{(from the 2nd equation)}$$

$$\begin{aligned} \text{Hence,} \quad \frac{x}{5(-19)-2(-9)} &= \frac{y}{2 \times 13 - (-6)(-19)} \\ &= \frac{1}{(-6)(-9)-5 \times 13} \end{aligned}$$

$$\text{or} \quad \frac{x}{-95+18} = \frac{y}{26-114} = \frac{1}{54-65}$$

$$\text{or} \quad \frac{x}{-77} = \frac{y}{-88} = \frac{1}{-11}$$

Hence, $x=7$, and $y=8$

$$\begin{array}{l} 8 \quad -7x+5y-11=0 \\ \quad 8x-5y-19=0 \end{array} \quad \text{(from the 2nd equation)}$$

$$\begin{aligned} \text{Hence,} \quad \frac{x}{5(-19)-11(-5)} &= \frac{y}{11 \times 8 - (-7)(-19)} \\ &= \frac{1}{(-7)(-5)-8 \times 5} \end{aligned}$$

$$\text{or} \quad \frac{x}{-95+55} = \frac{y}{88-133} = \frac{1}{35-40}$$

$$\text{or} \quad \frac{x}{-40} = \frac{y}{-45} = \frac{1}{-5}$$

Hence, $x=8$, and $y=9$

$$\begin{array}{l} 9 \quad 4x-11y+6=0 \\ \quad 9x-13y-10=0 \end{array} \quad \text{(from the 2nd equation)}$$

$$\begin{aligned} \text{Hence} \quad \frac{x}{(-11)(-10)-6(-13)} &= \frac{y}{6 \times 9 - 4(-10)} \\ &= \frac{1}{4(-13)-(-11)9} \end{aligned}$$

$$\text{or} \quad \frac{x}{110+78} = \frac{y}{54+40} = \frac{1}{-52+99}$$

$$\text{or} \quad \frac{x}{188} = \frac{y}{94} = \frac{1}{47}$$

Hence, $x=4$, and $y=2$

- 10 From the 1st equation, $8x - 7y - 19 = 0$
and from the 2nd " $10x - 9y - 23 = 0$

$$\text{Hence, } \frac{x}{(-7)(-23) - (-19)(-9)} = \frac{y}{(-19)10 - 8(-23)}$$

$$= \frac{1}{8(-9) - (-7)10}$$

$$\text{or } \frac{x}{161 - 171} = \frac{y}{-190 + 184} = \frac{1}{-72 + 70}$$

$$\text{or } \frac{x}{-10} = \frac{y}{-6} = \frac{1}{-2}$$

Hence, $x = 5$, and $y = 3$

11. $-12x + 17y + 16 = 0$
 $9x - 13y - 11 = 0$ (from the 2nd equation)

$$\text{Hence, } \frac{x}{17(-11) - 16(-13)} = \frac{y}{16 \times 9 - (-12)(-11)}$$

$$= \frac{1}{(-12)(-13) - 17 \times 9}$$

$$\text{or } \frac{x}{-187 + 208} = \frac{y}{144 - 132} = \frac{1}{156 - 153}$$

$$\text{or } \frac{x}{21} = \frac{y}{12} = \frac{1}{3}$$

Hence $x = 7$, and $y = 4$

- 12 $14x - 11y + 18 = 0$
 $11x - 7y + 1 = 0$

$$\text{Hence, } \frac{x}{(-11)1 - 18(-7)} = \frac{y}{18 \times 11 - 14 \times 1}$$

$$= \frac{1}{14(-7) - (-11)11}$$

$$\text{or } \frac{x}{-11 + 126} = \frac{y}{198 - 14} = \frac{1}{-98 + 121}$$

$$\text{or } \frac{x}{115} = \frac{y}{184} = \frac{1}{23}$$

Hence $x = 5$, and $y = 8$

- 13 From the 2nd equation, we have

$$\frac{z}{2} = \frac{y}{3} = k \text{ (suppose),}$$

$$z = 2k, \text{ and } y = 3k$$

(A)

Substituting these values of x and y in the 1st equation, we have

$$34k - 21k = 52$$

$$\text{or } 13k = 52, \quad k = 4$$

Hence, from A, $x = 8$, and $y = 12$,

14 From the 2nd equation, we have

$$\frac{1}{3} = \frac{y'}{7} = k \text{ (suppose),}$$

$$x = 3', \text{ and } y = 7k \quad (A)$$

Substituting these values of x and y in the 1st equation, we have

$$27k + 35k = 124$$

$$\text{or } 62k = 124, \quad k = 2$$

Hence, from A, $x = 6$, and $y = 14$

15 From the 2nd equation, we have

$$\frac{2}{4} = \frac{y'}{9} = k \text{ (suppose),}$$

$$x = 4k, \text{ and } y = 9k \quad (A)$$

Substituting these values of x, y in the 1st equation, we have

$$60k + 63k = 246$$

$$\text{or } 123k = 246, \quad k = 2$$

Hence, from A, $x = 8$, and $y = 18$

16 From the 1st equation, we have

$$\frac{x}{8} = \frac{y'}{9} = k \text{ (suppose),}$$

$$x = 8k, \text{ and } y = 9k \quad (A)$$

Substituting these values of x, y in the 2nd equation, we have

$$80k + 207k = 287, \quad k = 1$$

Hence, from A, $x = 8$, and $y = 9$

17 From the first equation, we have $4x = 3y$

$$\text{or } \frac{x}{3} = \frac{y}{4} = k \text{ (suppose),}$$

$$x = 3k, \text{ and } y = 4k \quad (A)$$

Substituting the values of x, y in the 2nd equation, we have

$$21k - 44k + 92 = 0$$

$$\text{or } -23k = -92, \quad k = 4$$

Hence, from A, $x = 12$, and $y = 16$

Substituting these values of x, y in the 1st equation, we have

$$\frac{1}{2}(33k + 5k) + \frac{1}{2}(33k - 5k) = 59$$

$$\text{or} \quad 11k + 1^2k - 7k = 59$$

$$\text{or} \quad 4k = 59$$

$$k = 3$$

Hence, from A , $x = 90$ and $y = 15$

22 From the 1st equation, we have

$$4x + 5y = 40x - 40y$$

$$\text{or} \quad 36x = 45y$$

$$\text{or} \quad 4x = 5y$$

$$\text{or} \quad \frac{x}{5} = \frac{y}{4} = k \quad (\text{suppose}),$$

$$x = 5k, \text{ and } y = 4k \quad (A)$$

Substituting these values of x, y in the 2nd equation, we have

$$\frac{10k - 4k}{3} + 8k = 20,$$

$$\text{or} \quad 10k = 20, \quad k = 2$$

Hence, from A , $x = 10$, and $y = 8$

23 From the 1st equation we have

$$3y + xy = 7x + xy$$

$$\text{or} \quad 3y = 7x$$

$$\text{or} \quad \frac{y}{7} = \frac{x}{3} = k \quad (\text{suppose}), \quad (A)$$

$$x = 3k, \text{ and } y = 7k$$

Substituting these values of x, y in the 2nd equation, we have

$$12k + 9 = 35k - 14$$

$$\text{or} \quad 23k = 23, \quad k = 1$$

Hence from A , $x = 3$, and $y = 7$

24 From the 1st equation we have

$$4y - 6 = 2x + 2y$$

$$\text{or} \quad 2x - 2y + 6 = 0$$

$$\text{or} \quad x - y + 3 = 0 \quad (1)$$

From the 2nd equation, we have

$$8x - 5 = 9y - 9x$$

$$\text{or} \quad 17x - 9y - 5 = 0 \quad (2)$$

Hence, from (1) and (2),

$$\frac{x}{(-1)(-5)-3(-9)} = \frac{y}{3 \times 17 - 1(-5)}$$

$$= \frac{1}{1(-9) - (-1)17}$$

or $\frac{x}{5+27} = \frac{y}{51+5} = \frac{1}{-9+17}$

or $\frac{x}{32} = \frac{y}{56} = \frac{1}{8}$

Hence, $x=4$, and $y=7$

25 From the 1st equation, we have

$$xy + 5y + 7x + 35 = xy + y - 9x - 9 + 112$$

or $16x + 4y - 68 = 0$ (1)

From the 2nd equation, we have

$$2x - 3y + 9 = 0 \quad (2)$$

Hence, from (1) and (2),

$$\frac{x}{4 \times 9 - (-68)(-3)} = \frac{y}{(-68)2 - 16 \times 9}$$

$$= \frac{1}{16(-3) - 4 \times 2}$$

or $\frac{x}{36-204} = \frac{y}{-136-144} = \frac{1}{-48-8}$

or $\frac{x}{-168} = \frac{y}{-280} = \frac{1}{-56}$

Hence, $x=3$, and $y=5$

26 From the 1st and the 2nd equations, we have

$$\frac{x}{(-5)4 - (-7)2} = \frac{y}{2 \times 2 - 4 \times 4}$$

$$= \frac{z}{4(-7) - 2(-5)}$$

or $\frac{x}{-20+14} = \frac{y}{4-16} = \frac{z}{-28+10}$

or $\frac{x}{-6} = \frac{y}{-12} = \frac{z}{-18}$

or $x = \frac{y}{2} = \frac{z}{3} = k$ (suppose)

$$x=k, y=2k, z=3k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$\begin{aligned} k + 2k + 3k &= 6 \\ \text{or } 6k &= 6, \quad k = 1 \end{aligned}$$

Hence, from A , $x = 1, y = 2$ and $z = 3$

27 From the 1st and the 2nd equations, we have

$$\begin{aligned} \frac{x}{6 \times 6 - 4 \times 8} &= \frac{y}{8 \times 3 - 6 \times 5} = \frac{z}{5 \times 4 - 3 \times 6} \\ \text{or } \frac{x}{36 - 32} &= \frac{y}{24 - 30} = \frac{z}{20 - 18} \\ \text{or } \frac{x}{4} &= \frac{y}{-6} = \frac{z}{2} \\ \text{or } \frac{x}{2} &= \frac{y}{-3} = \frac{z}{1} = k \quad (\text{suppose}) \\ x &= 2k, \quad y = -3k, \quad z = k \end{aligned} \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$\begin{aligned} 2k - 15k + 16k &= 3 \\ \text{or } 3k &= 3, \quad k = 1 \end{aligned}$$

Hence, from A , $x = 2, y = -3, z = 1$

28 From the 1st two equations, we have

$$\begin{aligned} \frac{x}{(-7)7 - (-8)11} &= \frac{y}{11 \times 6 - 2 \times 7} = \frac{z}{2(-8) - 6(-7)} \\ \text{or } \frac{x}{-49 + 88} &= \frac{y}{66 - 14} = \frac{z}{-16 + 42} \\ \text{or } \frac{x}{39} &= \frac{y}{52} = \frac{z}{26} \\ \text{or } \frac{x}{3} &= \frac{y}{4} = \frac{z}{2} = k \quad (\text{suppose}) \\ x &= 3k, \quad y = 4k \text{ and } z = 2k \end{aligned} \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$9k + 15k + 10k = 35, \quad \text{or, } 35k = 35, \quad k = 1$$

Hence, from A , $x = 3, y = 4, z = 2$

29 From the 1st. two equations, we have

$$\frac{x}{3 \times 8 - (-7) \times (-8)} = \frac{y}{(-8) \times 5 - 8 \times 7} = \frac{z}{7(-7) - 5 \times 3}$$

$$\text{or} \quad \frac{x}{24 - 56} = \frac{y}{-40 - 56} = \frac{z}{-49 - 15}$$

$$\text{or} \quad \frac{x}{-32} = \frac{y}{-96} = \frac{z}{-64}$$

$$\text{or} \quad x = \frac{y}{3} = \frac{z}{2} = k \text{ (suppose)}$$

$$x = k, \quad y = 3k, \quad z = 2k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$3k + 15k - 14k = 64, \quad \text{or,} \quad 32k = 64 \\ k = 2$$

Hence, from A , $x = 2, \quad y = 6, \quad z = 4$

30 From the 1st. two equations, we have

$$\frac{x}{(-2)3 - (-8)1} = \frac{y}{1 \times 9 - 3 \times 1} = \frac{z}{1(-8) - 9(-2)}$$

$$\text{or} \quad \frac{x}{-6 - 8} = \frac{y}{9 - 3} = \frac{z}{-8 - 18}$$

$$\text{or} \quad \frac{x}{-14} = \frac{y}{6} = \frac{z}{-26}$$

$$\text{or} \quad x = \frac{y}{3} = \frac{z}{5} = k \text{ (suppose)}$$

$$x = k, \quad y = 3k, \quad z = 5k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$2k - 9k + 25k = 36 \quad \text{or,} \quad 36k = 36 \quad k = 1.$$

Hence, from A , $x = 1, \quad y = 3, \quad z = 5$

31. From the 1st. equation, we have

$$8x - 18y = 14z - 7z, \quad \text{or,} \quad 8x - 11y - 7z = 0. \quad (1)$$

From the 2nd equation, we have

$$7x - 14y = 8z - 8z, \quad \text{or} \quad 7x - 6y - 8z = 0 \quad (2)$$

Hence, from (1) and (2)

$$\frac{x}{4(-8) - 6(-7)} = \frac{y}{(-7)7 - (-8)8} = \frac{z}{8 \times 6 - 7 \times 4}$$

$$\text{or } \frac{x}{-32+42} = \frac{y}{-49+64} = \frac{z}{48-28}$$

$$\text{or } \frac{x}{10} = \frac{y}{15} = \frac{z}{20}$$

$$\text{or } \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = k \quad (\text{suppose})$$

$$x=2k, \quad y=3k, \quad z=4k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$6k + 12k + 20k = 38, \quad \text{or,} \quad 38k = 38, \quad k = 1$$

Hence, from A , $x=2, y=3, z=4$

32 From the 1st equation, we have

$$4x + 4y = 6z - 3y, \quad \text{or,} \quad 4x + 7y - 6z = 0 \quad (1)$$

From the 2nd equation, we have

$$5x - 10y = 6y - 9z, \quad \text{or,} \quad 5x - 16y + 9z = 0 \quad (2)$$

Hence, from (1) and (2),

$$\frac{x}{7 \times 9 - (-16)(-6)} = \frac{y}{(-6)5 - 9 \times 4} = \frac{z}{4(-16) - 5 \times 7}$$

$$\text{or } \frac{x}{63-96} = \frac{y}{-30-36} = \frac{z}{-64-35}$$

$$\text{or } \frac{x}{-33} = \frac{y}{-66} = \frac{z}{-99}$$

$$\text{or } \frac{x}{2} = \frac{y}{3} = \frac{z}{9} = k \quad (\text{suppose})$$

$$x=2k, \quad y=3k, \quad z=9k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$6(k-2) + 7(2k-3) + 8(3k-4) = 67$$

$$\text{or } 6k - 12 + 14k - 21 + 24k - 32 = 67$$

$$\text{or } 44k = 67 + 12 + 21 + 32 = 132, \quad k = 3$$

Hence, from A , $x=3, y=6, z=9$

33 From the 1st equation, we have

$$5x - 2y = 0$$

From the 2nd equation, we have

$$7y - 5z = 0$$

Hence,

$$\frac{x}{(-2)(-5)-7 \times 0} = \frac{y}{0-(-5)5} = \frac{z}{5 \times 7 - (-2)0}$$

$$\text{or} \quad \frac{x}{10} = \frac{y}{25} = \frac{z}{35}$$

$$\text{or} \quad \frac{x}{2} = \frac{y}{5} = \frac{z}{7} = k \quad (\text{suppose})$$

$$x=2k, \quad y=5k, \quad z=7k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$8k+25k+42k=150$$

$$\text{or} \quad 75k=150, \quad k=2$$

Hence, from A , $x=4, y=10, z=14$

34 We have $15x=10y=6z=k$ (suppose)

$$x=\frac{k}{15}, \quad y=\frac{k}{10}, \quad z=\frac{k}{6} \quad (A)$$

Substituting these values of x, y, z in the last equation, we have

$$\frac{7k}{15} + \frac{4k}{5} + \frac{3k}{2} = 332$$

$$\text{or} \quad 14k+24k+45k=9960$$

$$\text{or} \quad 83k=9960$$

$$k=120$$

Hence, from A , $x=8, y=12, z=20$

35 From the 1st two equations, we have

$$\frac{x}{(-13)(-9)-6 \times 8} = \frac{y}{8 \times 7 - (-9)4} = \frac{z}{4 \times 6 - 7(-13)}$$

$$\text{or} \quad \frac{x}{117-48} = \frac{y}{56+36} = \frac{z}{24+91}$$

$$\text{or} \quad \frac{x}{69} = \frac{y}{92} = \frac{z}{115}$$

$$\text{or} \quad \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k \quad (\text{suppose})$$

$$x=3k, \quad y=4k, \quad z=5k \quad (A)$$

Substituting these values of x, y, z in the 3rd equation,

we have

$$\frac{5}{3\lambda} + \frac{8}{4\lambda} + \frac{15}{5\lambda} = \frac{20}{3}$$

$$\text{or } \frac{5}{3\lambda} + \frac{2}{\lambda} + \frac{3}{\lambda} = \frac{20}{3}$$

$$\text{or } 5 + 6 + 9 = 20\lambda, \text{ or, } 20\lambda = 20, \quad \lambda = 1$$

Hence, from A , $x=3$, $y=4$, $z=5$

Exercise (74)

(Note) In all examples of this exercise the 1st, 2nd and the 3rd equations will be denoted by (1), (2) and (3) respectively

1 Multiplying (3) by 5, we have

$$-20x + 15y + 5z = 25,$$

$$\text{also, } 2x - 3y + 5z = 11 \quad (1)$$

Hence, by subtraction, $22x - 18y = -14$

$$\text{or } 11x - 9y + 7 = 0 \quad (4)$$

Again, Multiplying (3) by 7, we have

$$-28x + 21y + 7z = 35,$$

$$\text{also, } 5x + 2y - 7z = -12 \quad (2)$$

Hence, by addition, $-23x + 23y = 23$

$$\text{or } x - y + 1 = 0 \quad (5)$$

Thus we have $11x - 9y + 7 = 0 \quad (4)$

$$\text{and } x - y + 1 = 0 \quad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-9+7} = \frac{y}{7-11} = \frac{1}{-11+9}$$

$$\text{or } \frac{x}{-2} = \frac{y}{-4} = \frac{1}{-2}$$

$$x=1, \text{ and } y=2$$

Substituting these values of x, y in (3), we have

$$-4 + 6 + z = 5, \quad z=3$$

Thus we have $x=1, \quad y=2, \quad z=3$

2 Multiplying (1) by 2 and (3) by 5, we have

$$6x + 4y + 10z = 64$$

$$\text{and } 25x + 15y + 10z = 135$$

Hence, by subtraction,

$$19x + 11y = 71$$

$$\text{or } 19x + 11y - 71 = 0 \quad (4)$$

Again, multiplying (2) by 2 and (3) by 3, we have

$$4x + 10y + 6z = 62$$

$$\text{and } 15x + 9y + 6z = 81$$

Hence, by subtraction,

$$11x - y = 19$$

$$\text{or } 11x - y - 19 = 0 \quad (5)$$

$$\text{Thus we have } 19x + 11y - 71 = 0 \quad (4)$$

$$\text{and } 11x - y - 19 = 0 \quad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-209 - 71} = \frac{y}{-781 + 361} = \frac{1}{-19 - 121}$$

$$\text{or, } \frac{x}{-280} = \frac{y}{-420} = \frac{1}{-140}$$

$$x = 2, \text{ and } y = 3$$

Substituting these values of x, y in (1), we have

$$6 + 6 + 5z = 32$$

$$\text{or } 5z = 20,$$

$$z = 4$$

$$\text{Thus we have } x = 2, \quad y = 3, \quad z = 4$$

3 From (1) and (2) by subtraction, we have

$$7x + 2y - 5z = 0 \quad (4)$$

Again from (1) and (3) by subtraction, we have

$$5x + 2y - 4z = 0 \quad (5)$$

$$\text{Thus we have } 7x + 2y - 5z = 0 \quad (4)$$

$$\text{and } 5x + 2y - 4z = 0 \quad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-8 + 10} = \frac{y}{-25 + 28} = \frac{z}{14 - 10}$$

$$\begin{aligned} \text{or} \quad \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = k \text{ (suppose)} \\ x = 2k, \quad y = 3k, \quad z = 4k \quad (A) \end{aligned}$$

Substituting these values of x, y, z in (1), we have

$$2k + 3k - 4k = 1$$

$$\text{or} \quad k = 1$$

Hence, from A , $x = 2, \quad y = 3, \quad z = 4$

4 From (1) and (3) by subtraction, we have

$$2x - 2z = -4,$$

$$\text{or} \quad x - z + 2 = 0 \quad (4)$$

Again, by multiplying (1) by 2 and (2) by 3, we have

$$4x + 6y + 8z = 58$$

$$\text{and} \quad 9x + 6y + 15z = 96$$

Hence, by subtraction,

$$5x + 7z = 38,$$

$$\text{or} \quad 5x + 7z - 38 = 0 \quad (5)$$

Thus we have $5x + 7z - 38 = 0 \quad (5)$

$$\text{and} \quad x - z + 2 = 0 \quad (4)$$

Therefore, by cross multiplication,

$$\frac{x}{14 - 38} = \frac{z}{-38 - 10} = \frac{1}{-5 - 7}$$

$$\text{or} \quad \frac{x}{-24} = \frac{z}{-48} = \frac{1}{-12},$$

$$x = 2, \text{ and } z = 4$$

Substituting these values of x, z in (1), we have

$$4 + 3y + 16z = 29$$

$$\text{or} \quad 3y = 9, \quad y = 3$$

Thus we have $x = 2, y = 3, z = 4$

5 Multiplying (2) by 2, we have

$$6x + 4y - 10z = 16$$

$$\text{also,} \quad 2x + 3y + 4z = 16 \quad (1)$$

Hence, by subtraction,

$$4x + y - 14z = 0 \quad (4)$$

Again, multiplying (2) by 3 and (3) by 4, we have

$$\begin{aligned} 9x-6j-15z &= 24 \\ \text{and } 20x-24j+12z &= 24 \end{aligned}$$

Hence, by subtraction,

$$11x-30j-27z=0 \quad (5)$$

$$\text{also,} \quad 4x+j-14z=0 \quad (4)$$

Therefore,

$$\frac{x}{420-27} = \frac{j}{108-154} = \frac{z}{11-120}$$

$$\text{or} \quad \frac{x}{393} = \frac{j}{262} = \frac{z}{131}$$

$$\text{or} \quad \frac{x}{3} = \frac{j}{2} = z = k \quad (\text{suppose})$$

$$\therefore \quad x=3k, \quad j=2k, \quad z=k. \quad (A)$$

Substituting these values of x, j, z in (1) we have

$$6k+6k-4k=16, \quad \text{or } 16k=16, \quad k=1.$$

Therefore, $x=3, \quad j=2, \quad z=1$

6 Multiplying (1) by 3 and (3) by 2 we have

$$\begin{aligned} 12x-9j-6z &= 24 \\ \text{and } -12x+10j-14z &= -2 \end{aligned}$$

Hence, by addition,

$$j+20z=22$$

$$\text{or} \quad j-20z-22=0 \quad (4')$$

Again, multiplying (2) by 2, we have

$$6x-8j+10z=12$$

$$\text{also,} \quad -6x+5j-7z=-1 \quad (3)$$

Hence, by addition,

$$-3j+17z=11$$

$$\text{or} \quad 3j-17z+11=0 \quad (5)$$

$$\text{also} \quad j-20z-22=0 \quad (4')$$

Therefore,

$$\frac{j}{371-220} = \frac{z}{11-66} = \frac{1}{60-17}$$

$$\text{or} \quad \frac{j}{154} = \frac{z}{77} = \frac{1}{77}$$

$$j=2 \text{ and } z=1$$

Substituting these values of y, z in (1), we have

$$4x - 6 + 2 = 4$$

$$\text{or} \quad 4x = 12, \quad x = 3$$

$$\text{Thus, we have} \quad x = 3, \quad y = 2, \quad z = 1$$

7 By adding (1) and (2), we have

$$x - 2y + z = 0 \quad \dots (4)$$

Multiplying (2) by 2, we have

$$-14x + 10y + 12z = -2$$

$$\text{also,} \quad 12x - 8y - 11z = 2 \quad (3)$$

Hence, by addition,

$$-2x + 2y + z = 0 \quad (5)$$

$$\text{also,} \quad x - 2y + z = 0 \quad (4)$$

$$\text{Therefore,} \quad \frac{x}{2+2} = \frac{y}{1+2} = \frac{z}{4-2}$$

$$\text{or} \quad \frac{x}{4} = \frac{y}{3} = \frac{z}{2} = k \quad (\text{suppose})$$

$$x = 4k, \quad y = 3k, \quad z = 2k$$

Substituting these values of x, y, z in (1), we have

$$32k - 21k - 10k = 1, \quad k = 1$$

$$\text{Therefore,} \quad x = 4, \quad y = 3, \quad z = 2$$

8 Multiplying (2) by 2, we have

$$6x - 4y + 4z = 28$$

$$\text{also,} \quad x + 5y - 4z = 5 \quad (1)$$

Hence, by addition,

$$7x + y = 33$$

$$\text{or} \quad 7x + y - 33 = 0 \quad (4)$$

Again, multiplying (3) by 2, we have

$$-20x + 16y + 2z = 12,$$

$$\text{also,} \quad 3x - 2y + 2z = 14$$

Hence, by subtraction,

$$-23x + 18y = -2$$

$$\text{or} \quad 23x - 18y - 2 = 0 \quad (5)$$

$$\text{also,} \quad 7x + y - 33 = 0 \quad (4)$$

Therefore, $\frac{x}{594+2} = \frac{y}{-14+759} = \frac{1}{23+126}$

or $\frac{x}{596} = \frac{y}{745} = \frac{1}{149}$

$x=4$ and $y=5$

Substituting these values of x, y in (3), we have

$-40+40+z=6, \quad z=6$

Thus, we have $x=4, \quad y=5, \quad z=6$

9 Multiplying (1) by 3 and (2) by 2 we have

$6x+12y+15z=147$

and $6x+10y+12z=128$

Hence, by subtraction,

$2y+3z=19$

or $2y+3z-19=0 \quad (4)$

Again, multiplying (1) by 2, we have

$4x+8y+10z=98,$

also, $4x+3y+4z=55 \quad (5)$

Hence, by subtraction,

$5y+6z=43$

or $5y+6z-43=0 \quad (5)$

also, $2y+3z-19=0 \quad (4)$

Therefore $\frac{y}{-114+129} = \frac{z}{-86+95} = \frac{1}{15-12}$

or $\frac{y}{15} = \frac{z}{9} = \frac{1}{3}$

$y=5, \quad z=3$

Substituting these values of y, z in (1), we have

$2x+20+15=49$

or $2x=14, \quad x=7$

Thus, we have $x=7, \quad y=5, \quad z=3$

10 Multiplying (1) by 3, we have

$3x+9y+15z=30$

also, $3x+5y+7z=14$

Hence, by subtraction,

$$4y + 8z = 16$$

$$\text{or } y + 2z - 4 = 0 \quad (4)$$

Again, multiplying (1) by 5, we have

$$5x + 15y + 25z = 50,$$

$$\text{also, } 5x + 7y + 8z = 15$$

Hence, by subtraction,

$$8y + 17z = 35$$

$$\text{or } 8y + 17z - 35 = 0 \quad (5)$$

$$\text{also, } y + 2z - 4 = 0 \quad (4)$$

$$\text{Therefore, } \frac{y}{-68+70} = \frac{z}{-35+32} = \frac{1}{16-17}$$

$$\text{or } \frac{y}{2} = \frac{z}{-3} = -1$$

$$y = -2, \quad z = 3$$

Substituting these values of y, z in (1), we have

$$x - 6 + 15 = 10$$

$$\text{or } x = 1$$

Thus, we have $x = 1, y = -2, z = 3$

11 Multiplying (1) by 2 and (2) by 3, we have

$$24x + 16y - 22z = -6$$

$$\text{and } 33x - 39y - 3z = 6$$

Hence, by addition,

$$57x - 23y - 25z = 0 \quad (4)$$

Again, by adding (2) and (3), we have

$$19x + 4y - 13z = 0 \quad (5)$$

$$\text{also, } 57x - 23y - 25z = 0 \quad (4)$$

Therefore, by cross multiplication,

$$\frac{x}{-100-299} = \frac{y}{-741+475} = \frac{z}{-437-228}$$

$$\text{or } \frac{x}{-399} = \frac{y}{-266} = \frac{z}{-665}$$

$$\text{or } \frac{x}{3} = \frac{y}{2} = \frac{z}{5} = k \quad (\text{suppose})$$

$$x = 3k, \quad y = 2k, \quad z = 5k$$

Substituting these values of x, y, z in (1), we have

$$\begin{aligned} 36k + 16k - 55k &= -3 \\ \text{or} \quad -3k &= -3, & k &= 1 \\ r &= 3, & y &= 2, & z &= 5 \end{aligned}$$

12 Multiplying (1) by 3 and (2) by 2, we have

$$\begin{aligned} 15x - 12y + 27z &= 57 \\ \text{and} \quad 14x + 12y - 24z &= 32 \end{aligned}$$

Hence, by addition,

$$29x + 3z = 89 \quad (4)$$

Again, multiplying (1) by 2, we have

$$\begin{aligned} 10x - 8y + 18z &= 38, \\ \text{also,} \quad -9x + 8y + 15z &= -13 \end{aligned} \quad (3)$$

Hence, by addition, $x + 33z = 25$

$$\text{or} \quad x + 33z - 25 = 0 \quad (5)$$

$$\text{also,} \quad 29x + 3z - 89 = 0 \quad (4)$$

$$\text{Therefore,} \quad \frac{x}{-2937 + 75} = \frac{z}{-725 + 89} = \frac{1}{3 - 957}$$

$$\begin{aligned} \text{or} \quad \frac{1}{-2862} &= \frac{z}{-636} = \frac{1}{-954} \\ x &= 3, & z &= \frac{2}{3} \end{aligned}$$

Substituting these values of x, z in (1), we have

$$15 - 4y + 6 = 19, \text{ or, } 4y = 2, \quad y = \frac{1}{2}$$

Thus, we have $x = 3, y = \frac{1}{2}, z = \frac{2}{3}$

13 Adding (1) and (2), we have

$$2x + z = 25 \quad (4)$$

Again, multiplying (2) by 6, we have

$$\begin{aligned} 6y + 6x + 12z &= 240, \\ \text{also,} \quad 4x - 5x - 6y &= -150 \end{aligned} \quad (3)$$

Hence, by addition,

$$\begin{aligned} x + 16z &= 90 \\ \text{or} \quad x + 16z - 90 &= 0 \quad (5) \\ \text{also,} \quad 2x + z - 25 &= 0 \quad (4) \end{aligned}$$

Therefore,
$$\frac{z}{-400+90} = \frac{z}{-180+25} = \frac{1}{1-32}$$

or
$$\frac{z}{-310} = \frac{z}{-155} = \frac{1}{-31}$$

or
$$z=10, z=5$$

Substituting these values of z, z in (1), we have

$$10-y-5=-15$$

or
$$-y=-20, \quad y=20$$

Thus, we have
$$z=10, \quad y=20, \quad z=5$$

14 From (1) and (2), we have

$$2x-2y-3z=-2$$

and
$$x-3y-3z=-1$$

Hence, by subtraction,

$$z+y=-1$$

or
$$z+y+1=0 \quad (4)$$

Again, from (2) and (3), we have

$$z-3y-3z=-1$$

and
$$2z+4y+3z=4$$

Hence, by addition,

$$3z+y=3$$

or
$$3z+y-3=0 \quad (5)$$

also,
$$z+y+1=0 \quad (4)$$

Therefore,
$$\frac{z}{1+3} = \frac{y}{-3-3} = \frac{1}{3-1}$$

or
$$\frac{x}{4} = \frac{y}{-6} = \frac{1}{2}, \quad x=2, y=-3$$

Substituting these values of x, y in (2), we have

$$2-3z=-9-1$$

or
$$-3z=-12, \quad z=4$$

Thus, we have $x=2, y=-3, z=4$

15 From (2) and (3), by subtraction, we have

$$z+4y=29$$

or
$$z+4y-29=0 \quad (4)$$

Again, multiplying (1) by 6, we have

$$\begin{aligned} 18x + 12y - 6z &= 120, \\ \text{also,} \quad x - y + 6z &= 41 \end{aligned} \quad (3)$$

Hence, by addition,

$$\begin{aligned} 19x + 11y &= 161 \\ \text{or} \quad 19x + 11y - 161 &= 0 \quad (5) \\ \text{also,} \quad x + 4y - 29 &= 0 \quad (4) \end{aligned}$$

$$\text{Therefore,} \quad \frac{x}{-319+644} = \frac{y}{-161+551} = \frac{1}{76-11}$$

$$\begin{aligned} \text{or} \quad \frac{x}{325} &= \frac{y}{320} = \frac{1}{65} \\ x &= 5, y = 6 \end{aligned}$$

Substituting these values of x, y in (1), we have

$$15 + 12 - z = 20, \quad z = 7$$

Thus, we have $x=5, y=6, z=7$

16 From (1) and (2), we have

$$\begin{aligned} -4x + 4y - 5z &= -22 \\ \text{and} \quad 4x - 6y + 3z &= 2 \end{aligned}$$

Hence, by addition,

$$\begin{aligned} -2y - 2z &= -20 \\ \text{or} \quad y + z - 10 &= 0 \quad (4) \end{aligned}$$

Again, multiplying (2) by 5 and (3) by 2, we have

$$\begin{aligned} 20x - 30y + 15z &= 10 \\ \text{and} \quad 20x - 6y + 2z &= 28 \end{aligned}$$

Hence, by subtraction,

$$\begin{aligned} -24y + 13z &= -18 \\ \text{or} \quad 24y - 13z - 18 &= 0 \quad (5) \\ \text{also,} \quad y + z - 10 &= 0 \quad (4) \end{aligned}$$

$$\text{Therefore,} \quad \frac{y}{130+18} = \frac{z}{-18+240} = \frac{1}{24+13}$$

$$\text{or} \quad \frac{y}{148} = \frac{z}{222} = \frac{1}{37}, \quad y=4, z=6$$

Substituting these values of y, z in (2), we have

$$18 + 4x = 24 + 2$$

$$\text{or} \quad 4x = 6, \quad x = 2$$

Thus, we have $x = 2, y = 4, z = 6$

17 Multiplying (2) by 10, we have

$$5x + 8y - z = 40,$$

$$\text{also,} \quad 5x + 2y + z = 30 \quad (1)$$

Hence, by addition, $10x + 10y = 70$

$$\text{or} \quad x + y - 7 = 0 \quad (4)$$

Again, multiplying (1) by 10, we have

$$50x + 20y + 10z = 300$$

$$\text{also,} \quad 2x + 5y + 10z = 129$$

Hence, by subtraction,

$$48x + 15y = 171$$

$$\text{or} \quad 16x + 5y - 57 = 0 \quad (5)$$

$$\text{also,} \quad x + y - 7 = 0 \quad (4)$$

$$\text{Therefore,} \quad \frac{x}{-35 + 57} = \frac{y}{-57 + 112} = \frac{1}{16 - 5}$$

$$\text{or} \quad \frac{x}{22} = \frac{y}{55} = \frac{1}{11},$$

$$x = 2, y = 5$$

Substituting these values of x, y in (1), we have

$$10 + 10 + z = 30$$

$$z = 30$$

Thus, we have $x = 2, y = 5, z = 10$

18 From (1), we have

$\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 12$, and multiplying this by 12, we have

$$3x + 4y + 3z = 144$$

$$\text{also,} \quad -\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 8 \quad (2)$$

Hence, by subtraction,

$$\frac{1}{2}x - \frac{1}{3}z = 10$$

$$\text{or} \quad 11x - z - 120 = 0 \quad (4)$$

$$\text{also,} \quad 3x + 2z - 60 = 0 \quad (5)$$

(multiplying (3) by 6)

Therefore, $\frac{1}{60+240} = \frac{2}{-360+660} = \frac{1}{22+3}$

or $\frac{1}{300} = \frac{2}{300} = \frac{1}{25}$
 $1 = 12, 2 = 12$

Substituting these values of x, y in (2), we have

$$1y + 4 - 2 = 8, \text{ or, } 1y = 6, \quad y = 12$$

Thus, we have $1 = y = 2 = 12$

19 From (1) and (3), by subtraction, we have

$$\frac{5}{2} - \frac{10}{2} = -\frac{5}{12}$$

or $\frac{1}{2} - \frac{2}{2} + \frac{1}{12} = 0$ (4)

Again, multiplying (1) by 4 and (2) by 5, we have

$$\frac{4}{2} + \frac{20}{y} - \frac{16}{2} = \frac{1}{3}$$

and $\frac{15}{2} - \frac{20}{y} + \frac{25}{2} = \frac{95}{24}$

Hence, by addition, we have

$$\frac{19}{2} + \frac{9}{2} = \frac{103}{24}$$

or $\frac{19}{x} + \frac{9}{2} - \frac{103}{24} = 0$ (5)

also, $\frac{1}{x} - \frac{2}{2} + \frac{1}{12} = 0$ (4)

Therefore, by cross multiplication,

$$\frac{\frac{1}{x}}{\frac{3}{4} - \frac{103}{12}} = \frac{\frac{1}{2}}{-\frac{103}{24} - \frac{19}{12}} = \frac{1}{-38-9}$$

or $\frac{\frac{1}{x}}{\frac{9-103}{12}} = \frac{\frac{1}{2}}{\frac{-103-38}{24}} = \frac{1}{-47}$

$$\text{or} \quad \frac{-12}{94\tau} = \frac{-24}{141\varepsilon} = \frac{-1}{47}$$

$$\text{or} \quad \frac{12}{2\tau} = \frac{24}{3\varepsilon} = 1, \quad \tau = 6, \quad \varepsilon = 8$$

Substituting these values of τ , ε in (1), we have

$$\frac{1}{6} + \frac{5}{y'} - \frac{1}{2} = \frac{1}{12}, \quad \text{or,} \quad \frac{5}{y'} = \frac{5}{12}, \quad y' = 12$$

Thus, we have $\tau = 6$, $y' = 12$, $\varepsilon = 8$

20 Multiplying (1) by 2, we have

$$\frac{6}{\tau} - \frac{8}{5y'} + \frac{2}{\varepsilon} = 15\frac{1}{2},$$

$$\text{also,} \quad \frac{1}{3\tau} + \frac{1}{2y'} + \frac{2}{\varepsilon} = 10\frac{1}{10} \quad (2)$$

Hence, by subtraction,

$$\frac{17}{3\tau} - \frac{21}{10y'} = 5\frac{1}{10}$$

$$\text{or} \quad \frac{170}{\tau} - \frac{63}{y'} - 151 = 0 \quad (4)$$

Again, multiplying (2) by 2, we have

$$\frac{2}{3\tau} + \frac{1}{y'} + \frac{4}{\varepsilon} = 20\frac{1}{10}$$

$$\text{also,} \quad \frac{17}{5\tau} - \frac{1}{2y'} + \frac{4}{\varepsilon} = 16\frac{1}{10} \quad (3)$$

Hence, by subtraction,

$$-\frac{2}{15\tau} + \frac{3}{2y'} = 4\frac{7}{10}$$

$$\text{or} \quad \frac{1}{\tau} - \frac{45}{y'} + 127 = 0 \quad (5)$$

$$\text{also,} \quad \frac{170}{\tau} - \frac{63}{y'} - 151 = 0 \quad (4)$$

$$\text{Therefore,} \quad \frac{\frac{1}{\tau}}{6795 + 8001} = \frac{\frac{1}{y'}}{21590 + 604}$$

$$= \frac{1}{-252 + 7650}$$

$$\text{or} \quad \frac{1}{14796\tau} = \frac{1}{22194y'} = \frac{1}{7398}$$

$$\tau = \frac{1}{2}, \quad y' = \frac{1}{1}$$

Substituting these values of x & y in (1), we have

$$6 - \frac{12}{5} + \frac{1}{z} = 7\frac{3}{5} \quad \text{or,} \quad \frac{1}{z} = 4, \quad z = \frac{1}{4}$$

Thus, we have $x = \frac{1}{5}$, $y = \frac{1}{5}$, $z = \frac{1}{4}$

21 Multiplying (2) by 4, we have

$$8y - 4z = 44,$$

$$\text{also} \quad 3x + 4z = 57$$

Hence, by addition

$$3x + 8y = 101$$

$$\text{or} \quad 3x + 8y - 101 = 0 \quad (4)$$

$$\text{also,} \quad 5x + 3y - 65 = 0 \quad (1)$$

$$\text{Therefore,} \quad \frac{x}{-520+303} = \frac{y}{-505+195} = \frac{1}{9-40}$$

$$\text{or} \quad \frac{x}{-217} = \frac{y}{-310} = \frac{1}{-31}$$

$$x = 7 \quad y = 10$$

Substituting the value of y in (2), we have

$$20 - z = 11, \quad \text{or} \quad z = 9$$

Thus, we have $x = 7$, $y = 10$, $z = 9$

22 Multiplying (1) by 2, we have

$$\frac{4}{x} + \frac{2}{y} = 3$$

$$\text{also} \quad \frac{3}{z} - \frac{2}{y} = 2$$

Hence, by addition,

$$\frac{4}{x} + \frac{3}{z} = 5$$

$$\text{or} \quad \frac{4}{x} + \frac{3}{z} - 5 = 0 \quad (4)$$

$$\text{also} \quad \frac{1}{x} + \frac{1}{z} - \frac{4}{3} = 0 \quad (3)$$

$$\text{Therefore,} \quad \frac{\frac{1}{x}}{-4+5} = \frac{\frac{1}{z}}{-5+\frac{1}{3}} = \frac{1}{4-3}$$

$$\text{or} \quad \frac{1}{x} = \frac{1}{\frac{1}{3}z} = 1$$

$$x = 1, \quad z = 3$$

Substituting the value of z in (7), we have

$$2 + \frac{1}{y'} = \frac{3}{2}$$

$$\text{or } \frac{1}{y'} = -\frac{1}{2}, \quad y' = -2$$

Thus, we have $x = 1$, $y = -2$, $z = 3$

23 Multiplying (2) by b and (3) by a , we have

$$bax + abz = b^2$$

$$\text{and } abz + acy = a^2$$

Hence, by subtraction,

$$bax - acy = b^2 - a^2$$

$$\text{or } bax - acy + (a^2 - b^2) = 0 \quad (4)$$

$$\text{also, } bx + ay - z = 0 \quad (1)$$

Therefore,

$$\frac{a}{ax^2 - a^2 + ab^2} = \frac{y}{a^2b - b^3 + ac^2}$$

$$= \frac{1}{abc + abx}$$

$$\text{or } \frac{1}{a(b^2 + c^2 - a^2)} = \frac{y}{b(a^2 - b^2 + c^2)} = \frac{1}{2abc}$$

$$x = \frac{b^2 + c^2 - a^2}{2bc}, \quad y = \frac{a^2 - b^2 + c^2}{2ac}$$

Substituting the value of x in (2), we have

$$\frac{b^2 + c^2 - a^2}{2b} + az = b$$

$$\text{or } az = b - \frac{b^2 + c^2 - a^2}{2b}$$

$$= \frac{2b^2 - b^2 - c^2 + a^2}{2b} = \frac{b^2 - c^2 + a^2}{2b},$$

$$z = \frac{b^2 - c^2 + a^2}{2ab}$$

$$\text{Thus, we have } x = \frac{b^2 + c^2 - a^2}{2bc}, \quad y = \frac{c^2 + a^2 - b^2}{2ac}, \quad z = \frac{a^2 + b^2 - c^2}{2ab}$$

24 Multiplying (1) by 5 and (2) by 4, we have

$$15x + 20y - 55 = 0$$

$$\text{and } 20x - 24z + 32 = 0$$

Hence, by subtraction,

$$15x + 24z - 87 = 0 \quad (4)$$

also, $-8x + 7z - 13 = 0 \quad (3)$

therefore, $\frac{x}{-312 + 609} = \frac{z}{696 + 195} = \frac{1}{105 + 192}$

or, $\frac{x}{297} = \frac{z}{891} = \frac{1}{297}$

$$x = 1, z = 3$$

Substituting the value of x in (1), we have

$$3 + 4y - 11 = 0$$

or $4y = 8$

$$y = 2$$

Thus we have $x = 1, y = 2, z = 3$

25 Multiplying (1) by 4 and (2) by 3, we have

$$4x + 12y - 8 = 0$$

and $-3x - 12y + 9z - 45 = 0$

Hence, by addition,

$$x + 9z - 53 = 0 \quad (4)$$

also, $2x + 7z - 7 = 0 \quad (3)$

therefore, $\frac{x}{-63 + 371} = \frac{z}{-106 + 7} = \frac{1}{7 - 18}$

or $\frac{x}{308} = \frac{z}{-99} = \frac{1}{-11}$,

$$x = -28, z = 9$$

Substituting the value of x in (1), we have

$$3y - 28 - 2 = 0$$

or $3y = 30$

$$y = 10$$

Thus we have $x = -28, y = 10, z = 9$

Exercise 75.

1 Adding together the given equations, we have

$$2\left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c}\right) = 3$$

or $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \frac{3}{2} \quad (A)$

Subtracting the 3rd equation from (A) we have

$$\frac{1}{a} = \frac{1}{2}, \quad 1 = \frac{a}{2}$$

Similarly, we have $y = \frac{b}{2}$, and $z = \frac{c}{2}$

2 Adding together the given equations, we have

$$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = a + b + c$$

$$\text{or} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{a+b+c}{2} \quad (A)$$

Subtracting the 3rd equation from (A), we have

$$\frac{1}{x} = \frac{a+b-c}{2}$$

$$x = \frac{2}{a+b-c}$$

Similarly,

$$y = \frac{2}{a-b+c},$$

and

$$z = \frac{2}{c+b-a}$$

3 From (1), we have

$$\frac{1}{y} + \frac{1}{z} = \frac{1}{a}$$

$$\text{or} \quad \frac{1}{y} + \frac{1}{z} = \frac{1}{a} \quad (4)$$

From (2), we have

$$\frac{z+1}{z^2} = \frac{1}{b}$$

$$\text{or} \quad \frac{1}{z} + \frac{1}{x} = \frac{1}{b} \quad (5)$$

From (3) we have

$$\frac{x+y}{xy} = \frac{1}{c}$$

$$\text{or} \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{c} \quad (6)$$

From (4), (5), (6), by addition, we have

$$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

$$\text{or} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \quad (7)$$

Subtracting (4) from (7),

$$\frac{1}{x} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) - \frac{1}{a} = \frac{1}{2} \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a} \right),$$

$$x = \frac{2abc}{ab + ac - bc}$$

Subtracting (5) from (7),

$$\frac{1}{y'} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) - \frac{1}{b} = \frac{1}{2} \left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c} \right)$$

$$y' = \frac{2abc}{bc - ac + ab}$$

Subtracting (6) from (7),

$$\frac{1}{z} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) - \frac{1}{c} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c} \right)$$

$$z = \frac{2abc}{bc + ac - ab}$$

4 From (1) and (2), we have

$$axy - bcx - acy = 0$$

and
$$bxy - acx + bcy = 0$$

Therefore, by cross multiplication,

$$\frac{xy}{-b^2c^3 - a^2c^3} = \frac{x}{-abc - abc} = \frac{y'}{-a^2c + b^2c}$$

$$x = \frac{a^2c^2 + b^2c^2}{a^2c - b^2c} = \frac{c(a^2 + b^2)}{a^2 - b^2}$$

and
$$y = \frac{a^2c^2 + b^2c^2}{2abc} = \frac{c(a^2 + b^2)}{2ab}$$

5 From (1), we have $\frac{x+y}{xy} = \frac{3}{4}$

or
$$\frac{1}{x} + \frac{1}{y} = \frac{3}{4} \quad (4)$$

From (2),
$$\frac{x+z}{xz} = \frac{2}{3}$$

or
$$\frac{1}{x} + \frac{1}{z} = \frac{2}{3} \quad (5)$$

From (3),
$$\frac{y+z}{yz} = \frac{5}{12}$$

or
$$\frac{1}{y} + \frac{1}{z} = \frac{5}{12} \quad (6)$$

From (4), (5) and (6), by addition,

$$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{22}{12}$$

$$\text{or} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{12} \quad (7)$$

Subtracting (6), (5) and (4) respectively from (7), we have

$$\frac{1}{x} = \frac{1}{2}, \quad \frac{1}{y} = \frac{1}{4} \text{ and } \frac{1}{z} = \frac{1}{6} \text{ respectively,}$$

$$x = 2, \quad y = 4, \text{ and } z = 6$$

(6) Adding together (1), (2), and (3), we have

$$2(x + y + z) = 4 + 6 + 8 = 18, \quad x + y + z = 9 \quad (4)$$

Subtracting (1) from (4), $x = 5$

$$(2) \quad , \quad y = 3$$

$$(3) \quad , \quad z = 1$$

Thus we have $x = 5, \quad y = 3, \quad z = 1$

7 Adding together the three equations, we have

$$x + y + z = 6 + 10 + 14 = 30 \quad (4)$$

Subtracting (1) from (4), we have

$$2x = 24, \quad x = 12$$

Subtracting (2) from (4), we have

$$2y = 20, \quad y = 10$$

Subtracting (3) from (4), we have

$$2z = 16, \quad z = 8$$

8 Adding together the three equations, we have

$$-2(x + y + z) = -60, \quad x + y + z = 30 \quad (4)$$

Subtracting (3) from (4), we have

$$5x = 65, \quad x = 13$$

Subtracting (1) from (4), we have

$$5y = 40, \quad y = 8$$

Subtracting (2) from (4), we have

$$5z = 45, \quad z = 9$$

9 Adding together the three equations, we have

$$5(x + y + z) - 80 = 0$$

$$\text{or} \quad x + y + z - 16 = 0 \quad (4)$$

Subtracting (1) from (4), we have

$$8x - 32 = 0, \quad x = 4$$

Subtracting (2) from (4), we have

$$8y - 40 = 0, \quad y = 5$$

Subtracting (3) from (4), we have

$$8z - 56 = 0, \quad z = 7$$

Thus we have $x = 4$, $y = 5$, and $z = 7$

10 Adding together the two equations, we have

$$\begin{aligned} x(a^2 + 2ab + b^2) + (b^2 + a^2 + 2ab)y \\ = 2a^2b + 2ab^2 + a^3 + a^2b + ab^2 + b^3 \end{aligned}$$

$$\begin{aligned} \text{or } x(a+b)^2 + y(a+b)^2 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ &= (a+b)^3 \end{aligned}$$

$$\text{or} \quad x + y = a + b \quad (3)$$

From (3), we have

$$b^2x + b^2y = b^2(a+b)$$

$$\text{also,} \quad a^2x + b^2y = 2ab(a+b)$$

Hence, by subtraction,

$$x(a^2 - b^2) = b(a+b)(2a-b)$$

$$\text{or} \quad x(a-b) = b(2a-b),$$

$$x = \frac{b(2a-b)}{a-b}$$

Substituting this value in 3, we have

$$\begin{aligned} y &= a + b - \frac{b(2a-b)}{a-b} \\ &= \frac{a^2 - b^2 - 2ab + b^2}{a-b} = \frac{a^2 - 2ab}{a-b} = \frac{a(2b-a)}{b-a} \end{aligned}$$

$$\begin{aligned} 11 \quad \text{Since} \quad & ax + by + cz = 0 \quad (2) \\ \text{and} \quad & a^2x + b^2y + c^2z = 0 \quad (3) \end{aligned} \quad \left. \vphantom{\begin{matrix} 11 \\ \text{Since} \\ \text{and} \end{matrix}} \right\},$$

therefore, by cross multiplication,

$$\frac{x}{bc^2 - b^2c} = \frac{y}{a^2c - ac^2} = \frac{z}{ab^2 - a^2b}$$

$$\text{or} \quad \frac{x}{bc(c-b)} = \frac{y}{ac(a-c)} = \frac{z}{ab(b-a)} = k \text{ (suppose)}$$

$$x = bc(c-b)k, \quad y = ac(a-c)k, \text{ and}$$

$$z = ab(b-a)k \quad (B)$$

Substituting these values of x, y, z in (1), we have

$$-k\{ab(a-b) + ac(c-a) + bc(b-c)\} = A$$

$$\text{or} \quad k(a-b)(b-c)(c-a) = A$$

$$k = \frac{A}{(a-b)(b-c)(c-a)},$$

from (B),

$$x = \frac{Abc}{(a-b)(a-c)},$$

$$y = \frac{Aac}{(b-a)(b-c)},$$

$$\text{and} \quad z = \frac{Aab}{(c-a)(c-b)}$$

$$\begin{aligned} 12 \quad & \text{Since} \quad x+y+z=0 \quad (1) \\ & \text{and} \quad (a+b)x + (a+c)y + (b+c)z = 0 \quad (2) \end{aligned} \quad \left. \vphantom{\begin{aligned} 12 \quad & \text{Since} \quad x+y+z=0 \quad (1) \\ & \text{and} \quad (a+b)x + (a+c)y + (b+c)z = 0 \quad (2) \end{aligned}} \right\},$$

therefore, by cross multiplication,

$$\frac{z}{(b+c)-(a+c)} = \frac{y}{(a+b)-(b+c)} = \frac{x}{(a+c)-(a+b)}$$

$$\text{or} \quad \frac{x}{b-a} = \frac{y}{a-c} = \frac{z}{c-b} = k \text{ (suppose)}$$

$$x = k(b-a),$$

$$y = k(a-c) \text{ and } z = k(c-b) \quad (B)$$

Substituting these values of x, y, z in (3), we have

$$-k\{ab(a-b) + ac(c-a) + bc(b-c)\} = 1$$

$$\text{or} \quad k(a-b)(b-c)(c-a) = 1,$$

$$k = \frac{1}{(a-b)(b-c)(c-a)}$$

Therefore, from (B),

$$x = \frac{1}{(b-c)(a-c)},$$

$$y = \frac{1}{(a-b)(c-b)},$$

$$\text{and} \quad z = \frac{1}{(c-a)(b-a)}$$

$$\begin{aligned} 13 \quad & \text{Since} \quad x+y+z=0 \quad (1) \\ & \text{and} \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0 \quad (2) \end{aligned} \quad \left. \vphantom{\begin{aligned} 13 \quad & \text{Since} \quad x+y+z=0 \quad (1) \\ & \text{and} \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0 \quad (2) \end{aligned}} \right\},$$

therefore, by cross multiplication,

$$\frac{x}{\frac{1}{c}-\frac{1}{b}} = \frac{y}{\frac{1}{a}-\frac{1}{c}} = \frac{z}{\frac{1}{b}-\frac{1}{a}} = k \text{ (suppose)}$$

$$x = k\left(\frac{1}{c}-\frac{1}{b}\right), \quad y = k\left(\frac{1}{a}-\frac{1}{c}\right) \text{ and}$$

$$z = k\left(\frac{1}{b}-\frac{1}{a}\right) \quad (B)$$

Substituting these values of x, y, z in (3), we have

$$-1 \left\{ \frac{1}{a^2} \left(\frac{1}{b} - \frac{1}{c} \right) + \frac{1}{b^2} \left(\frac{1}{c} - \frac{1}{a} \right) + \frac{1}{c^2} \left(\frac{1}{a} - \frac{1}{b} \right) \right\} = 1$$

$$\text{or} \quad k \left(\frac{1}{a} - \frac{1}{b} \right) \left(\frac{1}{b} - \frac{1}{c} \right) \left(\frac{1}{c} - \frac{1}{a} \right) = 1,$$

$$k = \frac{1}{\left(\frac{1}{a} - \frac{1}{b} \right) \left(\frac{1}{b} - \frac{1}{c} \right) \left(\frac{1}{c} - \frac{1}{a} \right)}$$

Therefore, from (B),

$$x = \frac{1}{\left(\frac{1}{a} - \frac{1}{b} \right) \left(\frac{1}{a} - \frac{1}{c} \right)} = \frac{a^2 bc}{(b-a)(c-a)},$$

$$y = \frac{1}{\left(\frac{1}{a} - \frac{1}{b} \right) \left(\frac{1}{c} - \frac{1}{b} \right)} = \frac{ab^2 c}{(b-a)(b-c)},$$

$$\text{and } z = \frac{1}{\left(\frac{1}{b} - \frac{1}{c} \right) \left(\frac{1}{a} - \frac{1}{c} \right)} = \frac{abc^2}{(c-b)(c-a)}$$

14 From (1) and (2), by subtraction, we have

$$y(a-b) - z(a^2 - b^2) = -(a^3 - b^3)$$

$$\text{or} \quad y - z(a+b) = -(a^2 + ab + b^2) \quad (4)$$

From (2) and (3), by subtraction, we have

$$y(b-c) - z(b^2 - c^2) = -(b^3 - c^3)$$

$$\text{or} \quad y - z(b+c) = -(b^2 + bc + c^2) \quad (5)$$

Again, from (4) and (5), by subtraction, we have

$$\begin{aligned} z(a-c) &= a^2 + ab - bc - c^2 \\ &= (a^2 - c^2) + (ab - bc) \\ &= (a+c)(a-c) + b(a-c) \end{aligned}$$

$$=(a+b+c)(a-c),$$

$$z=a+b+c$$

Substituting this value of z in (4), we have

$$\begin{aligned} y &= (a+b)(a+b+c) - (a^2 + ab + b^2) \\ &= a^2 + 2ab + b^2 + ac + bc - a^2 - ab - b^2 \\ &= ab + ac + bc \end{aligned}$$

Hence, from (1),

$$1 - a(ab + ac + bc) + a^2(a + b + c) = a^3$$

or

$$\begin{aligned} 1 &= a^3 + a^2b + a^2c + abc - a^3 - a^2b - a^2c \\ &= abc \end{aligned}$$

Thus we have

$$x = abc, y = ab + ac + bc,$$

and

$$z = a + b + c$$

15 Since $ax + by + cz = 0$ (1)

and $(b+c)x + (c+a)y + (a+b)z = 0$ (2)

therefore, by cross multiplication,

$$\begin{aligned} \frac{x}{b(a+b) - c(c+a)} &= \frac{y}{c(b+c) - a(a+b)} \\ &= \frac{z}{a(c+a) - b(b+c)} \end{aligned}$$

or $\frac{x}{(b^2 - c^2) + (ab - ac)} = \frac{y}{(c^2 - a^2) + b(c - a)}$

$$= \frac{z}{(a^2 - b^2) + c(a - b)}$$

or $\frac{x}{(b-c)(b+c+a)} = \frac{y}{(c-a)(c+a+b)} = \frac{z}{(a-b)(a+b+c)}$

or $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b} = k$ (suppose)

$$x = k(b-c), y = k(c-a), z = k(a-b)$$

Substituting these values of x, y, z in (3), we have

$$\begin{aligned} k\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\ = a^2(b-c) + b^2(c-a) + c^2(a-b), \end{aligned}$$

$$k = 1$$

Therefore,

$$x = b - c, y = c - a,$$

and

$$z = a - b$$

16 Since

$$a_1x + b_1y + c_1 = 0 \quad (1)$$

and

$$a_2x + b_2y + c_2 = 0 \quad (2)$$

therefore, by cross multiplication,

$$\frac{\tau}{b_1c_2 - b_2c_1} = \frac{j'}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - b_1a_2}$$

$$\tau = \frac{b_1c_2 - b_2c_1}{a_1b_2 - b_1a_2}$$

and

$$j' = \frac{a_2c_1 - a_1c_2}{a_1b_2 - b_1a_2}$$

In order that the three equations may be consistent the 3rd equation will also be satisfied by the values of τ , j'

We have from (3),

$$\frac{a_2(b_1c_2 - b_2c_1)}{a_1b_2 - b_1a_2} + \frac{b_2(a_2c_1 - a_1c_2)}{a_1b_2 - b_1a_2} + c_2 = 0$$

$$a_2(b_1c_2 - b_2c_1) + b_2(a_2c_1 - a_1c_2) + c_2(a_1b_2 - b_1a_2) = 0$$

is the required condition

17 Multiplying (2) by 3, we have

$$9x - 3y + 12z = 60,$$

also,

$$2x - 3y + 5z = 18 \quad (1)$$

Hence, by subtraction,

$$7x + 7z = 42$$

or

$$x + z = 6 \quad (5)$$

Again, multiplying (2) by 2, we have

$$6x - 2y + 8z = 40$$

also,

$$4x + 2y - z = 5 \quad (3)$$

Hence, by addition,

$$10x + 7z = 45 \quad (6)$$

and from (5),

$$7x + 7z = 42$$

Hence, by subtraction,

$$3x = 3$$

$$x = 1$$

Therefore, from (5),

$$z = 6 - 1 = 5$$

and from (2),

$$3 - j + 20 = 20,$$

$$j = 3$$

In order the four equations may be consistent the fourth equation also will be satisfied by these values of x , j , z

from (4), we have

$$(a+1) + (a-2)3 - (a+3)5 = 76$$

$$\text{or} \quad a - 3a + 3a = 76 - 1 - 6 - 15$$

$$\text{or} \quad 9a = 54$$

$$a = 6$$

18 Multiplying (3) by 1 and (4) by 3, we have

$$8x - 12z = 156$$

$$\text{and} \quad 12j - 9z = 123$$

Hence, by subtraction

$$8x - 9z = 33 \quad (5)$$

Multiplying (2) by 8 and (5) by 3, we have

$$40x - 56z = 88,$$

$$\text{and} \quad 40x - 45z = 165$$

Hence, by subtraction,

$$-11z = -77$$

$$z = 7$$

Substituting this value of z in (2), we have

$$5x = 11 + 49 = 60$$

$$x = 12$$

Substituting this value of x in (3), we have

$$3j = 39 - 24 = 15$$

$$j = 5$$

Substituting this value of j in (1), we have

$$3w = 2 - 10 = -8, \quad w = -\frac{8}{3}$$

Thus we have

$$x = 12, \quad j = 5, \quad z = 7 \text{ and } w = -\frac{8}{3}$$

19 Multiplying (3) by 3, we have

$$12x - 9x + 6w = 15,$$

$$\text{also,} \quad 9x - 2z + 7w = 11 \quad (1)$$

Hence, by addition,

$$12x - 2z + 7w = 56 \quad (6)$$

Again, multiplying (2) by 3, we have

$$21x - 15z - 3t = 36$$

$$\text{also} \quad 3x - 4w + 5t = 7 \quad (4)$$

Hence, by addition,

$$24x - 15z - 4w = 43 \dots (7)$$

also, $24y - 4z + 14w = 112$ (multiplying (6) by 2)

Hence by subtraction,

$$11z + 18w = 69 \quad (8)$$

Again, multiplying (5) by 11 and (8) by 7 we have

$$77z - 55w = 121$$

and $77z + 126w = 483$

Hence, by subtraction

$$181w = 362, \quad w = 2$$

Substituting this value of w in (8), we have

$$11z + 36 = 69$$

or $11z = 33 \quad z = 3$

Again substituting these values of w, z in (1), we have

$$9x - 6 + 2 = 41$$

or $9x = 45 \quad x = 5$

Substituting the values of x, w in (3) we have

$$4y - 15 + 4 = 5$$

or $4y = 16, \quad y = 4$

Substituting the values of y, w in (4) we have

$$12 - 8 + 3t = 7$$

or $3t = 3 \quad t = 1$

Thus we have $x = 5, y = 4, z = 3, w = 2$ and $t = 1$

20 Multiplying both sides of (2) by abc

and those of (1) by b , we have

$$cx + ay + bz = 3abc$$

and $bx + by + bz = ab^2 + b^2c + abc$

Hence, by subtraction,

$$(c-b)x + (a-by) = 2abc - ab^2 - b^2c \quad (4)$$

also, $(c-b)x + (a-b)y + (c-a)z = 2abc - ab^2 - b^2c + ac^2 - a^2c \quad (3)$

Hence, by subtraction,

$$(c-a)z = ac^2 - a^2c = ac(c-a)$$

$$z = ac$$

Substituting this value of z in (1), we have

$$x + y + ac = ab + bc + ac$$

or $x + y = ab + bc \quad (5)$

and in (2) we have

$$\frac{1}{ab} + \frac{z}{bc} + 1 = 3$$

$$\text{or} \quad c(1 + z) = 2abc \quad (6)$$

$$\text{also} \quad c(1 + cy) = abc + bc^2 \quad (\text{multiplying (5) by } c)$$

Hence by subtraction,

$$y(a - c) = abc - bc^2 = bc(a - c),$$

$$y = bc$$

Therefore, from (5), we have

$$1 + bc = ab + bc,$$

$$1 = ab$$

Thus we have $1 = ab$, $y = bc$ and $z = ac$

Exercise 76

1 Let $\frac{x}{y}$ represent the fraction

$$\text{Then we have } \frac{2x}{x+7} = \frac{2}{3}$$

$$\text{or} \quad x = \frac{1+7}{3} \quad (1)$$

$$\text{and} \quad \frac{x+2}{2x} = \frac{3}{5}$$

$$\text{or} \quad x+2 = \frac{6x}{5}$$

$$\text{or} \quad x = \frac{6x}{5} - 2 = \frac{6x - 10}{5} \quad (2)$$

Therefore, from (1) and (2),

$$\frac{x+7}{3} = \frac{6x-10}{5}$$

$$\text{or} \quad 5x + 35 = 18x - 30$$

$$\text{or} \quad 13x = 65, \quad x = 5$$

$$\text{Hence,} \quad x = \frac{5+7}{3} = 4$$

Thus the fraction is 4

- 2 Let x and y be the required numbers

Then we have $x + 5y = 52$ (1)

and $y + 8x = 65$ (2)

Multiplying (2) by 5, we have

$$5y + 40x = 325,$$

also, $x + 5y = 52$ (1)

Hence, by subtraction,

$$39x = 273, \quad x = 7$$

Hence, $y + 56 = 65, \quad y = 9$

Thus the numbers are 7 and 9

- 3 Let x, y be the two numbers of which x is the greater

Then we have $5x - 4y = 22$ (1)

and $3x + 7y = 32$ (2)

Multiplying (1) by 3 and (2) by 5, we have

$$15x - 12y = 66,$$

and $15x + 35y = 160$

Hence, by subtraction, $47y = 94, \quad y = 2$

Hence, from (1), $5x - 8 = 22$

or $5x = 30, \quad x = 6$

Thus the numbers are 6 and 2

- 4 Let x and y be the numbers of which x is the greater

Then we have $x - y = 45$ (1)

and $\frac{x}{y} = 4$ (2)

From (2), we have $x = 4y$

Hence, from (1), we have

$$4y - y = 45$$

or $3y = 45, \quad y = 15$

Therefore, $\frac{x}{15} = 4, \text{ or } x = 60$

Thus the numbers are 60 and 15

- 5 Let x and y be the numbers of which x is the greater

Then we have $\frac{x}{4} + \frac{y}{3} = 11$ (1)

and $\frac{x}{8} - \frac{y}{5} = 0$ (2)

Multiplying (2) by 2, we have

$$\frac{x}{4} - \frac{2y}{5} = 0,$$

also,

$$\frac{x}{4} + \frac{y}{3} = 11 \quad (1)$$

Hence, by subtraction,

$$-\frac{11}{15}y = -11, \quad y = 15$$

Therefore, from (1), $\frac{x}{4} + 5 = 11$

$$\text{or} \quad \frac{x}{4} = 6, \quad x = 24$$

Thus the numbers are 24 and 15

6 Let $\frac{x}{j}$ represent the fraction

Then we have $\frac{x}{j-1} = \frac{1}{2} \quad (1)$

$$\text{and} \quad \frac{x+7}{j} = 1 \quad (2)$$

From (1), we have

$$2x = j - 1$$

$$\text{or} \quad 2x - j = -1 \quad (3)$$

From (2), we have $x + 7 = j$

$$\text{or} \quad x - j = -7 \quad (4)$$

Subtracting (4) from (3), we have

$$x = 6$$

Therefore, from (1), we have

$$\frac{6}{j-1} = \frac{1}{2}$$

$$\text{or} \quad j - 1 = 12 \quad j = 13$$

Thus the fraction is $\frac{6}{13}$

7 Let $\frac{x}{j}$ represent the fraction

Then we have $\frac{x+1}{j} = 1 \quad (1)$

$$\text{and} \quad \frac{x}{j+1} = \frac{1}{2} \quad (2)$$

From (1), we have $r+1=3$

From (2) we have $2r=1+1$

Hence, by subtraction,

$$x-1=1 \quad r=2$$

Therefore, from (1), $\frac{2+1}{3}=1, \quad 1=3$

Thus the fraction is $\frac{2}{3}$

8 Let $\frac{x}{y}$ represent the fraction

Then we have $\frac{r+1}{3}=\frac{1}{2} \quad (1)$

and $\frac{x}{3+1}=\frac{1}{3} \quad (2)$

From (1) we have $2r-2=3$

From (2) we have $3x=3+1$

Hence by subtraction,

$$x-2=1, \quad x=3$$

Therefore, from (1), $\frac{3+1}{3}=\frac{1}{2}$

or $\frac{4}{3}=\frac{1}{2} \quad y=8$

Thus the fraction is $\frac{3}{8}$

9 Let A and B have Rs x and Rs y respectively

Then we have $x+y=39 \quad (1)$

and $x\left(1-\frac{2}{3}\right)+y\left(1-\frac{3}{4}\right)=11 \quad (2)$

From (2), we have

$$\frac{x}{3}+\frac{y}{4}=11$$

or $4x+3y=132,$

also, $3x+3y=117$ (multiplying (1) by 3)

Hence by subtraction, $x=15$

Therefore, from (1),

$$15+y=39,$$

$$y=24$$

Thus A has Rs 15 and B has Rs 24

10 Let x and y be the numbers of which x is the greater

Then we have $y + 7 = 2x$ (1)

and $x - 2 = 3y$. (2)

Multiplying (1) by 3 we have

$$3y + 21 = 6x$$

or $3y = 6x - 21$

also $3y = x + 2$. (2)

Hence, $5x - 23 = 0$, $x = 5$

Therefore, from (1),

$$y + 7 = 10, \quad y = 3$$

Thus the numbers are 5 and 3

11 Let the rates be x and y miles per hour respectively

Then we have $9(x - y) = 27$. (1)

and $3(x + y) = 27$. (2)

From (1), we have $x - y = 3$.

From (2), we have $x + y = 9$

Hence, by addition $2x = 12$ $x = 6$

Also, $y = 9 - x$

$$= 9 - 6$$

or $y = 3$

Thus the rates are 6 miles & 3 miles per hour

12 Let x be the number of sovereigns and $2x$ the number of half-crowns

Then we have $x - \left(\frac{5}{2 \times 20}\right) \times 2x = 10$

or $x - \frac{x}{4} = 10$

or $\frac{3}{4}x = 10$ $x = 8$

Thus the number of sovereigns is 8 and of half crowns is 16

13 Let one man can do the work in x days and one boy

can do it in y days Therefore, in one day one man can do $\frac{1}{x}$ of the work and one boy can do $\frac{1}{y}$ of the work

Then we get
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{15} \quad (1)$$

$$\frac{7}{1} + \frac{9}{y} = \frac{1}{2} \quad (2)$$

From (1), we have
$$\frac{9}{x} + \frac{9}{y} = \frac{3}{5}$$

also,
$$\frac{7}{x} + \frac{9}{y} = \frac{1}{2} \quad (2)$$

Hence, by subtraction,

$$\frac{2}{x} = \frac{1}{10}, \quad x = 20$$

Thus one man would do the work in 20 days

14 Let x and y yards be the length and breadth respectively of the rectangle

Then we have the area of the rectangle

$$= xy = (x+6)(y-4) = (x+8)(y-5)$$

Hence, $xy = xy + 6y - 4x - 24 \quad (1)$

and $xy = xy + 8y - 5x - 40 \quad (2)$

From (1), we have $6y - 4x - 24 = 0$

From (2), we have $8y - 5x - 40 = 0$

Therefore,
$$\frac{y}{160-120} = \frac{x}{192+240} = \frac{1}{-30+32}$$

or
$$\frac{y}{40} = \frac{x}{48} = \frac{1}{2}$$

$$y = 20, \quad x = 24$$

Thus the area $= xy = 20 \times 24 = 480$ sq yds

15 Let $\text{£}x$ and $\text{£}y$ be the price of tea and coffee respectively per pound

Then we have

$$15x + 17y = 3\frac{66}{240} = \frac{131}{40} \quad (1)$$

and $25x + 13y = 4\frac{74}{240} = \frac{517}{120}$ (2)

Multiplying (1) by 5 and (2) by 3, we have

$$75x + 85y = \frac{131}{8},$$

and $75x + 39y = \frac{517}{40}$

Hence by subtraction,

$$46y = \frac{655 - 517}{40} = \frac{138}{40}$$

$$y = \frac{3}{40}$$

Therefore, from (1),

$$15x + \frac{51}{40} = \frac{131}{40}$$

or $15x = \frac{131}{40} - \frac{51}{40} = \frac{80}{40} = 2,$

$$x = \frac{2}{15}$$

Thus the price of tea per lb is £ $\frac{2}{15}$ or 2s 8d and the price of coffee per lb is £ $\frac{3}{40}$ or 1s 6d

16 Let the rates of A and B be x and y miles respectively per hour

Then we have

$$\frac{30}{x} - 3 = \frac{30}{y} \quad (1)$$

Also, we have $\frac{30}{2x} + 2 = \frac{30}{y} \quad (2)$

Hence, by subtraction,

$$\frac{30}{2x} - 5 = 0$$

or $\frac{30}{2x} = 5$

or $10x = 30$

$$x = 3$$

Therefore, from (1),

$$\frac{30}{3} - 3 = \frac{30}{y'}$$

or $10 - 3 = \frac{30}{y'}$

or $7 = \frac{30}{y'}$

$$y' = \frac{30}{7} = 4\frac{2}{7}$$

Thus *A*'s rate is 3 miles and *B*'s rate is $4\frac{2}{7}$ miles

17 Let *x* and *y* be the number of marbles belonging to Charles and William respectively

Then we have

$$x + 10 = 2(y - 10) \quad (1)$$

and $y + 10 = 3(x - 10) \quad (2)$

From (1), we have $x - 2y + 30 = 0$

From (2), we have $3x - y - 40 = 0$

Therefore,

$$\frac{x}{80 + 30} = \frac{y'}{90 + 40} = \frac{1}{-1 + 6}$$

or $\frac{x}{110} = \frac{y}{130} = \frac{1}{5}$

$$x = 22 \text{ and } y = 26$$

Thus Charles had 22 marbles and William has 26 marbles

18 Let *A*, *B*, *C* receive *x*, *y*, *z* rupees respectively

Then we have

$$x + y + z = 1100 \quad (1)$$

$$y + 200 = 2(x - 200) = 3z \quad (2)$$

Multiplying (1), by 2 we have

$$2x + 2y + 2z = 2200$$

and from (2), $2x - y = 600$

Hence, by subtraction

$$3y + 2z = 1600,$$

also, $-3y + 9z = 600$ [from (2)]

Hence, by addition $11z = 2200$, $z = 200$

Therefore, from (2), $y + 200 = 600$,

$$y = 400$$

Therefore, from (1)

$$1 + 400 + 200 = 1100,$$

$$x = 500$$

Thus A receives Rs 500, B Rs 400 and C Rs 200

19 Let x and y be the digits in the tens and units place respectively

Evidently the number $10x + y$

Then we have

$$10x + y = 6(x + y) + 3$$

$$\text{or} \quad 4x - 5y - 3 = 0 \quad (1)$$

$$\text{and} \quad 10y + 1 = 4(1 + y) + 9$$

$$\text{or} \quad -3x + 6y - 9 = 0$$

$$\text{or} \quad x - 2y + 3 = 0 \quad (2)$$

Multiplying (2) by 4, we have

$$4x - 8y + 12 = 0$$

$$\text{also,} \quad 4x - 5y - 3 = 0 \quad (1)$$

Hence, by subtraction

$$3y = 15,$$

$$y = 5$$

Therefore, from (2),

$$1 - 10 + 3 = 0,$$

$$x = 7$$

Thus the number is 75

20 Let the digits be x and y in the tens and units place respectively

Evidently the number $= 10x + y$

Then we have

$$(10x + y) + (10y + x) = 121$$

$$\text{or} \quad 11x + 11y = 121$$

$$\text{or} \quad x + y = 11 \quad (1)$$

$$\text{and} \quad (10x + y) - (10y + 1) = 9$$

$$\text{or} \quad 9x - 9y = 9$$

$$\text{or} \quad x - y = 1 \quad (2)$$

Adding (1) and (2), we have

$$\begin{aligned} 2x &= 12, \\ x &= 6 \end{aligned}$$

Therefore, from (1),

$$6 + y = 11, \quad y = 5$$

Thus the number is 65

21 Let x and y be the numbers of crowns and half-guineas respectively

Then we have

$$5x + \frac{21y}{2} = 21 \times 25$$

$$\text{or} \quad 10x + 21y = 1050 \quad (1)$$

$$\text{and} \quad 2y = 3x + 17$$

$$\text{or} \quad 3x - 2y = -17 \quad (2)$$

Multiplying (1) by 3, and (2) by 10, we have

$$30x + 63y = 3150,$$

$$\text{and} \quad 30x - 20y = -170$$

Hence, by subtraction,

$$83y = 3320, \quad y = 40$$

Therefore, from (2),

$$3x - 80 = -17$$

$$\text{or} \quad 3x = 63, \quad x = 21$$

Thus there were 21 crowns and 40 half-guineas

22 Let the price of each horse be £ x and that of each cow £ y

Then we have

$$9x + 7y = 300 \quad (1)$$

$$\text{and} \quad 6x + 13y = 300 \quad (2)$$

Multiplying (1) by 2, and (2) by 3, we have

$$18x + 14y = 600,$$

$$\text{and} \quad 18x + 39y = 900$$

Hence, by subtraction,

$$25y = 300, \quad y = 12$$

Therefore, from (1),

$$9x + 84 = 300$$

$$\text{or} \quad 9x = 216, \quad x = 24$$

Thus the price of each horse = £24,
and cow = £12

23 Let x and y be the respective wages per day of A and B

Now since $£5 \quad 17s = 217s$
we have $15x + 14y = 217 \quad (1)$

also, $4x - 3y = 11 \quad (2)$

Multiplying (1) by 4 and (2) by 15, we have

$$60x + 56y = 868,$$

$$\text{and } 60x - 45y = 165$$

Hence, by subtraction,

$$101y = 303, \quad y = 3$$

Therefore, from (2), $4x - 9 = 11$

$$\text{or} \quad 4x = 20, \quad x = 5$$

Thus the wages of A and B are 5s and 3s respectively

24 Let A and B do the work in x and y days respectively
Evidently they can do

$$\left(\frac{1}{x} + \frac{1}{y}\right) \text{ of the work in one day}$$

Then we have $\frac{1}{x} + \frac{1}{y} = \frac{1}{16} \quad (1)$

$$\text{and} \quad \frac{1}{x} + \frac{4}{y} + \frac{36}{y} = 1$$

$$\text{or} \quad \frac{4}{x} + \frac{40}{y} = 1$$

$$\text{or} \quad \frac{1}{x} + \frac{10}{y} = \frac{1}{4} \quad (2)$$

Subtracting (1) from (2), we have

$$\frac{9}{y} = \frac{3}{16}$$

$$\text{or} \quad 3y = 144, \quad y = 48$$

I therefore, from (1), $\frac{1}{x} + \frac{1}{48} = \frac{1}{16}$

$$\text{or} \quad \frac{1}{x} = \frac{1}{16} - \frac{1}{48} = \frac{3-1}{48} = \frac{2}{48} = \frac{1}{24},$$

$$x = 24$$

Thus x and y would each separately do the work in 24 and 48 days respectively

25 Let the fraction be represented by $\frac{x}{y}$

Then we have

$$\frac{x+2}{y+1} = \frac{5}{8}$$

or $8x+16=5y+5$

or $8x-5y=-11$ (1)

and $\frac{x-1}{y-1} = \frac{1}{2}$

or $2x-2=y-1$

or $2x-y=1$ (2)

Multiplying (2) by 5, we have

$$10x-5y=5$$

also, $8x-5y=-11$ (1)

Hence, by subtraction

$$2x=16, \quad x=8$$

Therefore, from (2), $16-y=1, \quad y=15$

Thus the fraction is $\frac{8}{15}$

26 Let x miles be the distance and y hours the time taken by the traveller to walk the distance. Evidently his rate is $\frac{x}{y}$ miles per hour

Then we have

$$\frac{x}{\frac{1}{y} + \frac{1}{2}} = \frac{4y}{5}$$

or $5x=4x+2y$

or $x-2y=0$ (1)

and $\frac{x}{\frac{1}{y} - \frac{1}{2}} = y + 2\frac{1}{2}$

or $\frac{2xy}{2x-y} = \frac{2y+5}{2}$

or $4xy=4xy-2y^2+10x-5y$

or $10x-5y-2y^2=0$ (2)

From (1), we have $x = 2y$

Therefore, from (2),

$$20y - 5y - 2y^2 = 0$$

or $15y = 2y^2$

or $15 = 2y$, $y = \frac{15}{2} = 7\frac{1}{2}$

Therefore $x = 2y = 15$

Thus 15 miles is the distance

27 As the number lies between 10 and 100, the number of digits must be two and let them be x and y in the tens and units place respectively

Then we have

$$10x + y = 8(x + y)$$

or $2x - 7y = 0$ (1)

and $10x + y - 45 = 10y + 1$

or $x - 9y = 45$

or $x - y = 5$ (2)

Multiplying (2) by 2, we have

$$2x - 2y = 10,$$

also, $2x - 7y = 0$ (1)

Hence, by subtraction,

$$5y = 10 ; \quad y = 2$$

Therefore, from (2), $x - 2 = 5$, $x = 7$

Thus the number is 72

28 Let A and B have x and y shillings respectively

Then we have

$$x - 10 = 2(y + 10) - 25$$

or $x - 10 = 2y + 20 - 25$

or $x - 2y = 5$ (1)

and $x + (x + 10) = y - 10$

or $5x + 50 = 17y - 170$

or $5x - 17y = -220$ (2)

Multiplying (1) by 5, we have

$$5x - 10y = 25,$$

also, $5x - 17y = -220$ (2)

Hence, by subtraction,

$$7y = 245 , \quad y = 35$$

Therefore, from (1),

$$x - 70 = 5 \quad x = 75$$

Thus A and B have 75s and 35s respectively

29 $\pounds 2\ 2s = \pounds 2\ 1\ 0 = \pounds 2\ 1\ 0$, and $\pounds 1\ 8s = \pounds 1\ 8\ 0 = \pounds 1\ 8\ 0$

Let x be the number of sheep and y the number of pounds the farmer had

Then we have

$$y = \frac{21x}{10} - \frac{7}{5} \quad (1)$$

and $y = 2x + 2 \quad (2)$

Therefore, from (1) and (2), we have

$$\frac{21x}{10} - \frac{7}{5} = 2x + 2$$

or $x\left(\frac{21}{10} - 2\right) = 2 + \frac{7}{5} = \frac{17}{5}$

or $\frac{x}{10} = \frac{17}{5}, \quad x = 34$

Therefore, from (2), $y = 68 + 2 = 70$

Thus there were 34 sheep and the farmer had 70 pounds

30 Let the digits be x and y in the tens and units place respectively

The number is evidently $10x + y$

Then we have

$$10x + y = 3(x + y)$$

or $7x = 2y$

or $x = \frac{2}{7}y \quad (1)$

and $3(10x + y) = (x + y)^2 \quad (2)$

Substituting in (2), the value of x as found in (1), we have

$$3\left(\frac{20}{7}y + y\right) = \left(\frac{2}{7}y + y\right)^2$$

or $3y\left(\frac{20}{7} + 1\right) = y^2\left(\frac{2}{7} + 1\right)^2$

or $3\left(\frac{27}{7}\right) = y\left(\frac{9}{7}\right)^2$

or $\frac{81}{7} = y \cdot \frac{81}{49}$,

$$y = \frac{81}{7} \times \frac{49}{81} = 7$$

Therefore, from (1), $x = \frac{2}{7} \times 7 = 2$

Thus the number is 27

Exercise 77.

1 Let x , y and z be the digits in the hundreds, tens and units place respectively

$$\text{the required number} = 100x + 10y + z$$

Then we have

$$100x + 10y + z = 25(x + y + z)$$

$$\text{or } 75x - 15y - 24z = 0$$

$$\text{or } 25x - 5y - 8z = 0 \quad (1)$$

$$\text{Again } 100x + 10y + z + 198 = 100z + 10y + x$$

$$\text{or } 99x - 99z + 198 = 0$$

$$\text{or } x - z + 2 = 0 \quad (2)$$

$$\text{Also } x + y + z = 1$$

$$\text{or } x + y + z = 1 \quad (3)$$

Multiplying (3) by 5, we have

$$5x + 5y + 5z = 5$$

$$\text{also } 25x - 5y - 8z = 0 \quad (1)$$

by subtraction

$$20x - 13z = -5 \quad (4)$$

Multiplying (2) by 13, we have

$$13x - 13z = -26$$

$$\text{also } 20x - 13z = -5 \quad (4)$$

by subtraction,

$$7x = 21 \quad x = 3$$

$$\text{From (2) } 3 - z + 2 = 0 \quad z = 5$$

Therefore from (3)

$$3 + y + 5 = 1 \quad y = 7$$

Thus the number is 375

2 Let x lbs be the weight and x s per pound be the prime cost of the article.

xy s is the cost of the whole.

$$\text{We have } 32x = xy + 100 \quad (1)$$

$$\text{and } 22x = xy - 300 \quad (2)$$

$$\text{by subtraction } 8x = 400, \quad x = 50$$

From (1), $1500 = 50y + 100$

or $30 = y + 2$, $y = 28$

Thus the weight is 50 lbs and the cost per pound is 28 s

3 Let A and B have x and y shillings respectively

Then we have

$$x + \frac{x}{2} + 15 = 3\left(y - \frac{x}{2}\right)$$

or $3x - 3y + 15 = 0$

or $x - y + 5 = 0$ (1)

and $y - \frac{x}{2} + 10 = 2\left(x + \frac{x}{2} - 10\right)$

or $y - \frac{7x}{2} + 30 = 0$

or $7x - 2y - 60 = 0$ (2)

Multiplying (1) by 2, we have

$$2x - 2y + 10 = 0,$$

also, $7x - 2y - 60 = 0$

by subtraction,

$$5x - 70 = 0, \quad x = 14$$

From (1), $14 - y + 5 = 0$, $y = 19$

Thus A had 14s and B had 19s

4 Suppose A can do the work in x days, B in y days and C in z days

Therefore, in one day A , B and C can respectively do

$\frac{1}{x}$, $\frac{1}{y}$ and $\frac{1}{z}$ of the work

Then we have $\frac{12}{x} + \frac{12}{y} = 1$ (1)

$$\frac{15}{y} + \frac{30}{z} = 1$$
 (2)

and $\frac{10}{x} + \frac{10}{y} + \frac{10}{z} = 1$ (3)

Multiplying (3) by 3, we have

$$\frac{30}{x} + \frac{30}{y} + \frac{30}{z} = 3,$$

A has $3s\ 6d$,
and B has $4s\ 2d$

- 6 Suppose A can do the work in x days and B in y days
in one day A can do $\frac{1}{x}$ of the work and B $\frac{1}{y}$ of the work

Then we have
$$\frac{m}{x} + \frac{m}{y} = 1 \quad (1)$$

and
$$\frac{n}{x} + \frac{n+p}{y} = 1 \quad (2)$$

Multiplying (1) by n and (2) by m , we have

$$\frac{mn}{x} + \frac{mn}{y} = n,$$

and
$$\frac{mn}{x} + \frac{mn+mp}{y} = m$$

Hence, by subtraction,

$$\frac{mp}{y} = m - n$$

$$y = \frac{mp}{m - n}$$

From (1),
$$\frac{m}{x} + \frac{m - n}{p} = 1$$

or
$$\frac{m}{x} = 1 - \frac{m - n}{p} = \frac{p - m + n}{p},$$

$$x = \frac{mp}{p - m + n}$$

Thus A can do the work in $\frac{mp}{p - m + n}$ days and B in $\frac{mp}{m - n}$

days

- 7 Let A have Rs x , B Rs y and C Rs z

Then we have

$$x + 700 = 2(y - 700) \quad (1)$$

$$y + 1400 = 3(z - 1400) \quad (2)$$

$$z + 420 = 5(x - 420) \quad (3)$$

From (1), we have

$$x - 2y + 2100 = 0,$$

also, $2y - 6z + 11200 = 0$ [multiplying (2) by 2]

Hence, by addition,

$$x - 6z + 13300 = 0 \quad (4)$$

Multiplying (4) by 5, we have

$$5x - 30z + 66500 = 0$$

Also, $5x - z - 2520 = 0$ [from (3)]

by subtraction,

$$29z - 69020 = 0,$$

$$z = 2380,$$

from (3), $2380 + 420 = 5x - 2100,$

or, $5x = 4900,$

$$x = 980,$$

from (1), $980 + 700 = 2y - 1400$

or, $2y = 3080, \quad y = 1540$

Thus A has Rs 980, B Rs 1540 and C Rs 2380

8 Suppose the man walked x hours at the rate of 4 miles an hour and y hours at the rate of 5 miles an hour,

he walked for $(x + y)$ hours,

We have $4x + 5y = 35$ (1)

and $5x + 4y = 37$ (2)

or, $4x + 5y - 35 = 0,$

and $5x + 4y - 37 = 0$

$$\frac{x}{-185 + 140} = \frac{y}{-175 + 148} = \frac{1}{16 - 25},$$

or, $\frac{x}{-45} = \frac{y}{-27} = \frac{1}{-9},$

$$\therefore x = 5, \text{ and } y = 3,$$

$$x + y = 8$$

Thus he walked for 8 hours

9 Let x miles be the required distance and y miles per hour the rate at which the train travelled

Then the journey occupied $\frac{x}{y}$ hours

We have $\frac{x}{y+6} = \frac{x}{y} - 4$ (1)

Multiplying (2) by 3, we have

$$\frac{3x}{4} + \frac{5y}{2} = \frac{21}{2},$$

$$\text{also, } \frac{3x}{4} + \frac{y}{6} = \frac{7}{2} \quad (1)$$

$$\text{Hence, by subtraction, } \frac{14y}{6} = 7,$$

$$y = 3,$$

$$x = 4$$

Therefore, 4 gallons and 3 gallons were drawn out from the 1st and the 2nd vessels respectively

11 Let x be the digit in the hundreds' place and y the digit in the units' place

$$\text{the digit in the tens' place} = (x + y)$$

$$\begin{aligned} \text{The required number} &= 100x + 10(x + y) + y \\ &= 110x + 11y \end{aligned}$$

$$\text{We have } x + (x + y) + y = 10$$

$$\text{or, } 2x + 2y = 10$$

$$\text{or, } x + y = 5 \quad (1)$$

$$\text{Also we have } 100y + 10(x + y) + x = 99 + 110x + 11y$$

$$\text{or, } 110y + 11x = 99 + 110x + 11y$$

$$\text{or, } 99x - 99y = -99$$

$$\text{or, } x - y = -1 \quad (2)$$

$$\text{by adding (1) and (2), } 2x = 64,$$

$$x = 2,$$

$$\text{from (1), } y = 3$$

Thus the number is 253

12 Let the man have x half-crowns, y shillings and z six pences

$$\text{Then we have } x + y + z = 20 \quad (1)$$

$$\frac{5x}{2} + y + \frac{z}{2} = 20 \quad (2)$$

$$\text{and } x + 2y + 6z = 73 \quad (3)$$

From (1) and (2) by subtraction, we have

$$\frac{3x}{2} - \frac{z}{2} = 0,$$

or, $3x - z = 0$ (4)

Multiplying (1) by 2, we have

$$2x + 3y + 2z = 40,$$

also, $x + 2y + 6z = 73$ (5)

by subtraction,

$$x - 4z = -33$$
 (5)

and from (4), $12x - 4z = 0,$

by subtraction,

$$-11x = -33,$$

$$x = 3.$$

from (5), $3 - 4z = 33,$

or, $-4z = -36,$

$$z = 9.$$

from (1), $3 - y + 9 = 20$

$$y = 8$$

Thus the man has 3 half-crowns, 8 shillings and 9 six-pences

13 Let there be x men and let each receive y shillings

the sum divided is 77 shillings

Then we have

$$(x+4)(y-1) = 77,$$

or $4y - x - 4 = 0$ (1)

and $(x-5)(y+2) = 77,$

or, $2x - 5y - 10 = 0$ (2)

From (1), $8y - 2x - 8 = 0,$

by addition, $3y - 18 = 0$

$$y = 6,$$

from (1), $24 - x - 4 = 0,$

or, $-x = -20,$

$$x = 20$$

Thus there are 20 persons and each receives 6s

14 Let each of the equal stop cocks fill the cistern in x hours and the remaining one in y hours. Evidently each of the equal cocks can fill $\frac{1}{x}$ and the third cock $\frac{1}{y}$ of the cistern in one hour.

Then we have
$$\frac{8}{x} + \frac{4}{y} = \frac{5}{12} \quad (1)$$

and
$$\frac{10^2}{x} + \frac{10^1}{y} = \frac{7}{9}$$

or,
$$\frac{32}{x} + \frac{32}{y} = \frac{7}{3}$$

Multiplying (1) by 4, we have

$$\frac{32}{x} + \frac{16}{y} = \frac{5}{3}$$

also
$$\frac{32}{x} + \frac{32}{y} = \frac{7}{3} \quad (2)$$

by subtraction,

$$\frac{16}{y} = \frac{2}{3}$$

or
$$2y = 48,$$

$$y = 24$$

from (1),
$$\frac{8}{x} + \frac{1}{6} = \frac{5}{12}$$

or
$$\frac{8}{x} = \frac{1}{4},$$

$$x = 32$$

Therefore, each of the equal cocks can fill the cistern in 32 hours and the remaining cock in 24 hours.

15 Let x shillings and y shillings be the price per bushel of wheat and barley respectively, also let the person offer to sell m bushels of wheat.

Then we have

$$12x = 8y + 56 \quad (1)$$

and
$$xm = 200, \quad (2)$$

also,
$$ym = 200 - 75 = 125 \quad (3)$$

Dividing (2) by (3), we have

$$\frac{x}{y} = \frac{200}{125} = \frac{8}{5}$$

or, $5x = 8y$

from (1),

$$12x = 5x + 56$$

or, $7x = 56$

$$x = 8$$

from (1),

$$96 = 8y + 56$$

or, $8y = 40$

$$y = 5$$

Thus there were 8 bushels of wheat and 5 bushels of barley

16 Let him take x quarts from the 1st sort and y quarts from the 2nd

Then we have

$$x + y = 100 \quad (1)$$

and $2x + 3\frac{1}{2}y = 2\frac{1}{2} \times 100$

or, $6x + 10y = 700$

or, $3x + 5y = 350 \quad (2)$

also, from (1), $3x + 3y = 300,$

by subtraction

$$2y = 50,$$

$$y = 25,$$

from (1) $x + 25 = 100,$

$$x = 75$$

Therefore, he must take 75 quarts from the 1st sort and 25 quarts from the 2nd sort

17 Let the corn rent be x quarters of wheat and y quarters of barley

Then we have

$$55x = 33y$$

or, $5x = 3y \quad (1)$

and $65x + 41y = 55x + 33y + 140$

$$\begin{aligned}
 \text{or,} & \quad 10x + 8y = 140 \\
 \text{or,} & \quad 5x + 4y = 70, \\
 \text{also,} & \quad 5x - 3y = 0 \quad (1) \\
 & \text{by subtraction,} \\
 & \quad 7y = 70, \\
 & \quad y = 10, \\
 \text{from (1)} & \quad 5x = 30, \\
 & \quad x = 6
 \end{aligned}$$

Thus the corn rent is 6 quarters of wheat and 10 quarters of barley

18 Let x and y miles per hour be the rates of the larger and the smaller trains respectively

Then we have

$$\begin{aligned}
 & \frac{60 + 72}{1760(x - y)} = \frac{12}{60 \times 60} \\
 \text{or} & \quad \frac{132}{1760(x - y)} = \frac{12}{3600} \\
 \text{or,} & \quad \frac{11}{176(x - y)} = \frac{1}{360} \\
 \text{or,} & \quad \frac{1}{16(x - y)} = \frac{1}{360} \\
 \text{or,} & \quad \frac{1}{x - y} = \frac{1}{45} \\
 \text{or,} & \quad 2x - 2y = 45 \quad (1) \\
 \text{and} & \quad \frac{60 + 72}{1760\left(x - \frac{3y}{2}\right)} = \frac{24}{60 \times 60} \\
 \text{or,} & \quad \frac{1}{2\left(x - \frac{3y}{2}\right)} = \frac{2}{45} \\
 \text{or,} & \quad 4x - 6y = 45 \quad (2) \\
 \text{also,} & \quad 4x - 4y = 90 \quad (1) \\
 & \text{by subtraction, } 2y = 45, \\
 & \quad y = 22\frac{1}{2},
 \end{aligned}$$

and from (1), $2x - 45 = 45$ or, $2x = 90$, $x = 45$

Thus the rates are 45 and $22\frac{1}{2}$ miles per hour

19. Let x bushels of rye and y bushels of wheat be mixed with the 28 bushels of barley.

$$\text{Then } x + y = 100 - 28 = 72. \quad (1)$$

$$\text{and } 28 \times \frac{1}{7} + 3x + 4y = 100 \times \frac{1}{3}$$

$$\text{or, } 9x + 12y = 1000 - 196 = 804$$

$$\text{or, } 3x + 4y = 268 \quad (2)$$

$$\text{also, from (1), } 3x + 3y = 216,$$

$$\text{by subtraction, } y = 52,$$

$$\text{from (1) } x = 20$$

Thus he must mix 20 bushels of rye and 52 bushels of wheat

20 Suppose the person had x guineas and y crowns at first, and z guineas and w crowns after he had paid off the debt

The amount left with the person after paying off the debt

$$= £27 \ 6s - £14 \ 17s = 546s - 297s = 249s$$

Then we have

$$21x + 5y = 546 \quad (1)$$

$$21z + 5w = 249 \quad (2)$$

$$z = y - w \quad (3)$$

$$\text{and } w = x - z \quad (4)$$

From (3) and (4), we have

$$y = z + w = z$$

Hence, from (1),

$$21x + 5z = 546$$

$$\text{or, } 26z = 546$$

$$z = 21,$$

$$\text{and } y = 21,$$

$$\text{Again, since } z + w = 21,$$

$$\text{we have } 5z + 5w = 105,$$

$$\text{also, } 21z + 5w = 249$$

$$\text{Hence, } 16z = 144,$$

$$z = 9$$

Hence

$$w = 21 - 9 = 12$$

Thus the man had 21 guineas and 21 crowns at first and had 9 guineas and 12 crowns left

21 Let x miles per hour be the rate of the tide in the middle of the river and y miles per hour the rate of the man independent of the tide

$$\begin{aligned} \text{Then we have} \quad & \frac{1}{2}(x + y) = 18 \\ \text{or,} \quad & x + y = 36 \quad (1) \\ \text{and} \quad & 2\frac{1}{2}(y - x) = 18 \\ \text{or} \quad & 5y - 3x = 40 \quad (2) \end{aligned}$$

Multiplying (1) by 5, we have

$$5x + 5y = 60$$

$$\text{also} \quad 5y - 3x = 40 \quad (2)$$

by subtraction,

$$8x = 20, \quad x = 2\frac{1}{2}$$

Thus the rate of the tide in the middle is $2\frac{1}{2}$ miles per hour

22 Let A run a mile in x minutes and B in y minutes

Then we have

$$\begin{aligned} & \frac{1}{2} - \frac{44y}{1760} - \frac{51}{60} = 1 \\ \text{or,} \quad & 120y - 3y - 102 = 1201 \\ \text{or,} \quad & 117y - 120x = 102 \\ \text{or} \quad & 39y - 40x = 34 \quad (1) \\ \text{and} \quad & \frac{1}{2} - 1\frac{1}{4} = x - \frac{88}{1760}x = x - \frac{1}{20} \\ \text{or,} \quad & 20y - 25 = 20x - x \\ \text{or,} \quad & 20y - 19x = 25 \quad (2) \end{aligned}$$

From (1) and (2), we have

$$40x - 39y + 34 = 0$$

$$\text{and} \quad 19x - 20y + 25 = 0,$$

$$\frac{x}{-975 + 680} = \frac{y}{646 - 1000} = \frac{1}{-800 + 741}$$

$$\text{or,} \quad \frac{x}{-295} = \frac{y}{-354} = \frac{1}{-59},$$

$$x = 5 \text{ and } y = 6$$

Thus A can run a mile in 5 minutes and B in 6 minutes

23 Let A run x miles an hour and B , miles an hour

Then we have

$$\frac{2}{x} - \frac{2}{j} = \frac{2}{60}$$

or,
$$\frac{1}{x} - \frac{1}{j} = \frac{1}{60} \quad (1)$$

and
$$\frac{2}{j-2} - \frac{2}{x-2} = \frac{2}{60}$$

or
$$\frac{1}{j-2} - \frac{1}{x-2} = \frac{1}{60} \quad (2)$$

From (1), we have $60(j-x) = xj$

From (2) we have $60(j-2)(x-2) = (x-2)(j-2)xj$

or $60(x-j) - 240 = xj - 2(j-x) - 4$

or, $62(x-j) = xj - 244$

Hence, by subtraction

$$122(j-x) = 244$$

$\therefore j-x=2 \quad (3)$

from (1),
$$\frac{j-x}{xj} = \frac{1}{60}$$

or
$$\frac{2}{xj} = \frac{1}{60} \quad xj = 120$$

From (3)
$$xj - x^2 = 2x$$

or
$$120 - x^2 = 2x$$

or
$$2x - x^2 - 120 = 0$$

or
$$(x-1)^2 = 121$$

or,
$$x+1=11, \quad x=10$$

from (3),

$$j-10=2$$

$$j=12$$

Thus A ran 10 miles an hour and B 12 miles an hour

24 Let x miles per hour be the required rate, and j miles the distance from Cambridge where the accident happened. Therefore, with the original speed the train could have reached Cambridge in $\frac{j}{x}$ from the place of the accident.

Then we have
$$\frac{\frac{y'}{1} \times x}{n} = \frac{j}{1} + a$$

or,
$$\frac{ny'}{1} = \frac{j}{1} + a$$

or,
$$j(n-1) = ax \quad (1)$$

and
$$\frac{\frac{j-b}{1} \times x}{n} = \frac{y-b}{1} + c$$

or,
$$\frac{n(j-b)}{1} = \frac{y-b}{1} + c$$

or,
$$(n-1)(j-b) = cx \quad (2)$$

Now dividing (1) by (2), we have

$$\frac{j}{j-b} = \frac{a}{c}$$

or,
$$cy = ay - ab$$

or,
$$j(a-c) = ab,$$

$$j = \frac{ab}{a-c}$$

from (1),

$$\frac{ab}{a-c}(n-1) = ax$$

$$x = \frac{b(n-1)}{a-c}$$

Thus $\frac{b(n-1)}{a-c}$ miles per hour is the rate of the train before

the accident

25. Let x miles be the distance of the place of accident from the terminus and j miles per hour be the rate of travelling before the accident

Then we have
$$\frac{x}{3j} = \frac{x}{j} + (3-1)$$

or,
$$\frac{5x}{3j} = \frac{x}{j} + 2$$

$$\text{or,} \quad \frac{2}{3} \frac{x}{y} = 2$$

$$\text{or,} \quad x = 3y \quad (1)$$

$$\text{and} \quad \frac{x-50}{\frac{3y}{5}} = \frac{x-50}{y} + (2 - 1\frac{1}{3})$$

$$\text{or,} \quad \frac{2}{3} \frac{x-50}{y} = \frac{2}{3}$$

$$\text{or,} \quad x - 50 = y$$

$$\text{or,} \quad x = y + 50 \quad (2)$$

from (1) and (2),

$$3y = y + 50$$

$$\text{or,} \quad 2y = 50,$$

$$y = 25$$

$$x = 75$$

Thus the train travelled at first at the rate of 25 miles per hour

Therefore, the length of the journey before the accident = 25 miles and the total length of the journey is (25 + 75) or 100 miles

Miscellaneous Exercises (3).

I.

$$\begin{aligned} 1 \quad & (a^2 + ax + x^2)(a^2 - ax + x^2) \\ & = \{(a^2 + x^2) + ax\} \{(a^2 + x^2) - ax\} \\ & = (a^2 + x^2)^2 - a^2x^2 \\ & = a^4 + a^2x^2 + x^4 \end{aligned}$$

$$\begin{aligned} 2 \quad & (ac - bd)^2 + (ad + bc)^2 \\ & = a^2c^2 - 2acbd + b^2d^2 + a^2d^2 + 2adbc + b^2c^2 \\ & = a^2(c^2 + d^2) + b^2(c^2 + d^2) \\ & = (a^2 + b^2)(c^2 + d^2) \end{aligned}$$

Each of these factors being equal to 1 the given expression is equal to 1

$$\begin{aligned} 3 \quad & x^3 + y^3 + 3xy - 1 \\ & = x^3 + y^3 + (-1)^3 - 3xy(-1) \end{aligned}$$

$$\begin{aligned}
 &= \{x+y+(-1)\}\{x^2+y^2+(-1)^2-xy-x(-1)-y(-1)\} \\
 &= (x+y-1)(x^2+y^2+1-xy+x+y)
 \end{aligned}$$

4

$$\begin{aligned}
 &\frac{2bc+(b^2+c^2-a^2)}{2bc-(b^2+c^2-a^2)} \\
 &= \frac{(b^2+c^2+2bc)+a^2}{a^2-(b^2+c^2-2bc)} \\
 &= \frac{(b+c)^2-a^2}{a^2-(b-c)^2} \\
 &= \frac{(b+c+a)(b+c-a)}{(a+b-c)(a-b+c)} \\
 &= \frac{(b+c+a)(b+c-a-2a)}{(a+b+c-2c)(a+b+c-2b)} \\
 &= \frac{2s(2s-2a)}{(2s-2c)(2s-2b)} = \frac{s(s-a)}{(s-c)(s-b)}
 \end{aligned}$$

5 The given expression

$$\begin{aligned}
 &= \left(\frac{x^6}{x^2-1} - \frac{1}{x^2-1} \right) - \left(\frac{x^4}{x^2+1} - \frac{1}{x^2+1} \right) \\
 &= \frac{x^6-1}{x^2-1} - \frac{x^4-1}{x^2+1} \\
 &= (x^4+x^2+1) - (x^2-1) \\
 &= x^4+2
 \end{aligned}$$

6

$$\begin{aligned}
 &\frac{x+4a+b}{x+a+b} + \frac{4x+a+2b}{x+a-b} = 5 \\
 \text{or, } &\frac{x+4a+b}{x+a+b} - 1 + \frac{4x+a+2b}{x+a-b} - 4 = 0 \\
 \text{or, } &\frac{x+4a+b-x-a-b}{x+a+b} + \frac{4x+a+2b-4x-4a-4b}{x+a-b} = 0 \\
 \text{or, } &\frac{3a}{x+a+b} + \frac{6b-3a}{x+a-b} = 0 \\
 \text{or, } &\frac{a}{x+a+b} = \frac{a-2b}{x+a-b} \\
 \text{or, } &ax+a^2-ab=ar+a^2+ab-2bx-2ab-2b^2 \\
 \text{or, } &-2bx-2b^2=0, \\
 &x=-b
 \end{aligned}$$

2 Putting a for $x+y$, b for $-(y+z)$ and c for $(z-x)$,

we have $a+b+c = x+y-y-z+z-x = 0$,

$$(x+y)^3 - (y+z)^3 + (z-x)^3$$

$$= a^3 + b^3 + c^3$$

$$= (a^3 + b^3 + c^3 - 3abc) + 3abc$$

$$= (a+b+c)(a^2+b^2+c^2-ab-ac-bc) + 3abc$$

$$= 3abc \quad [a+b+c=0]$$

$$= -3(x+y)(y+z)(z-x)$$

$$= 3(x+y)(y+z)(x-z)$$

3 We have $\left(x + \frac{1}{x}\right)\left(y + \frac{1}{y}\right)$

$$= 2(a+m)2(c+n),$$

$$xy + \frac{x}{y} + \frac{y}{x} + \frac{1}{xy} = 4(a+m)(c+n) \quad (1)$$

Again $\left(x - \frac{1}{x}\right)\left(y - \frac{1}{y}\right) = 2b2d$,

$$xy - \frac{x}{y} + \frac{y}{x} + \frac{1}{xy} = 4bd \quad (2)$$

By adding (1) and (2) we have

$$2xy + \frac{2}{xy} = 4(a+m)(c+n) + 4bd,$$

$$xy + \frac{1}{xy} = 2(a+m)(c+n) + 2bd$$

$$4 \quad \frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$$

$$\text{or, } \frac{1}{x-1} - \frac{1}{x-3} = \frac{3}{x-3} - \frac{2}{x-2}$$

$$\text{or, } \frac{x-3-1+1}{(x-1)(x-3)} = \frac{2(x-2-x+3)}{(x-3)(x-2)}$$

$$\text{or, } -2(x-2) = 2(x-1)$$

$$\text{or, } x-2-x+1=0$$

$$\text{or, } 2x-3=0, \quad x = \frac{3}{2}$$

$$5 \quad ax + 1 = by + 1 = ay + bx$$

From the first two, we have

$$ax = by$$

$$\text{or,} \quad y = \frac{ax}{b}$$

$$\text{Then we have} \quad ax + 1 = a \frac{ax}{b} + bx$$

$$= \frac{a^2x + b^2x}{b}$$

$$\text{or,} \quad abx + b = a^2x + b^2x$$

$$\text{or,} \quad x(a^2 + b^2 - ab) = b$$

$$x = \frac{b}{a^2 + b^2 - ab}$$

$$y = \frac{a}{b} \frac{b}{a^2 + b^2 - ab} = \frac{a}{a^2 + b^2 - ab}$$

$$6 \quad \text{We have} \quad x + y + z = 1 \quad (1)$$

$$\text{and} \quad 2x + 3y + z = 4 \quad (2)$$

$$\text{Hence,} \quad x + 2y = 3 \quad (4)$$

$$\text{Again,} \quad 2x + 3y + z = 4 \quad (2)$$

$$\text{also,} \quad 4x + 9y + z = 16 \quad (3)$$

$$\text{Hence,} \quad 2x + 6y = 12$$

$$\text{or,} \quad x + 3y = 6 \quad (5)$$

$$\text{from (4) and (5), } y = 3$$

$$\text{from (4), } x = 3 - 6 = -3$$

$$\text{and from (1), } z = 1 + 3 - 3 = 1$$

$$\text{Thus } x = -3, y = 3 \text{ and } z = 1$$

7 In one hour two pipes separately will fill $\frac{1}{a}$ and $\frac{1}{b}$ of the cistern

if x hours be the time required to fill it with the two pipes together we have

$$\frac{x}{a} + \frac{x}{b} = 1$$

$$\text{or,} \quad \frac{x(a+b)}{ab} = 1, \quad x = \frac{ab}{a+b}$$

The third pipe can empty $\frac{1}{c}$ of the cistern in 1 hour

if y hours be the time required to empty the cistern while the three pipes are working,

we have
$$\frac{y'}{c} - \frac{y'}{a} - \frac{y'}{b} = 1$$

or,
$$\frac{y(ab - bc - ac)}{abc} = 1, \quad y = \frac{abc}{ab - ac - bc}$$

Thus the cistern will be filled in $\frac{ab}{a+b}$ hours and emptied in $\frac{ab - ac - bc}{abc}$ hours

7 Let the fraction be represented by $\frac{x}{y}$

Then we have
$$\frac{x+6}{y} = \frac{3}{4}$$

or,
$$4x + 24 = 3y$$

or,
$$4x - 3y + 24 = 0 \quad (1)$$

and
$$\frac{x}{y-2} = \frac{1}{2}$$

or,
$$2x = y - 2$$

or,
$$2x - y + 2 = 0 \quad (2)$$

From (1) and (2) we get by cross multiplication,

$$\frac{x}{-6+24} = \frac{y}{48-8} = \frac{1}{-4+6}$$

or,
$$\frac{x}{18} = \frac{y}{40} = \frac{1}{2},$$

$$x = 9 \text{ and } y = 20$$

Thus the fraction is $\frac{9}{20}$

III

1 Putting a for $x^{\frac{1}{3}}$, b for $y^{\frac{1}{3}}$ and c for $z^{\frac{1}{3}}$, the divisor becomes $a+b+c$ and the dividend $= a^3 + b^3 + c^3 - 3abc$

$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca),$$

$$\begin{aligned}
 \text{the quotient} &= a^2 + b^2 + c^2 - ab - bc - ca \\
 &= (x^{\frac{1}{3}})^2 + (y^{\frac{1}{3}})^2 + (z^{\frac{1}{3}})^2 - x^{\frac{1}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{1}{3}} - y^{\frac{1}{3}}z^{\frac{1}{3}} \\
 &= x^{\frac{2}{3}} + y^{\frac{2}{3}} + z^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{1}{3}} - y^{\frac{1}{3}}z^{\frac{1}{3}}
 \end{aligned}$$

2 The first expression

$$\begin{aligned}
 &= a^2x^3 - 2abx^3 + b^2x^3 + a^3 + a^3b^3 - 2a^4b \\
 &= x^3(a^2 - 2ab + b^2) + a^3(a^2 + b^2 - 2ab) \\
 &= (x^3 + a^3)(a^2 - 2ab + b^2) \\
 &= (x + a)(x^2 - ax + a^2)(a - b)^2
 \end{aligned}$$

The second expression

$$\begin{aligned}
 &= a^2(2x^4 - 5a^2x^2 + 3a^4) - b^2(2x^4 - 5a^2x^2 + 3a^4) \\
 &= (a^2 - b^2)(2x^4 - 5a^2x^2 + 3a^4) \\
 &= (a^2 - b^2)\{2x^2(x^2 - a^2) - 3a^2(x^2 - a^2)\} \\
 &= (a + b)(a - b)(2x^2 - 3a^2)(x^2 - a^2) \\
 &= (a + b)(a - b)(2x^2 - 3a^2)(x + a)(x - a)
 \end{aligned}$$

Thus the H C F is $(a - b)(x + a)$

3 The given expression

$$\begin{aligned}
 &= \frac{1}{bc(a-b)(a-c)} - \frac{1}{ca(b-c)(a-b)} + \frac{1}{ab(a-c)(b-c)} \\
 &= \frac{a(b-c) - b(a-c) + c(a-b)}{abc(a-b)(b-c)(a-c)} \\
 &= 0
 \end{aligned}$$

4 We have $x - 1 = \frac{a}{a+b} - 1 = \frac{-b}{a+b}$

and $y + 1 = \frac{b}{a-b} + 1 = \frac{a}{a-b}$

$$\begin{aligned}
 \frac{x}{y} + \frac{x-1}{y+1} &= \frac{a(a-b)}{b(a+b)} - \frac{b(a-b)}{a(a+b)} \\
 &= \frac{a^2(a-b) - b^2(a-b)}{ab(a+b)} \\
 &= \frac{(a-b)(a^2 - b^2)}{ab(a+b)} = \frac{(a-b)^2}{ab}
 \end{aligned}$$

5 From the given equation

$$\begin{aligned}\frac{7x-29}{5x-12} &= \frac{8x+19}{18} - \frac{41+3}{9} \\ &= \frac{8x+19-81-6}{18} = \frac{13}{18},\end{aligned}$$

$$\text{or, } 18(7x-29) = 13(5x-12)$$

$$\text{or, } 126x - 65x = 522 - 156$$

$$\text{or, } 61x = 366, \quad x = 6$$

$$6 \quad \frac{3}{x} + \frac{2}{y} - 13 = 0$$

$$\text{and } \frac{7}{x} + \frac{3}{y} - 27 = 0,$$

$$\frac{1}{x(-54+39)} = \frac{1}{y(-91+81)} = \frac{1}{9-14}$$

$$\begin{aligned}\text{or, } \frac{1}{x(-15)} &= \frac{1}{y(-10)} = \frac{1}{-5}, \\ x &= 3 \text{ and } y = 1\end{aligned}$$

7 Let x minutes past 5 o'clock be the required time. At 5 o'clock the hands are 25 minute divisions apart, they will coincide when the minute-hand will travel 25 minute-divisions more than the hour-hand, and we also know that the minute-hand travels 12 times faster than the hour-hand

$$\text{we have } 12(x-25) = x$$

or,

$$11x = 300$$

$$x = \frac{300}{11} = 27\frac{1}{11}$$

Thus the hands will coincide at $27\frac{1}{11}$ minutes past 5 o'clock.

8 Let x be the digit in the hundreds' place. Then the digit in the tens' place is $\frac{5+x}{2}$,

$$\text{the required number} = 100x + 5(5+x) + 5 = 105x + 30$$

$$\text{Then we have } 105x + 30 + 108 = 100 \cdot \frac{(5+x)}{2} + 50 + x$$

$$= 250 + 50x + 50 + x$$

or

$$54x = 300 - 138 = 162,$$

$$x = 3$$

Thus the number is 345

IV.

1 The given expression

$$\begin{aligned}
 &= \{(a^2 + b^2) + ab \sqrt{2}\} \{(a^2 + b^2) - ab \sqrt{2}\} \\
 &= (a^2 + b^2)^2 - (ab \sqrt{2})^2 \\
 &= a^4 + 2a^2b^2 + b^4 - 2a^2b^2 \\
 &= a^4 + b^4
 \end{aligned}$$

2 (i) The 1st expression

$$\begin{aligned}
 &= (x^2 - ax^2) - (px^2 - apx) + (qx - aq) \\
 &= x^2(r - a) - px(x - a) + q(1 - a) \\
 &= (x - a)(r^2 - px + q)
 \end{aligned}$$

The 2nd expression

$$\begin{aligned}
 &= (x^3 - a^2x) + (ax^2 - 2a^2x + a^3) \\
 &= x(x^2 - a^2) + a(x^2 - 2ax + a^2) \\
 &= x(x + a)(x - a) + a(x - a)^2 \\
 &= (x - a)\{x(x + a) + a(x - a)\} \\
 &= (x - a)(x^2 + 2ax - a^2)
 \end{aligned}$$

Thus the H C F is $x - a$

(ii) The 1st expression

$$\begin{aligned}
 &= x^3 + (-y)^3 + (-z)^3 - 3x(-y)(-z) \\
 &= (x - y - z)(x^2 + y^2 + z^2 + xy + yz + xz)
 \end{aligned}$$

The 2nd expression

$$\begin{aligned}
 &= x^2 + (-y)^2 + (-z)^2 + 2x(-y) + 2x(-z) + 2(-y)(-z) \\
 &= (x - y - z)^2
 \end{aligned}$$

Thus the H C F is $x - y - z$

3 (i) The given expression

$$\begin{aligned}
 &= \frac{(a + b + c)^3}{a^3 - (b^3 + c^3 + 2bc)} \\
 &= \frac{(a + b + c)^3}{a^3 - (b + c)^3} = \frac{(a + b + c)^3}{(a + b + c)(a - b - c)} = \frac{a + b + c}{a - b - c}
 \end{aligned}$$

(ii)

$$\begin{aligned}
 &\frac{\frac{a}{a+b} + \frac{b}{a-b}}{\frac{a}{a-b} - \frac{b}{a+b}} = \frac{\frac{a^2 - ab + ab + b^2}{a^2 - b^2}}{\frac{a^2 + ab - ab + b^2}{a^2 - b^2}} \\
 &= \frac{a^2 + b^2}{a^2 - b^2} \times \frac{a^2 - b^2}{a^2 + b^2} = 1
 \end{aligned}$$

4 The given expression

$$\begin{aligned} &= \frac{(a+b)^2 + (a-b)^2}{a^2 - b^2} \cdot \frac{(a^{\frac{1}{2}} + b^{\frac{1}{2}})^2 - (a^{\frac{1}{2}} - b^{\frac{1}{2}})^2}{a - b} \\ &= \frac{2(a^2 + b^2)}{a^2 - b^2} \times \frac{a - b}{4a^{\frac{1}{2}}b^{\frac{1}{2}}} = \frac{a^2 + b^2}{2a^{\frac{1}{2}}b^{\frac{1}{2}}(a + b)} \end{aligned}$$

5 (i) From the given equation

$$56 + x - x^2 - 2\frac{1}{2}x = 17r + 1 - x^2$$

$$\text{or,} \quad 18\frac{1}{2}x = 55 \quad r = 3$$

$$(ii) \quad ax + y = x + by = \frac{1}{2}(x + y) + 1$$

$$\text{We have} \quad 2ax + 2y = x + y + 2$$

$$\text{or} \quad x(2a - 1) + y - 2 = 0 \quad (1)$$

$$\text{Again we have} \quad 2x + 2by = x + y + 2$$

$$\text{or,} \quad r + y(2b - 1) - 2 = 0 \quad (2)$$

From (1) and (2) by cross multiplication, we have

$$\frac{x}{-2 + 4b - 2} = \frac{y}{-2 + 4a - 2} = \frac{1}{4ab - 2a - 2b + 1 - 1}$$

$$\text{or,} \quad \frac{x}{4(b - 1)} = \frac{y}{4(a - 1)} = \frac{1}{2(2ab - a - b)},$$

$$x = \frac{2(b - 1)}{2ab - a - b} \quad \text{and} \quad y = \frac{2(a - 1)}{2ab - a - b}$$

6 Putting x for $(a + b)$, y for $(b + c)$ and z for $(c + a)$ we have

$$x + y + z = 2(a + b + c),$$

$$(x + y + z)^2 = 8(a + b + c)^2$$

Thus the left-hand expression becomes

$$\begin{aligned} &= (x + y + z)^2 - (x^2 + y^2 + z^2) \\ &= \{x^2 + y^2 + z^2 + 3(x + y)(y + z)(z + x)\} - (x^2 + y^2 + z^2) \\ &= 3(x + y)(y + z)(z + x) \\ &= 3(a + b + b + c)(b + c + c + a)(c + a + a + b) \\ &= 3(2a + b + c)(a + 2b + c)(a + b + 2c) \end{aligned}$$

7 Let x minutes past 10 o'clock be the required time
Therefore, the minute-hand was x minute-divisions and the hour-

hand $50 + \frac{x}{12}$ minute-divisions apart from the 12th mark Now

because the person mistook the time to be 54 minutes earlier

$$\text{we must have } 50 + \frac{x}{12} - x = 60 - 54,$$

$$\text{or, } 50 - \frac{11}{12}x = 6$$

$$\text{or, } \frac{11}{12}x = 44, \quad x = 48$$

Thus the required time is 48 minutes past 10 o'clock

8 Let \mathcal{L}_1 and \mathcal{L}_2 be the required sums

Then we have

$$\frac{4x}{100} + \frac{7y}{100} = 196$$

$$\text{or, } 4x + 7y - 19600 = 0 \quad (1)$$

$$\text{and } \frac{5x}{100} + \frac{6y}{100} = 212$$

$$\text{or, } 5x + 6y - 21200 = 0 \quad (2)$$

From (1) & (2), by cross multiplication,

$$\frac{x}{400(-371+294)} = \frac{y}{400(-245+212)} = \frac{1}{24-35}$$

$$\text{or, } \frac{x}{400 \times (-77)} = \frac{y}{400 \times (-33)} = \frac{1}{-11}$$

$$x = 2800 \text{ and } y = 1200$$

Thus the required sums are $\mathcal{L}_1 2800$ and $\mathcal{L}_2 1200$

V

$$\begin{aligned} 1 \quad (i) \quad & 15x^2 - 41x + 14 \\ &= 15x^2 - 35x - 6x + 14 \\ &= 5x(3x-7) - 2(3x-7) \\ &= (3x-7)(5x-2) \end{aligned}$$

$$\begin{aligned} (ii) \quad & (2b^2 + a^2 - ac)(2b^2 + c^2 - ac) - b^2(a+c)^2 \\ &= \{(b^2 + a^2) + (b^2 - ac)\}\{(b^2 + c^2) + (b^2 - ac)\} - b^2(a+c)^2 \\ &= (b^2 - ac)^2 + (b^2 - ac)(b^2 + a^2 + b^2 - c^2) \\ &\quad + (b^2 + a^2)(b^2 + c^2) - b^2(a+c)^2 \end{aligned}$$

$$\begin{aligned}
 &= (b^2 - ac)^2 + (b^2 - ac)(a^2 + 2b^2 + c^2) \\
 &\quad + b^4 + b^2(a^2 + c^2) + a^2c^2 - b^2(a^2 + c^2 + 2ac) \\
 &= (b^2 - ac)^2 + (b^2 - ac)(a^2 + 2b^2 + c^2) + b^4 + a^2c^2 - 2acb^2 \\
 &= (b^2 - ac)^2 + (b^2 - ac)(a^2 + 2b^2 + c^2) + (b^2 - ac)^2 \\
 &= (b^2 - ac)(b^2 - ac + a^2 + 2b^2 + c^2 + b^2 - ac) \\
 &= (b^2 - ac)\{4b^2 + (a - c)^2\}
 \end{aligned}$$

2 We have evidently $x = ab$

$$\begin{aligned}
 \text{Now } x + y &= (r + j)^2 - 3xy(x + y) \\
 &= \left(ma + \frac{b}{m} \right)^2 - 3ab \left(ma + \frac{b}{m} \right) \\
 &= \left(ma + \frac{b}{m} \right)^2 - 3ma \frac{b}{m} \left(ma + \frac{b}{m} \right) \\
 &= (ma)^2 + \left(\frac{b}{m} \right)^2 = m^2 a^2 + \frac{b^2}{m^2}
 \end{aligned}$$

3 The given expression

$$\begin{aligned}
 &= \left\{ \frac{y^2 - jz + z^2}{x} + \frac{x^2}{y + z} - \frac{3jz}{j + z} \right\} \frac{\frac{3y + 2z}{yz}}{\frac{x + y + z}{xyz}} + (x + y + z)^2 \\
 &= \frac{y^3 + z^3 + x^3 - 3xyz}{x(y + z)} \frac{2x(y + z)}{x + y + z} + (x + y + z)^2 \\
 &= \frac{(x + y + z)(x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)}{x(y + z)} \frac{2x(y + z)}{x + y + z} + (x + y + z)^2 \\
 &= 2(x^2 + y^2 + z^2 - 2xy - 2yz - 2zx) + (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx) \\
 &= 3x^2 + 3y^2 + 3z^2 = 3(x^2 + y^2 + z^2)
 \end{aligned}$$

$$\begin{aligned}
 4 \quad A &= x^2 + (5m - 3)x^2 + 3m(2m - 5)x - 18m^2, \\
 B &= x^2 + (m - 3)x^2 - m(2m + 3)x + 6m^2, \\
 A - B &= 4mx^2 + (8m^3 - 12m)x - 24m^2 \\
 &= 4m\{x^2 + (2m - 3)x - 6m\} = 4mC \text{ (Suppose) }, \\
 \text{again, } B &= x^2 + (m - 3)x^2 - m(2m + 3)x + 6m^2 \\
 \text{and } mC &= mx^2 + m(2m - 3)x - 6m^2, \\
 B + mC &= x^2 + (2m - 3)x^2 - 6mx \\
 &= x\{x^2 + (2m - 3)x - 6m\} [= xC]
 \end{aligned}$$

Thus the required H C F $= x^2 + (2m - 3)x - 6m$

$$\begin{aligned}
 5 \quad x^3 - ar^2 + a^2x - a^3 & \\
 &= x^2(1-a) + a^2(x-a) = (1-a)(x^2+a^2) \\
 x^3 + ax^2 + a^2x + a^3 & \\
 &= x^2(1+a) + a^2(1+a) = (x^2+a^2)(1+a) \\
 x^3 + ax^2 - a^2x - a^3 & \\
 &= x^2(x+a) - a^2(1+a) = (x+a)(x+a)(1-a)
 \end{aligned}$$

Thus the required L C M

$$\begin{aligned}
 &= (x-a)(x+a)(x^2+a^2)(1+a) \\
 &= (x+a)(x^3-a^3)(x^2+a^2); \\
 &= (x+a)x^4-a^4 \\
 &= x^5+ax^4-a^4x-a^5
 \end{aligned}$$

$$6 \quad (1) \quad \frac{3}{7}(6x-7) + \frac{1-7x}{6} = 1$$

$$\text{or,} \quad 108x - 126 + 7 - 49x = 42x$$

$$\text{or,} \quad 108x - 91x = 119$$

$$\text{or,} \quad 17x = 119,$$

$$x = 7$$

(ii) By adding the three equations we have

$$10x = 10,$$

$$x = 1$$

Hence, from (1) & (2), we have

$$2y + 5z = 1 - 3 = -2$$

$$\text{and} \quad 3y - 2z = 2 - 5 = -3$$

$$\text{or,} \quad 4y + 10z = -4$$

$$\text{and} \quad 15y - 10z = -15$$

$$\text{Hence,} \quad 19y = -19 \quad y = -1$$

Therefore, from (1),

$$3 - 2 + 5z = 1$$

$$\text{or,} \quad 5z = 0, \quad z = 0$$

Thus $x = 1, y = -1$ and $z = 0$

7 Suppose the man bought x eggs at two a penny. Then he bought $4x$ and $5x$ eggs respectively, at $5d$ a dozen and at $8d$ a score. Therefore, on the whole he bought $(1+4x+5x)$ or $10x$ eggs

$$\begin{aligned}\text{His total cost of buying} &= \left(\frac{1}{2} + \frac{5}{12} \times 42 + \frac{8}{20} \times 52 \right) d \\ &= \left(\frac{7}{2} + \frac{5x}{3} + 22 \right) d = \frac{25x}{6} d\end{aligned}$$

$$\text{and by selling he got } \frac{101}{100} \times 44d = \frac{221}{5} d$$

$$\text{Then we have } \frac{25x}{6} + 42 = \frac{221}{5}$$

$$\text{or, } x \left(\frac{25}{5} - \frac{25}{6} \right) = 42$$

$$\text{or, } \left(\frac{132 - 125}{30} \right) x = 42$$

$$\text{or, } \frac{7x}{30} = 42, \quad x = 180$$

Therefore, he bought $(180 \times 10) = 1800$ eggs

8 Let x yds be the length of the course, y yds per minute the speed of A and z yds per minute the speed of B

$$\text{Then we have } \frac{1}{x} = \frac{3}{2}$$

$$\text{or, } 2y = 3z \quad (1)$$

$$\text{Also we have } \frac{1 - 30 - 20}{x} = \frac{2}{y} = 5$$

$$x = 5y \quad (2)$$

$$\text{and } x = 50 - 5z \quad (3)$$

$$\text{From (2) and (3), } 5y - 50 = 5z$$

$$\text{or, } y - 10 = z$$

$$\text{or, } y = z + 10$$

$$\text{or, } 2y = 2z + 20 = 3z, \quad z = 20,$$

$$\text{and from (1), } 2y = 60, \quad y = 30,$$

$$\text{also from (2), } x = 150$$

Thus the length of the course was 150 yds, A's speed was 30 yds per minute and B's speed 20 yds per minute

VI

1 (i) Putting x for $\left(\frac{a}{b} + \frac{b}{a}\right)$ and y for $\left(\frac{a}{b} - \frac{b}{a}\right)$, the given expression becomes

$$\begin{aligned} &= x^4 - 2x^2y^2 + y^4 \\ &= (x^2 - y^2)^2 \\ &= \left\{ \left(\frac{a}{b} + \frac{b}{a}\right)^2 - \left(\frac{a}{b} - \frac{b}{a}\right)^2 \right\}^2 = (4)^2 = 16 \end{aligned}$$

$$\begin{aligned} (ii) \quad & \frac{x^6 - y^6}{x^3 + 2x^2y + 2xy^2 + y^3} \\ &= \frac{(x^3 + y^3)(x^3 - y^3)}{(x^3 + y^3) + 2xy(x + y)} \\ &= \frac{(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)}{(x + y)(x^3 - xy^2 + y^3 + 2xy^2)} \\ &= \frac{(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)}{(x^2 + xy + y^2)} \\ &= (x^2 - xy + y^2)(x - y) \end{aligned}$$

$$\begin{aligned} 2 (i) \quad & x^6 - y^6 - (x - y)^6 \\ &= x^6 - y^6 - (x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6) \\ &= 6x^5y - 15x^4y^2 + 20x^3y^3 - 15x^2y^4 + 6xy^5 - y^6 \\ &= 6xy(x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4) \\ &= 6xy\{(x^3 - y^3) - 2xy(x - y)\} \\ &= 6xy\{(x - y)(x^2 + xy + y^2) - 2xy(x - y)\} \\ &= 6xy(x - y)(x^2 - xy + y^2) \end{aligned}$$

(ii) See Example I Art 9, Ch VIII

$$\begin{aligned} & \left. \begin{array}{l} 1 + p + q \\ 1 + p + q \end{array} \right) \frac{x^3 + (p + q)x + a}{x^3 + px^2 + qx^2} \left(\frac{x^3 - (p + q)x + a}{(p + q)\{(p + q) + 1\}} + 1 \right) \\ & \quad - \frac{(p + q)x^3 + (p + q)x + a}{x(p + q)\{(p + q) + 1\} + a} \\ & \quad - \frac{(p + q)x^2 - (p + q)^2x}{x(p + q)\{(p + q) + 1\} + a} \\ & \quad - \frac{1 + p + q\{(p + q) + 1\} + (p + q)^2\{(p + q) + 1\}}{- (p + q)^2\{(p + q) + 1\} + a} \end{aligned}$$

Therefore, the condition of divisibility is evidently

$$- (p + q)^2\{(p + q) + 1\} + a = 0$$

or $(p + q)^2 + (p + q)^2 - a = 0$

3 The given expression

$$\begin{aligned}
 &= \frac{(x-y) \times (x^2-x+y^2) \times (x^2-y^2) \times (x^4-y^4)}{(x+y) \times (x^2+xj+y^2) + (x^2+y^2) + (x-y)^4} \\
 &= \frac{(x-y)(x^2-x+y^2)(x^2-y^2)(x^2+y^2)(x+y)(x-y)}{(x+y)(x^2+xj+y^2)(x^2+y^2)(x-y)^2(x-y)} \\
 &= \frac{(x^2+y^2)(x^2-y^2)(x^2+y^2)}{(x^2-y^2)(x^2-y^2)(x^2+y^2)} = \frac{x^2+y^2}{x^2-y^2}
 \end{aligned}$$

4 The first expression $= x^2(x^2+x+1) + (x^2+x+1)$
 $= (x^2+1)(x^2+x+1)$
 $= (x^2-x+1)(x+1)(x^2+x+1)$

The second expression $= x^2(x^2-x+1) - (x^2-x+1)$
 $= (x^2-x+1)(x^2-1)$
 $= (x^2-x+1)(x-1)(x^2+x+1)$

Thus their L.C.M. $= (x^2-x+1)(x+1)(x^2-x+1)(x+1)$
 $= (x^2-1)(x^2+1) = x^4-1$

5 $\frac{1}{4} - \frac{2(x+1)}{9} + \frac{5(x-5)}{12} - 4 - \frac{x+1}{16}$
 or, $9(x-1) - 8(x+1) + 15(x-5) - 144 = 2(x+1)$
 or, $9x - 8x + 15x - 2x = 2 + 9 + 8 + 75 + 144$
 or, $14x = 238, \quad x = 17$

6 From (2), we have

$$12x - 9z = 42$$

also, $5y - 9z = 18 \quad (1)$

$$12x - 5y = 24$$

or, $12x - 5y - 24 = 0 \quad (4)$

and from (3), $7x - 6y - 14 = 0$

$$\frac{x}{70-141} = \frac{y}{-168+168} = \frac{1}{-72+35}$$

or, $\frac{x}{-74} = \frac{y}{0} = \frac{1}{-37},$

$$x=2 \text{ and } y=0$$

from (1), $-9z = 18, \quad z = -2$

Thus $x=2, y=0$ and $z=-2$

7 Let x miles be the distance between them after t days and let them come together after y days

Each day their distance diminishes by $(a-b)$ miles,

after t days the distance will diminish by $t(a-b)$ miles

Then we have $x = d - t(a-b)$ and $d = y(a-b)$

$$\text{or, } y = \frac{d}{a-b}$$

Thus the distance between them after t days is $\{d - t(a-b)\}$ miles and they will come together after $\left(\frac{d}{a-b}\right)$ days

(1) When $a=b$, their rates are equal and $x=d$, i.e., the distance will remain the same

(2) When $a=b$ and $d=0$, the rates being equal and they being in the same place they will walk together all the time and y becomes indeterminate and is capable of assuming any value whatever

8 Let it be x minutes past 6 o'clock when the hands are at right angles. Then at the required instant the minute-hand is at a distance of x minute-divisions from the 12th mark and the hour-hand which was 30 minute divisions apart at 6 o'clock is at a distance of $\left(30 + \frac{x}{12}\right)$ minute-divisions from the 12th mark. In order that the hands may be at right angles the minute hand must be 15 minute divisions apart from the hour-hand on either side

$$\text{Then we have } x = \left(30 + \frac{x}{12}\right) \pm 15$$

$$\text{or, } 12x = 360 + x \pm 180$$

$$\text{or, } 11x = 540 \text{ or } 180,$$

$$x = \frac{540}{11} \text{ or } \frac{180}{11},$$

$$= 49 \frac{1}{11} \text{ or } 16 \frac{4}{11}$$

Thus the hands are at right angles at $49 \frac{1}{11}$ and $16 \frac{4}{11}$ minutes past 6 o'clock

VII

$$\begin{array}{r}
 1 \quad (i) \quad a - a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}c^{\frac{1}{2}} + b^{\frac{1}{2}} - b^{\frac{1}{2}}c^{\frac{1}{2}} + c^{\frac{1}{2}} \\
 \quad \quad \quad a^{\frac{1}{2}} + b^{\frac{1}{2}} + c^{\frac{1}{2}} \\
 \hline
 a^{\frac{3}{2}} - ab^{\frac{1}{2}} - ac^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{3}{2}} - a^{\frac{1}{2}}b^{\frac{3}{2}}c^{\frac{1}{2}} + a^{\frac{1}{2}}c^{\frac{3}{2}} \\
 \quad \quad \quad ab^{\frac{3}{2}} \quad \quad - a^{\frac{1}{2}}b^{\frac{3}{2}} - a^{\frac{1}{2}}b^{\frac{3}{2}}c^{\frac{1}{2}} \quad \quad + b - b^{\frac{3}{2}}c^{\frac{1}{2}} + b^{\frac{3}{2}}c^{\frac{3}{2}} \\
 \quad \quad \quad \quad \quad \quad ac^{\frac{3}{2}} \quad \quad - a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{3}{2}} - a^{\frac{1}{2}}c^{\frac{3}{2}} \quad \quad + b^{\frac{3}{2}}c^{\frac{3}{2}} - b^{\frac{3}{2}}c^{\frac{3}{2}} + c^{\frac{3}{2}} \\
 \hline
 a^{\frac{3}{2}} \quad \quad \quad - 3a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}} \quad \quad \quad + b \quad \quad \quad + c^{\frac{3}{2}}
 \end{array}$$

Thus the product is $a^{\frac{3}{2}} - 3a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}} + b + c^{\frac{3}{2}}$

$$\begin{array}{r}
 (ii) \quad \text{Quotient} = \frac{x^2}{2} - \frac{3x}{4} + 6 \\
 \frac{2x^3}{3} - \frac{5x}{6} + 1 \quad \left) \frac{x^4}{3} - \frac{11x^3}{12} + \frac{41x^2}{8} - \frac{23x}{4} + 6 \right. \\
 \quad \quad \quad \frac{x^4}{3} - \frac{5x^3}{12} + \frac{x^2}{2} \\
 \hline
 \quad \quad \quad - \frac{x^3}{2} + \frac{37x^2}{8} - \frac{23x}{4} + 6 \\
 \quad \quad \quad - \frac{x^3}{2} + \frac{5x^2}{8} - \frac{3x}{4} \\
 \hline
 \quad \quad \quad \quad \quad 4x^2 - 5x + 6 \\
 \quad \quad \quad \quad \quad 4x^2 - 5x + 6
 \end{array}$$

$$\begin{aligned}
 2 \quad & \left(1 + \frac{x^2 + y^2 - z^2}{2xy} \right) - \left(1 + \frac{y^2 + z^2 - x^2}{2yz} \right) \\
 &= \frac{2xy + x^2 + y^2 - z^2}{2xy} - \frac{2yz + y^2 + z^2 - x^2}{2yz} \\
 &= \frac{(x+y)^2 - z^2}{2xy} - \frac{(y+z)^2 - x^2}{2yz} \\
 &= \frac{(x+y+z)(x+y-z)}{2xy} \times \frac{2yz}{(y+z+x)(y+z-x)} \\
 &= \frac{z(x+y-z)}{x(y+z-x)}
 \end{aligned}$$

3 The given expression

$$= \frac{x^2(x-1) - (1-1)}{x^4 + x^3 + x^2 - 3x^3 - 3x^2 - 3x + x^2 + x + 1}$$

A and B can respectively do $\frac{1}{2(x-1)}$ and $\frac{1}{2(x+2)}$ of the work in one day. Also they can do together $\frac{1}{x}$ of the work in one day.

Thus we have
$$\frac{1}{2(x-1)} + \frac{1}{2(x+2)} = \frac{1}{x}$$

or,
$$\frac{1}{x-1} + \frac{1}{x+2} = \frac{2}{x}$$

or,
$$\frac{x+2-x-1}{(x+2)(x-1)} = \frac{2}{x}$$

or,
$$x(2x+1) = 2(x^2+x-2)$$

or,
$$2x^2+x = 2x^2+2x-4$$

$$x = 4$$

Thus they can finish the work together in 4 days

VIII.

1 The given expression

$$\begin{aligned} &= \left\{ \frac{(a^2-j^2)(a^2+j^2)}{(a-j)^2} - \frac{a^2a-j^2}{a-j} \right\} \\ &\times \left\{ \frac{a^2(a^2-j^2)}{a^2+j^2} - \frac{a^2(a^2-2aj+j^2)}{a^2-aj+j^2} \right\} \\ &= \left\{ \frac{(a-j)(a+j)(a^2+j^2)}{(a-j)^2} \times \frac{2-j}{a(a+j)} \right\} \\ &\times \left\{ \frac{a^2(a-j)(a-j)}{(a+j)(a^2-aj+j^2)} \times \frac{a^2-aj+j^2}{a^2(a-j)^2} \right\} \\ &= \frac{a^2+j^2}{a} \times \frac{a}{a-j} = \frac{a^2-j^2}{a-j} \end{aligned}$$

2

$$\begin{array}{r} 3x^2 + 16ax^2 + 15x^2x - 4a^2 \quad \left(\begin{array}{l} 6x^2 - 5a^2x + 3a^2x^2 - 13a^2x - 3a^2 \\ 6x^2 + 32ax^2 + 30x^2x^2 - 8a^2x \\ - 27ax^2 - 27a^2x^2 - 21a^2x - 3a^2 \\ - 27ax^2 - 144a^2x^2 - 135a^2x + 36a^2 \\ 39a^2 \end{array} \right) \left(\begin{array}{l} 2x - 9 \\ 2x - 9 \\ 2x - 9 \\ 2x - 9 \\ 2x - 9 \end{array} \right) \\ \hline 3x^2 - 16ax - a^2 \end{array}$$

$$\begin{array}{r}
 3x^2 + 4ax - a^2 \quad \left) \begin{array}{l} 3x^2 + 16a^2x + 15a^2x - 4a^2 \\ 3x^2 + 4a^2x - a^2x \end{array} \right. \begin{array}{l} x - 4a \\ 12a^2x + 16a^2x - 4a^2 \\ 12ax^2 + 16a^2x - 4a^2 \end{array}
 \end{array}$$

Thus the H C F is $3x^2 + 4ax - a^2$

$$\begin{aligned}
 3 \quad \text{The 1st expression} &= 2x^2 - 2x + x - 1 \\
 &= 2x(x - 1) + (x - 1) \\
 &= (x - 1)(2x + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{The 2nd expression} &= 2x^2 + 2x + x + 1 \\
 &= 2x(x + 1) + (x + 1) \\
 &= (2x + 1)(x + 1)
 \end{aligned}$$

$$\text{The 3rd expression} = (x + 1)(x - 1)$$

$$\begin{aligned}
 \text{Therefore their L C M} &= (x - 1)(2x + 1)(x + 1) \\
 &= (2x + 1)(x^2 - 1)
 \end{aligned}$$

4 The given expression

$$\begin{aligned}
 &= \frac{a^2 + ab + ac + bc}{a^2 + ab + 2ac + 2ca} \times \frac{a^2 + (2c)^2}{a^2 + 8ac + a^2c^2 - 2ac^2 + 4c^2} \\
 &= \frac{a(a + b) + c(a + b)}{a(a + b) + 2c(a + b)} \times \frac{(a + 2c)(a^2 - 2ac + 4c^2)}{a^2(a^2 + (2c)^2) + c^2(a^2 - 2ac + (2c)^2)} \\
 &= \frac{(a + b)(a + c)}{(a + b)(a + 2c)} \times \frac{(a + 2c)(a^2 - 2ac + 4c^2)}{a^2(a + 2c)(a^2 - 2ac + 4c^2) + c^2(a^2 - 2ac + 4c^2)} \\
 &= \frac{(a + c)(a^2 - 2ac + 4c^2)}{(a^2 - 2ac + 4c^2)a(a + 2c) + c^2} = \frac{a + c}{a^2 + 2ac + c^2} = \frac{1}{a + c}
 \end{aligned}$$

5 Since $ap = bq = cr$,

$$\text{we have } (ap)^2 = bq \times cr \quad (1)$$

$$(bq)^2 = ap \times cr \quad (2)$$

$$\text{and } (cr)^2 = ap \times bq \quad (3)$$

$$\text{Now from (1), } a^2p^2 = bqcr \quad \text{or, } \frac{p^2}{qr} = \frac{bc}{a^2},$$

$$\text{from (2), } b^2q^2 = apr \quad \text{or, } \frac{q^2}{pr} = \frac{ac}{b^2},$$

$$\text{and from (3), } c^2r^2 = apbq \quad \text{or, } \frac{r^2}{pq} = \frac{ab}{c^2}$$

$$\text{Hence, } \frac{p^2}{q^2} + \frac{q^2}{p^2} + \frac{r^2}{p^2q^2} = \frac{bc}{a^2} + \frac{ac}{b^2} + \frac{ab}{c^2}$$

6

$$\begin{aligned}
 z+y &= \frac{a-b}{m-c} + \frac{b-c}{m-a} \\
 &= \frac{m(a-b) + m(b-c) - a(a-b) - c(b-c)}{(m-c)(m-a)} \\
 &= \frac{m(a-c) - (a^2 - c^2) + b(a-c)}{(m-c)(m-a)} \\
 &= \frac{(a-c)\{m - (a+c-b)\}}{(m-c)(m-a)},
 \end{aligned}$$

and $z+xyz = z(1+xy)$

$$\begin{aligned}
 &= \frac{c-a}{m-b} \left\{ 1 + \frac{(a-b)(b-c)}{(m-c)(m-a)} \right\} \\
 &= \frac{c-a}{m-b} \left\{ \frac{m^2 - (a+c)m + ac + ab - ac + bc - b^2}{(m-c)(m-a)} \right\} \\
 &= \frac{c-a}{m-b} \left\{ \frac{m^2 - (a+c)m + b(a+c-b)}{(m-c)(m-a)} \right\} \\
 &= \frac{c-a}{m-b} \times \frac{(m-b)\{m - (a+c-b)\}}{(m-c)(m-a)} \\
 &= \frac{-(a-c)\{m - (a+c-b)\}}{(m-c)(m-a)}
 \end{aligned}$$

Thus $z+y+z+xyz=0$ 7 From (2), we have $\frac{a}{b+y} = \frac{b}{a+x}$

$$\text{or,} \quad a^2 + ax = b^2 + by$$

$$\text{or,} \quad ax - by = b^2 - a^2,$$

$$\text{also, } ax + by = c^2 \quad (1)$$

$$\text{Hence, } 2ax = c^2 + b^2 - a^2, \quad x = \frac{c^2 + b^2 - a^2}{2a},$$

$$\begin{aligned}
 \text{from (1), } by &= c^2 - \frac{c^2 + b^2 - a^2}{2} \\
 &= \frac{a^2 - b^2 + c^2}{2}, \quad y = \frac{a^2 - b^2 + c^2}{2b}
 \end{aligned}$$

8 Let £ x be the sum. Then each received £ $\frac{x}{15}$. Therefore, each would receive £ $\frac{105x}{100 \times 15}$ or £ $\frac{7x}{100}$, if £2 more had been available

Thus we have $\frac{x+2}{15} = \frac{71}{100}$

or, $100x + 200 = 105x$

or, $5x = 200, \quad x = 40$

Thus the sum is £40

IX

1 Since $\frac{a}{b} + \frac{c}{d} = \frac{b}{a} + \frac{d}{c}$

or $\frac{ad+bc}{bd} = \frac{bc+ad}{ac}$
 $bd=ac$

Thus we must have

$$\frac{a^2}{b^2} + \frac{c^2}{d^2} = \frac{a^2d^2+b^2c^2}{b^2d^2} = \frac{a^2d^2+b^2c^2}{a^2c^2} = \frac{d^2}{c^2} + \frac{b^2}{a^2}$$

2 $1-a = \frac{a^2+ab+b^2}{a+b} - a = \frac{a^2+ab+b^2-a^2-ab}{a+b} = \frac{b^2}{a+b},$

$$1-b = \frac{a^2+ab+b^2}{a+b} - a = \frac{a^2+ab+b^2-a^2-ab}{a+b} = \frac{b^2}{a+b},$$

and $x-a-b = \frac{a^2+ab+b^2}{a+b} - (a+b)$
 $= \frac{a^2+ab+b^2-a^2-2ab-b^2}{a+b} = \frac{-ab}{a+b},$

$$\frac{(x-a)(x-b)}{(1-a-b)^2} = \frac{\frac{b^2}{(a+b)} \cdot \frac{a^2}{(a+b)}}{\frac{(-ab)^2}{(a+b)^2}} = \frac{a^2b^2}{(a+b)^2} \times \frac{(a+b)^2}{a^2b^2} = 1$$

3 The given expression

$$\begin{aligned} &= \frac{2x^2-2xy-x^2+2x+2y^2}{x^2-y^2} \times \frac{x+y}{xy} - \frac{3xy-2xy+2y^2+x^2-xy}{xy(x-y)} \\ &= \frac{x^2+2y^2}{x^2-y^2} \times \frac{x+y}{xy} - \frac{x^2+2y^2}{xy(x-y)} \\ &= \frac{x^2+2y^2}{(x+y)(x-y)} \times \frac{x+y}{xy} \times \frac{xy(x-y)}{x^2+2y^2} = 1 \end{aligned}$$

$$\begin{aligned} \text{also} \quad 7j + 2z &= 20 \\ 47z &= 141 & z &= 3 \end{aligned}$$

Therefore from (5), $j = 23 - 21 = 2$,

and from (1) $x = 14 - 4 - 9 = 1$

Thus $x=1$, $y=2$ and $z=3$

7 Suppose the woman bought $2x$ apples on the whole,

her total cost of buying $= \left(\frac{x}{3} + \frac{x}{4} \right)$ pence

and by selling she got $\frac{2 \times 2x}{7}$ i.e., $\frac{4x}{7}$ pence

Then we have $\frac{x}{3} + \frac{x}{4} = \frac{4x}{7} + 3$

or $28x + 21x = 48x + 252$,

$$x = 252,$$

$$\frac{4x}{7} = \frac{4 \times 252}{7} = 144 \text{ pence i.e. } 12s$$

Thus the woman sold them for 12s

8 Let A run x miles per hour, B run y miles per hour and C run z miles per hour. When A runs 1760 yds B runs $(1760 - 160)$ yds or 1600 yds and when B runs 1760 yds C runs $(1760 - 76\frac{2}{3})$ yds or, $1683\frac{1}{3}$ yds or, 38720 yds

Thus we have $\frac{1}{x} = \frac{1600}{1760y} = \frac{10}{11y}$

$$\text{or,} \quad 10x = 11y \quad (1)$$

Again $\frac{1}{y} = \frac{38720}{1760z + 23z} = \frac{22}{23z}$

$$\text{or,} \quad 22y = 23z \quad (2)$$

Also $\frac{1}{z} - \frac{1}{y} = \frac{1}{4 \times 60} \quad (3)$

Hence $\frac{1}{z} - \frac{22}{23z} = \frac{1}{240}$

$$\text{or} \quad \frac{1}{z} - \frac{1}{23} = \frac{1}{240}, \quad z = \frac{243}{23} = 10\frac{10}{23}$$

$$\text{or,} \quad \frac{1}{x-1} - \frac{1}{r-2} = \frac{2}{x-2} - \frac{2}{x-3}$$

$$\text{or,} \quad \frac{x-2-x+1}{(x-2)(x-2)} = 2 \left\{ \frac{r-3-x+2}{(x-2)(r-3)} \right\}$$

$$\text{or,} \quad \frac{-1}{x-1} = \frac{-2}{r-3}$$

$$\text{or,} \quad x-3 = 2r-2, \quad x = -1.$$

(11) From (1) and (3) we have by addition

$$x+2y+z = x+z+4a$$

$$\text{or,} \quad 2y = 4a, \quad y = 2a$$

Again from (2) and (3), we have

$$2x+y+z = y+z+2a$$

$$\text{or,} \quad 2x = 2a \quad x = a$$

Therefore from (3), we have $z = 3a$

Thus $x = a$, $y = 2a$ and $z = 3a$

7 Subtracting (2) from (1), we have

$$\frac{a-b}{x} - \frac{a-b}{y} = \frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$$

$$\text{or,} \quad \frac{1}{x} - \frac{1}{y} = \frac{1}{ab} \quad (3)$$

Again, by adding (1) and (2), we have

$$\frac{a+b-2}{x} - \frac{a+b-4}{y} = \frac{1}{b} + \frac{1}{a} = \frac{a+b}{ab},$$

$$\text{and, from (3),} \quad \frac{a+b}{r} - \frac{a+b}{j} = \frac{a+b}{ab}$$

$$\text{Hence} \quad \frac{2}{x} - \frac{4}{y} = 0$$

$$\text{or,} \quad \frac{1}{x} - \frac{2}{y} = 0 \quad (4)$$

from (3) and (4), we have

$$\frac{c}{x} - \frac{c}{y} - \left(\frac{1}{x} - \frac{2}{y} \right) = \frac{c}{ab}$$

$$\text{Thus} \quad \frac{c-1}{x} - \frac{c-2}{y} = \frac{c}{ab}$$

8 Let x miles and y miles per hour be the respective rates of the faster and the slower trains. When they are moving in opposite directions a distance of $(x+y)$ miles is covered per hour and when in the same direction $(x-y)$ miles per hour. In passing each other they shall have to traverse their lengths $(82+84)$ feet or 176 feet.

$$\text{Thus we have } \frac{176}{3 \times 1760} \cdot \frac{1}{x+y} = \frac{3}{2} \times \frac{1}{60 \times 60}$$

$$\text{or, } \frac{1}{30(x+y)} = \frac{1}{2 \times 20 \times 60}$$

$$\text{or, } x+y=80 \quad (1)$$

$$\text{Also we have } \frac{176}{3 \times 1760} \cdot \frac{1}{x-y} = \frac{6}{60 \times 60}$$

$$\text{or, } \frac{1}{30(x-y)} = \frac{1}{10 \times 60}$$

$$\text{or, } x-y=20 \quad (2)$$

Hence, from (1) and (2),

$$2x = 100 \quad x = 50,$$

$$\text{from (1) } y = 50 - 20 = 30$$

Thus the rates are 50 miles and 30 miles per hour.

XI

1 The given expression

$$= \{(x+y)^2 + z^2\}^2 - 2(x+y)^2 z^2$$

$$= (x+y)^4 + 2(x^2 + y^2)z^2 + z^4 - 2(x+y)^2 z^2$$

$$= (x+y)^4 + z^4$$

2

$$\begin{array}{r} 6x^5 - 9x^4 + 19x^3 - 12x^2 + 19x - 15 \\ 2 \\ \hline 4x^4 - 2x^3 + 10x^2 + x + 15 \end{array} \begin{array}{l} 12x^5 - 18x^4 + 38x^3 - 24x^2 + 38x - 30 \\ 12x^5 - 6x^4 + 30x^3 + 3x^2 + 45x \\ \hline -12x^4 + 8x^3 - 27x^2 - 7x - 30 \\ -12x^4 + 6x^3 - 30x^2 - 3x - 45 \\ \hline 2x^3 + 3x^2 - 4x + 15 \end{array} \begin{array}{l} 3x - 3 \\ 3x - 3 \\ \hline 3x - 3 \\ 3x - 3 \\ \hline 3x - 3 \end{array}$$

$$\begin{aligned}
 6 \quad (i) \quad & 27x^2 - 48x - 512 \\
 &= \frac{1}{4}(81x^2 - 144x - 1536) \\
 &= \frac{1}{4}\{(9x - 8)^2 - (40)^2\} \\
 &= \frac{1}{4}(9x + 32)(9x - 48) \\
 &= (9x + 32)(3x - 16)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & (a+b+c+d)^2 - (a-b+c-d)^2 \\
 &= (a+b+c+d+a-b+c-d)(a+b+c+d-a+b-c+d) \\
 &= (2a+2d)(2b+2c) = 4(a+d)(b+c)
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad & m^4 + m^2n^2 + n^4 \\
 &= m^4 + 2m^2n^2 + n^4 - m^2n^2 \\
 &= (m^2 + n^2)^2 - (mn)^2 \\
 &= (m^2 + mn + n^2)(m^2 - mn + n^2)
 \end{aligned}$$

7 Multiplying (3) by 2, we have $2x - 4y + 2z = 0$

$$\text{Also,} \quad x - y + 2z = 11 \quad (1)$$

$$x - 3y = -11 \quad (4)$$

$$\text{From (2) and (3) we have } x + y = 9 \quad (5)$$

$$\text{Hence, from (4) and (5), } 4y = 20, \quad y = 5$$

$$\begin{aligned} \text{From (5),} \quad & x = 9 - 5 = 4 \\ & \text{from (3) } z = 10 - 4 = 6 \end{aligned}$$

$$\text{Thus, } x = 4, y = 5 \text{ and } z = 6$$

8 Let the greyhound take x leaps to catch the hare and by that time the hare will take $\frac{5x}{4}$ leaps. Then the hound will have to pass over $\left(40 + \frac{5x}{4}\right)$ leaps of the hare or $\frac{3}{4}\left(40 + \frac{5x}{4}\right)$ leaps of his own

$$\text{Thus, we have } \frac{3}{4}\left(40 + \frac{5x}{4}\right) = x$$

$$\text{or, } 480 + 15x = 16x, \quad x = 480$$

Thus the grey-hound must take 480 leaps

XII

1 The given expression

$$\begin{aligned}
 &= \frac{y(r^3 - y^3)}{xy(y+r)} - \left\{ \frac{x^3(a^2 + xy + y^2)}{(x+y)^2(x-y)} \times \frac{(1+y)^2}{x^2} \right\} \\
 &= \frac{(x-y)(r^2 + xy + y^2)}{r(1+y)} - \left\{ \frac{(1^2 + 1y + y^2)}{(1+y)(1-y)^3} \right\} \\
 &= \frac{(x-y)(1^2 + 1y + y^2)}{1(x+y)} \times \frac{(1+y)(1-y)^3}{(1^2 + 1y + y^2)} = \frac{(1-y)^4}{1}
 \end{aligned}$$

$$2 \quad x^2 + 1 = \frac{(a+1)^2}{(a-1)^2} + 1 = \frac{(a+1)^2 + (a-1)^2}{(a-1)^2} = \frac{2(a^2+1)}{(a-1)^2}$$

$$\text{Similarly, } y^2 + 1 = \frac{2(b^2+1)}{(b-1)^2} \text{ and } z^2 + 1 = \frac{2(c^2+1)}{(c-1)^2}$$

$$\begin{aligned}
 \text{Again } xy + 1 &= \frac{(a+1)(b+1)}{(a-1)(b-1)} + 1 \\
 &= \frac{ab + (a+b) + 1 + ab - (a+b) + 1}{(a-1)(b-1)} = \frac{2(ab+1)}{(a-1)(b+1)}
 \end{aligned}$$

$$\text{Similarly, } yz + 1 = \frac{2(bc+1)}{(b-1)(c-1)} \quad \text{and} \quad zx + 1 = \frac{2(ac+1)}{(c-1)(a-1)}$$

Thus the left-hand expression

$$\begin{aligned}
 &= \frac{8(a^2+1)(b^2+1)(c^2+1)}{(a-1)^2(b-1)^2(c-1)^2} \\
 &= \frac{8(ab+1)(bc+1)(ca+1)}{(a-1)(b-1)(b-1)(c-1)(c-1)(a-1)} \\
 &= \frac{(a^2+1)(b^2+1)(c^2+1)}{(ab+1)(bc+1)(ca+1)}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & \left. \begin{aligned} 2x^4 + x^3 - 9x^2 + 8x - 2 \end{aligned} \right) \begin{aligned} & 2x^4 - 7x^3 + 11x^2 - 8x + 2 \\ & \hline & 21^4 + x^3 - 9x^2 + 8x - 2 \\ & -4 \mid -8x^3 + 20x^2 - 16x - 4 \\ & \hline & 21^3 - 51^2 + 4x - 1 \end{aligned} \left(\begin{aligned} & 1 \\ & \hline & \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned} 21^3 - 5x^2 + 4x - 1 \end{aligned} \right) \begin{aligned} & 2x^4 + x^3 - 9x^2 + 8x - 2 \\ & \hline & 21^3 - 5x^3 + 4x^2 - 1 \\ & \hline & 61^3 - 13x^3 + 9x - 2 \\ & \hline & 61^3 - 15x^3 + 12x - 3 \\ & \hline & 2x^3 - 3x + 1 \end{aligned} \left(\begin{aligned} & x + 3 \\ & \hline & \end{aligned} \right.
 \end{aligned}$$

$$\left. \begin{array}{l} 2x^2 - 3x - 1 \end{array} \right\} \frac{2x^2 - 5x^2 - 1x - 1}{2x^2 - 3x^2 - x} \left(\begin{array}{l} x - 1 \\ -2x^2 - 3x - 1 \\ -2x^2 - 3x - 1 \end{array} \right)$$

Thus the H C F is $2x^2 - 3x - 1$

The L.C.M. of the two expressions = $\frac{\text{the Product}}{\text{their H.C.F.}}$

$$\begin{aligned} &= \frac{(2x^4 - x^2 - 9x^4 - 8x - 2)(2x^3 - 7x^2 + 11x^2 - 8x - 2)}{2x^2 - 3x - 1} \\ &= (x^2 - 2x - 2)(2x^4 - 7x^2 - 11x^2 - 8x + 2) \\ &= 2x^6 - 3x^5 - 7x^4 + 28x^3 - 36x^2 - 20x - 2 \end{aligned}$$

4. The left hand expression

$$\begin{aligned} &= a^4 - b^4 + c^4 - a^4 - 2a^2b^2 - b^2(c^2 - a^2) - 2a^2b^2 + 2c^2d^2 \\ &\quad - a^2c^2 - 2c^2d^2 - 8abcd \\ &= (a^4 - b^4 - c^4 - a^4 - 2a^2b^2 - 2c^2d^2 - 8abcd) \\ &= (a^4 - b^4 - c^4 - a^4 - 2a^2b^2 - 2c^2d^2 - 2abcd) \\ &= (a^4 - b^4 - c^4 - a^4 - 2a^2b^2 - 2c^2d^2 - 2abcd) \\ &= (a^4 - b^4 - c^4 - a^4 - 2a^2b^2 - 2c^2d^2 - 2abcd) \\ &= (a^4 - b^4 - c^4 - a^4 - 2a^2b^2 - 2c^2d^2 - 2abcd) \end{aligned}$$

$$5. \quad x - yz = \frac{b^2 - c^2 - a^2}{2bc} - \frac{(c^2 - a^2 - b^2)(a^2 - b^2 - c^2)}{2a^2bc}$$

$$= \frac{2a^2b^2 - 2a^2c^2 - 2a^4 - a^4 - (b^2 - c^2)^2}{2a^2bc}$$

$$= \frac{2a^2b^2 - 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4}{2a^2bc}$$

$$y - zx = \frac{c^2 - a^2 - b^2}{2ac} + \frac{(a^2 + b^2 - c^2)(b^2 - c^2 - a^2)}{2a^2c}$$

$$= \frac{2b^2c^2 - 2a^2b^2 - 2b^4 - b^4 - (a^2 - c^2)^2}{2ab^2c}$$

$$= \frac{2b^2c^2 + 2a^2b^2 - 2c^2c^2 - a^4 - b^4 - c^4}{2ab^2c}$$

$$\frac{x + yz}{y + zx} = \frac{2ab^2c}{2ab^2c} = \frac{b}{a}$$

6

$$\begin{aligned}
& \frac{1}{a+bc} + \frac{1}{b+ca} + \frac{1}{c+ab} \\
&= \frac{1}{1-b-c+bc} + \frac{1}{1-a-c+ca} + \frac{1}{1-a-b+ab} \\
& \quad [a+b+c=1] \\
&= \frac{1}{(1-b)(1-c)} + \frac{1}{(1-a)(1-c)} + \frac{1}{(1-a)(1-b)} \\
&= \frac{1-a+1-b+1-c}{(1-a)(1-b)(1-c)} = \frac{3-(a+b+c)}{1-(a+b+c)+(ab+ac+bc)-abc} \\
&= \frac{3-1}{1-1+1-\frac{1}{2}} \quad \left[\begin{array}{l} \text{Since } ab+ac+bc=\frac{1}{2} \\ \text{and } abc=\frac{1}{2} \end{array} \right] \\
&= \frac{2}{\frac{1}{2}} = 4
\end{aligned}$$

7 By adding (2) and (3), we have

$$\begin{aligned}
& \frac{20}{x} - \frac{29}{y} = \frac{77}{12} \\
\text{or,} \quad & \frac{80}{x} - \frac{29}{y} = \frac{77}{3} \quad (4)
\end{aligned}$$

Multiplying (1) by 2 and (3) by 3, we have

$$\begin{aligned}
& \frac{8}{x} + \frac{12}{y} + \frac{3}{z} = \frac{48}{5} \\
\text{and} \quad & \frac{75}{x} - \frac{15}{y} - \frac{3}{z} = 8, \\
\text{hence,} \quad & \frac{83}{x} - \frac{3}{y} = \frac{88}{5} \quad (5)
\end{aligned}$$

Now multiplying (4) by 3 and (5) by 29, we have

$$\begin{aligned}
& \frac{240}{x} - \frac{87}{y} = 77 \\
\text{and} \quad & \frac{2407}{x} - \frac{87}{y} = \frac{2552}{5}, \\
\text{hence,} \quad & \frac{2167}{x} = \frac{2167}{5}, \quad x=5
\end{aligned}$$

$$\text{Therefore from (4)} \quad \frac{29}{y} = 16 - \frac{77}{3} = -\frac{29}{3}, \quad y = -3$$

$$\text{Therefore from (2)} \quad -1 + \frac{3}{4} + \frac{1}{z} = 3 \text{ or, } \frac{1}{z} = 4, \quad z = \frac{1}{4}$$

Thus $x=5$, $y=-3$ and $z=\frac{1}{4}$

8 Suppose A and B travelled at x and y miles per hour respectively. Then in 5 hours A walked $5x$ miles and B , $5y$ miles.

The distance is therefore $5x + 5y = 5(x + y)$

$$\text{Then we have } \frac{5x}{x+1} = 4 \quad (1)$$

$$\text{and } \frac{5y}{y-2} = 6 \quad (2)$$

$$\text{From (1), } 5x = 4x + 4, \quad x = 4$$

$$\text{From (2), } 5y = 6y - 6, \quad y = 6$$

$$5(x + y) = 5(4 + 6) = 50$$

Thus the distance between the two places is 50 miles.

Exercise (78)

$$1 \quad a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$$

$$2 \quad x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

$$3 \quad 3^{-\frac{1}{2}} = 3^{\frac{1}{2}} = 3\sqrt{3}$$

$$4 \quad x^{-\frac{1}{2}} \times 3a^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} \times \frac{3}{a^{\frac{1}{2}}} = \frac{3}{\sqrt{x} \sqrt{a}}$$

$$5 \quad 8m^{-2} \times m^{-1} = 8m^{-(2+1)} = 8m^{-3} = \frac{8}{m^3} = \frac{8}{\sqrt[3]{m^3}}$$

$$6 \quad x^{-\frac{1}{2}} - 3a^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} - \frac{3}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{x}} - \frac{3}{\sqrt{a}} = \frac{\sqrt{a} - 3\sqrt{x}}{\sqrt{a}\sqrt{x}}$$

$$7 \quad x^{-\frac{1}{2}} - 2x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} - \frac{2}{x^{\frac{1}{2}}} = \frac{1-2}{x^{\frac{1}{2}}} = \frac{-1}{x^{\frac{1}{2}}} = -\frac{1}{\sqrt{x}}$$

$$8 \quad \sqrt[5]{x^2} - \sqrt[5]{x^{-a}} = \frac{\sqrt[5]{x^2}}{\sqrt[5]{x^{-a}}} = \sqrt[5]{\frac{x^2}{x^{-a}}} = \sqrt[5]{x^{2+a}}$$

$$9 \quad \sqrt[2m]{a^{-5}} \times \sqrt[m]{a^8} = a^{\frac{-5}{2m}} \times a^{\frac{8}{m}} = a^{-\frac{5}{2m} + \frac{8}{m}} = a^{\frac{-5+16}{2m}} = a^{\frac{11}{2m}} = \sqrt[2m]{a^{11}}$$

$$10 \quad \sqrt[4a]{x^b} - \sqrt[2a]{x^{-6}} = x^{\frac{b}{4a}} - x^{\frac{-6}{2a}} = x^{\frac{b}{4a}} \times x^{\frac{6}{2a}} = x^{\frac{b}{4a} + \frac{6}{2a}} = x^{\frac{b+12}{4a}} = x^{\frac{4}{a}} = \sqrt[a]{x^4}$$

$$11 \quad (\sqrt[2]{x})^7 = (x^{\frac{1}{2}})^7 = x^{\frac{7}{2}}$$

$$12 \quad \left(\frac{1}{\sqrt{a}}\right)^{-6} = \left(a^{\frac{1}{2}}\right)^{-1} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$$

$$13 \quad \frac{1}{\sqrt[3]{x^{-2}}} = \frac{1}{x^{\frac{-2}{3}}} = x^{\frac{2}{3}}$$

$$14 \quad \frac{1}{(\sqrt[5]{a})^{-2}} = \frac{1}{(a^{\frac{1}{5}})^{-2}} = \frac{1}{a^{\frac{-2}{5}}} = a^{\frac{2}{5}}$$

$$15 \quad \sqrt[3]{x^4} - \left(\sqrt[6]{x^7}\right)^{-1} = x^{\frac{4}{3}} - \left(x^{\frac{7}{6}}\right)^{-1} \\ = x^{\frac{4}{3}} x^{-\frac{1}{6}} = x^{\frac{8}{6} - \frac{1}{6}} = x^{\frac{7}{6}}$$

$$16 \quad \sqrt[4]{a^{-3}} - \left(\sqrt[8]{a}\right)^{-12} = a^{-\frac{3}{4}} - \left(a^{\frac{1}{8}}\right)^{-12} \\ = a^{-\frac{3}{4}} - a^{-\frac{12}{8}} = a^{-\frac{3}{4}} - a^{-\frac{3}{2} + \frac{1}{2}} = a^{\frac{1}{4}}$$

$$17 \quad 4^{-\frac{2}{3}} = \frac{1}{4^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{4^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{8}$$

$$18 \quad 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$19 \quad 9^3 = \sqrt[3]{9^3} = \sqrt[3]{9 \times 9 \times 9} = 3 \times 3 \times 3 = 27$$

$$20 \quad 16^{\frac{5}{4}} = \sqrt[4]{16^5} = \sqrt[4]{(2^4)^5} = 2^5 = 32$$

$$21 \quad 81^{-\frac{2}{3}} = \frac{1}{81^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{81})^2} = \frac{1}{3^2} = \frac{1}{27}$$

$$22 \quad \frac{1}{6^{-2}} = 6^2 = 36$$

$$23 \quad (125)^{-\frac{2}{3}} = \frac{1}{(125)^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$$

$$24 \quad \left(\frac{1}{27}\right)^{-\frac{4}{3}} = 27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = (3)^4 = 81$$

$$25 \quad \left(\frac{1}{216}\right)^{-\frac{2}{3}} = (216)^{\frac{2}{3}} = \left(\sqrt[3]{216}\right)^2 = (6)^2 = 36$$

$$26 \quad \frac{x^{m+2n}x^{5m-8n}}{x^{6m-6n}} = \frac{x^{m+2n+5m-8n}}{x^{6m-6n}} = x^{4m-6n+6n} = x^{-m}$$

Exercise (79)

$$1 \quad (a^{-4})^3 = a^{(-4) \times 3} = a^{-(3 \times 4)} = a^{-12}$$

$$2 \quad (a^{-\frac{3}{4}}b^{\frac{5}{6}})^4 = a^{(-\frac{3}{4}) \times 4} b^{\frac{5}{6} \times 4} = a^{-3}b^{\frac{10}{3}}$$

$$3 \quad (a^{-\frac{1}{2}}b^{-3})^{-2} = a^{(-\frac{1}{2}) \times (-2)} b^{(-3) \times (-2)} = ab^6$$

$$4 \quad (a^6b^{\frac{5}{4}})^{-4} = a^{6 \times (-4)} b^{\frac{5}{4} \times (-4)} = a^{-24}b^{-5}$$

$$5 \quad (\sqrt[3]{a^4b^3})^6 = (a^{\frac{4}{3}}b^1)^6 = a^8b^6$$

$$6 \quad (\sqrt[3]{x^3y^{-8}})^{-3} = (x^1y^{-\frac{8}{3}})^{-3} = x^{-3}y^8$$

$$7 \quad \frac{8}{\sqrt{x^2} \sqrt[3]{x^{-3}}} = \frac{8}{\sqrt{x^2} x^{-\frac{1}{3}}} = \frac{8}{x^{\frac{1}{2}} x^{-\frac{1}{3}}} = \frac{8}{x^{\frac{1}{6}}} = 8x^{\frac{5}{6}}$$

$$8 \quad \sqrt[3]{a^{-3}b^4} \times \sqrt[4]{a^2b^{-8}} = a^{-\frac{1}{3}}b^{\frac{4}{3}} \times a^{\frac{1}{2}}b^{-2} = a^{-\frac{1}{3}-\frac{1}{2}}b^{\frac{4}{3}-2} = a^{-\frac{5}{6}}b^{-\frac{2}{3}}$$

$$9 \quad \sqrt[4]{x^{-2}} \sqrt[3]{y^6} \times \sqrt[5]{x^2} \sqrt[4]{y^3} = \sqrt[4]{x^{-2}} y^{\frac{6}{4}} \times \sqrt[5]{x^2} \sqrt[4]{y^3} = \sqrt[4]{x^{-2}} y^{\frac{3}{2}} \times \sqrt[5]{x^2} \sqrt[4]{y^3} = x^{-\frac{1}{2}} y^{\frac{3}{2}} \times x^{\frac{2}{5}} y^{\frac{3}{4}} = x^{-\frac{1}{2}+\frac{2}{5}} y^{\frac{3}{2}+\frac{3}{4}} = x^{-\frac{1}{10}} y^{\frac{15}{4}}$$

$$10 \quad (8x^3 - 27a^{-3})^{\frac{2}{3}} = \left(\frac{8x^3}{27a^{-3}}\right)^{\frac{2}{3}} = \left(\frac{8}{27} x^3 a^3\right)^{\frac{2}{3}} = \left(\frac{2^3}{3^3} x^3 a^3\right)^{\frac{2}{3}} = \frac{2^2}{3^2} x^2 a^2 = \frac{4}{9} x^2 a^2$$

$$11 \quad (64x^3 - 27a^{-3})^{-\frac{2}{3}} = \left(\frac{4^3 x^3}{3^3 a^{-3}}\right)^{-\frac{2}{3}} = \left(\frac{4^3 x^3 a^3}{3^3}\right)^{-\frac{2}{3}} = \left(\frac{3^3}{4^3} x^{-3} a^{-3}\right)^{\frac{2}{3}} = \frac{3^2}{4^2} x^{-2} a^{-2} = \frac{9}{16} x^{-2} a^{-2}$$

$$12 \quad \sqrt[3]{a^4b^{-2}c^{-4}} \times \sqrt[4]{a^{-6}b^4c^8} = a^2b^{-\frac{2}{3}}c^{-\frac{4}{3}} \times a^{-\frac{3}{2}}b^{-1}c^2$$

$$= a^{2-\frac{3}{2}}b^{-\frac{2}{3}-1}c^{-\frac{4}{3}+2} = a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{2}{3}}$$

$$13 \quad \sqrt{a^{-3}b^4c^{-1}} - \sqrt[3]{a^2b^4c^{-1}} = a^{-\frac{3}{2}}b^2c^{-\frac{1}{2}} - a^{\frac{2}{3}}b^{\frac{4}{3}}c^{-\frac{1}{3}}$$

$$= a^{-\frac{3}{2}-\frac{2}{3}}b^{2-\frac{4}{3}}c^{-\frac{1}{2}+\frac{1}{3}} = a^{-\frac{13}{6}}b^{\frac{2}{3}}c^{-\frac{1}{6}}$$

$$14 \quad \sqrt{ab^{-2}c^3} - (\sqrt[3]{a^3b^{-1}c^{-3}})^{-1} = (a^{\frac{1}{2}}b^{-1}c^{\frac{3}{2}}) \times (ab^{\frac{2}{3}}c^{-1})$$

$$= a^{\frac{1}{2}+1}b^{-1+\frac{2}{3}}c^{\frac{3}{2}-1} = a^{\frac{3}{2}}b^{-\frac{1}{3}}c^{\frac{1}{2}}$$

$$15 \quad \left(\frac{a^{-1}b^3}{a^3b^{-4}}\right)^7 - \left(\frac{a^2b^{-5}}{a^{-1}b^3}\right)^{-5} = \frac{a^{-7}b^{14}}{a^{14}b^{-28}} \times \frac{a^{15}b^{-25}}{a^{-10}b^{15}} = \frac{a^8b^{-11}}{a^4b^{-13}} = a^4b^3$$

Exercise (80)

$$1 \quad \begin{array}{r} 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 2x^{\frac{1}{6}} + 1 \\ x^{\frac{1}{3}} - 2x^{\frac{1}{6}} + 1 \\ \hline x + 2x^{\frac{5}{6}} + 3x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + x^{\frac{1}{3}} \\ - 2x^{\frac{5}{6}} - 4x^{\frac{2}{3}} - 6x^{\frac{1}{2}} - 4x^{\frac{1}{3}} - 2x^{\frac{1}{6}} \\ \hline + x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 2x^{\frac{1}{6}} + 1 \\ \hline x \qquad \qquad - 2x^{\frac{1}{2}} \qquad \qquad + 1 \end{array}$$

$$2 \quad \begin{array}{r} a^{\frac{2}{3}} + 3a^{\frac{1}{3}}b^{\frac{1}{3}} + 9b^{\frac{2}{3}} \\ a^{\frac{1}{3}} - 3b^{\frac{1}{3}} \\ \hline a + 3a^{\frac{2}{3}}b^{\frac{1}{3}} + 9a^{\frac{1}{3}}b^{\frac{2}{3}} \\ - 3a^{\frac{2}{3}}b^{\frac{1}{3}} - 9a^{\frac{1}{3}}b^{\frac{2}{3}} - 27b \\ \hline a \qquad \qquad \qquad - 27b \end{array}$$

$$3 \quad \begin{array}{r} 1 + ab^{-1} + a^3b^{-3} \\ 1 - ab^{-1} + a^3b^{-3} \\ \hline 1 + ab^{-1} + a^3b^{-3} \\ - ab^{-1} - a^3b^{-3} - a^3b^{-3} \\ \hline + a^3b^{-3} + a^3b^{-3} + a^4b^{-4} \\ \hline 1 \qquad \qquad + a^3b^{-4} \qquad \qquad + a^4b^{-4} \end{array}$$

4

$$\frac{r+2j^{-1}+3z^{-1}}{r-2j^{-1}+3z^{-1}} \cdot \frac{r^2+22j^{-1}+3xz^{-1}}{-2xj^{-1}-4j-6j^{-1}z^{-1}} \cdot \frac{+3xz^{-1}+6j^{\frac{1}{2}}z^{\frac{1}{2}}+9z^{\frac{2}{2}}}{r^2+6xz^{-1}-4j+9z^{\frac{1}{2}}}$$

5

$$\frac{x^{-1}+z^{-1}j^{-1}+j^{-1}}{z^{-1}-r^{-1}j^{-1}+j^{-1}} \cdot \frac{r^{-2}+r^{-1}j^{-1}+z^{-1}j^{-1}}{-z^{-1}j^{-1}-x^{-1}j^{-1}-z^{-1}j^{-1}} \cdot \frac{+r^{-1}j^{-1}+z^{-1}j^{-1}+j^{-2}}{+z^{-1}j^{-1}+j^{-2}}$$

6

$$\frac{a^{-1}-a^{-1}+1-a^{-1}+a^{-1}}{a^{-1}+1+a^{-1}} \cdot \frac{a-a^{-1}+a^{-1}-1+a^{-1}}{+a^{-1}-a^{-1}+1-a^{-1}+a^{-1}} \cdot \frac{+a^{-1}-1+a^{-1}-a^{-1}+a^{-1}}{a^{-1}+a^{-1}-1+a^{-1}+a^{-1}}$$

7

$$\frac{z^{-1}-r^{-1}j^{-1}-z^{-1}z^{-1}+j^{-1}-j^{-1}z^{-1}+z^{-1}}{z^{-1}+j^{-1}+z^{-1}} \cdot \frac{z^{-1}-z^{-1}j^{-1}-z^{-1}z^{-1}+r^{-1}j^{-1}-r^{-1}j^{-1}z^{-1}+r^{-1}z^{-1}}{+r^{-1}j^{-1}-z^{-1}j^{-1}-z^{-1}j^{-1}z^{-1}+j^{-1}-j^{-1}z^{-1}+j^{-1}z^{-1}} \cdot \frac{+z^{-1}z^{-1}-x^{-1}j^{-1}z^{-1}-r^{-1}z^{-1}+j^{-1}z^{-1}-j^{-1}z^{-1}+z}{r-3x^{-1}j^{-1}z^{-1}+j+z}$$

8

$$\begin{array}{r}
 a^m + 3b^n - 2c^p \\
 a^m - 3b^n + 2c^p \\
 \hline
 2a^m + 3a^m b^n - 2a^m c^p \\
 - 3a^m b^n \qquad - 9b^{2n} + 6b^n c^p \\
 \hline
 \qquad \qquad \qquad + 2a^m c^p \qquad + 6b^n c^p - 4c^{2p} \\
 \hline
 a^{2m} \qquad \qquad \qquad - 9b^{2n} + 12b^n c^p - 4c^{2p}
 \end{array}$$

9

$$\begin{array}{r}
 a^7 + 2a^2 b^3 + 4a^2 b^3 + 8ab^4 + 16a^1 b^4 + 32b^7 \\
 a^1 - 2b^1 \\
 \hline
 a^7 + 2a^2 b^3 + 4a^2 b^3 + 8a^1 b^4 + 16ab^4 + 32a^1 b^4 \\
 - 2a^1 b^1 - 4a^2 b^1 - 8a^1 b - 16ab^1 - 32a^1 b^1 - 64b^2 \\
 \hline
 a^7 \qquad \qquad \qquad - 64b^2
 \end{array}$$

10

$$\begin{array}{r}
 a^{\frac{1}{2}} + a^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + 1 \\
 a^{\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} - r^{-\frac{1}{2}} \\
 \hline
 a + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} \\
 - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} \\
 + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} + a^{\frac{1}{2}} r^{-\frac{1}{2}} \\
 - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} - a^{\frac{1}{2}} r^{-\frac{1}{2}} \\
 \hline
 a \qquad \qquad + a^{\frac{1}{2}} x^{-\frac{1}{2}} \qquad \qquad - a^{\frac{1}{2}} r^{-\frac{1}{2}} \qquad \qquad - r^{-1}
 \end{array}$$

11

$$\begin{array}{r}
 \text{Quotient} = r - 1 \\
 (r^2 - 4r + 2) \div (r^2 - 2r - 4r^2 + 6r - 2r^1) \\
 \hline
 \qquad \qquad \qquad - 4r^2 + 2r \\
 \hline
 \qquad \qquad \qquad - r^2 \qquad + 4r - 2r^1 \\
 \hline
 \qquad \qquad \qquad - r^2 \qquad + 4r - 2r^1
 \end{array}$$

12

$$\begin{array}{r}
 \text{Quotient} = 2 + 4x^{-1} + 2x^{-2} \\
 (4 - 2x^{-1} + x^{-2}) \div (8 + 12x^{-1} + 2x^{-2} + 2x^{-1}) \\
 \hline
 \qquad \qquad \qquad 8 - 4x^{-1} + 2x^{-2} \\
 \hline
 \qquad \qquad \qquad 16x^{-1} \qquad + 2x^{-4} \\
 \hline
 \qquad \qquad \qquad 16x^{-1} - 8x^{-2} + 4x^{-2} \\
 \hline
 \qquad \qquad \qquad 8x^{-2} - 4x^{-2} + 2x^{-4} \\
 \hline
 \qquad \qquad \qquad 8x^{-2} - 4x^{-2} + 2x^{-4}
 \end{array}$$

$$\begin{aligned}
 13 \quad \text{Quotient} &= y + x^{-1}y^{-1} + 1 \\
 & (1^{-1} + x^{-1}y^{-1} + y^{-1}) (x^{-1}y + 2x^{-1}y^{-1} + 3 + 2xy^{-1} + xy^{-1}) \\
 & \quad \frac{x^{-1}y + x^{-1}y^{-1} + 1}{x^{-1}y^{-1} + 2 + 2x^{-1}y^{-1} + xy^{-1}} \\
 & \quad \frac{x^{-1}y^{-1} + 1 + x^{-1}y^{-1}}{1 + x^{-1}y^{-1} + xy^{-1}} \\
 & \quad \frac{1 + x^{-1}y^{-1} + xy^{-1}}{1 + x^{-1}y^{-1} + xy^{-1}}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \text{Quotient} &= a + a^{-1}b^{-1} - b \\
 & (a^{-1} - ab^{-1} + a^{-1}b - b^{-1}) (a^{-1} - a^{-1}b + ab^{-1} - 2a^{-1}b^2 + b^{-5}) \\
 & \quad \frac{a^{-1} - a^2b^{-1} + a^{-1}b - ab^{-1}}{a^2b^{-1} - 2a^{-1}b + 2ab^{-1} - 2a^{-1}b^2 + b^{-5}} \\
 & \quad \frac{a^2b^{-1} - a^{-1}b + ab^{-1} - a^{-1}b^2}{-a^{-1}b + ab^{-1} + a^{-1}b^2 + b^{-5}} \\
 & \quad \frac{-a^{-1}b + ab^{-1} - a^{-1}b^2 + b^{-5}}{-a^{-1}b + ab^{-1} - a^{-1}b^2 + b^{-5}}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \text{Quotient} &= x^{2n} - 1 + x^{-2n} \\
 & (5x^n - 3x^{-n}) (5x^{2n} - 8x^n + 8x^{-n} - 3x^{-2n}) \\
 & \quad \frac{5x^{2n} - 3x^n}{-5x^n + 8x^{-n} - 3x^{-2n}} \\
 & \quad \frac{-5x^n + 8x^{-n} - 3x^{-2n}}{5x^{-n} - 3x^{-2n}} \\
 & \quad \frac{5x^{-n} - 3x^{-2n}}{5x^{-n} - 3x^{-2n}}
 \end{aligned}$$

16 Putting a for $2x^{\frac{1}{2}}$, b for $y^{-\frac{1}{2}}$ and c for $-z^{\frac{1}{2}}$, we have

$$\begin{aligned}
 & 8x^{\frac{1}{2}} + y^{-\frac{1}{2}} - z + 6x^{\frac{1}{2}}y^{-\frac{1}{2}}z^{\frac{1}{2}} \\
 & = a^2 + b^2 + c^2 - 3abc \\
 & = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc) \\
 & = (2x^{\frac{1}{2}} + y^{-\frac{1}{2}} - z^{\frac{1}{2}}) \{ (2x^{\frac{1}{2}})^2 + (y^{-\frac{1}{2}})^2 + (-z^{\frac{1}{2}})^2 \\
 & \quad - 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{-\frac{1}{2}}z^{\frac{1}{2}} \} \\
 & = (2x^{\frac{1}{2}} + y^{-\frac{1}{2}} - z^{\frac{1}{2}}) (4x + y^{-1} + z^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{-\frac{1}{2}}z^{\frac{1}{2}})
 \end{aligned}$$

Thus the quotient $= 4x - 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{-1} + y^{-\frac{1}{2}}z^{\frac{1}{2}} + z^{\frac{1}{2}}$

$$\text{Quotient} = x^4 - x^{\frac{15}{2}} a^{\frac{1}{2}} + r^{\frac{9}{2}} a^{\frac{9}{2}} - x^{\frac{3}{2}} a^{\frac{17}{2}} + a^4$$

$$\begin{array}{r}
 17 \quad x^4 + r^{\frac{1}{2}} a^{\frac{1}{2}} + a^4 \quad x^2 + x^{\frac{1}{2}} a^{\frac{1}{2}} + a^2 \\
 \hline
 \quad \quad \quad x^3 + x^{\frac{1}{2}} a^{\frac{1}{2}} + r^{\frac{9}{2}} a^{\frac{1}{2}} \\
 \hline
 \quad \quad \quad - r^{\frac{1}{2}} a^{\frac{1}{2}} - x^{\frac{1}{2}} a^{\frac{1}{2}} + x^{\frac{1}{2}} a^{\frac{1}{2}} + a^3 \\
 \hline
 \quad \quad \quad - r^{\frac{1}{2}} a^{\frac{3}{2}} - x^{\frac{1}{2}} a^{\frac{1}{2}} x - \frac{15}{2} a^{\frac{9}{2}} \\
 \hline
 \quad \quad \quad x^{\frac{15}{2}} a^{\frac{9}{2}} + x^{\frac{1}{2}} a^{\frac{1}{2}} + a^3 \\
 \hline
 \quad \quad \quad r^{\frac{15}{2}} a^{\frac{9}{2}} + x^{\frac{1}{2}} a^{\frac{1}{2}} - r^{\frac{9}{2}} a^{\frac{1}{2}} \\
 \hline
 \quad \quad \quad - x^{\frac{9}{2}} a^{\frac{15}{2}} + a^3 \\
 \hline
 \quad \quad \quad - x^{\frac{9}{2}} a^{\frac{15}{2}} - x^{\frac{1}{2}} a^{\frac{1}{2}} - r^{\frac{9}{2}} a^{\frac{15}{2}} \\
 \hline
 \quad \quad \quad x^{\frac{1}{2}} a^{\frac{9}{2}} + r^{\frac{1}{2}} a^{\frac{1}{2}} + a^3 \\
 \hline
 \quad \quad \quad x^{\frac{1}{2}} a^{\frac{1}{2}} + r^{\frac{1}{2}} a^{\frac{1}{2}} + a^3
 \end{array}$$

$$\begin{aligned}
 18 \quad & (x^{2^{n-1}} + a^{2^{n-1}})(x^{2^{n-1}} - a^{2^{n-1}}) \\
 = & (x^{2^{n-1}})^2 - (a^{2^{n-1}})^2 \\
 = & x^{2^{2^{n-1}}} - a^{2^{2^{n-1}}} = x^{2^{1+n-1}} - a^{2^{1+n-1}} = x^{2^n} + a^{2^n}
 \end{aligned}$$

$$\text{Quotient} = x^{2^{n-1}} - y^{2^{n-1}}$$

$$\begin{array}{r}
 19 \quad (x^{2^{n-1}} + y^{2^{n-1}}) x^{2^n} - y^{2^n} \\
 \hline
 \quad \quad \quad x^{2^n} + x^{2^{n-1}} y^{2^{n-1}} \\
 \hline
 \quad \quad \quad - x^{2^{n-1}} y^{2^{n-1}} - y^{2^n} \\
 \hline
 \quad \quad \quad - x^{2^{n-1}} y^{2^{n-1}} - y^{2^n}
 \end{array}$$

$$20 \quad \left(a^{\frac{1}{m}} \right)^{\frac{1}{n} - \frac{1}{m}} \left\{ \frac{1}{a^{\frac{1}{m}-1}} \right. \\ = \left. \left(a^{\frac{1}{m}} \right)^{\frac{1}{n} - \frac{1}{m}} \right\}^{\frac{1}{m-1}}$$

$$= a^{\frac{1}{m} \times \frac{m-1}{m-1}} \times \frac{1}{m-1} = a^{\frac{m-1}{m}} \times \frac{1}{m-1} = a^{\frac{1}{m}}$$

$$21 \quad \begin{array}{r} \text{Quotient} = x^{\frac{1}{2}} + 3x^{\frac{1}{2}} - 1 \\ (x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}) \overline{) x + 3x^{\frac{1}{2}} - 2x^{\frac{1}{2}} - 7x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}} \\ \underline{x \phantom{+ 3x^{\frac{1}{2}}} - 2x^{\frac{1}{2}}} \\ 3x^{\frac{1}{2}} \phantom{- 7x^{\frac{1}{2}}} + 2x^{-\frac{1}{2}} \\ \underline{3x^{\frac{1}{2}} \phantom{+ 2x^{-\frac{1}{2}}}} \\ -x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \\ \underline{-x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}} \\ 0 \end{array}$$

$$22 \quad \begin{array}{r} x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2} - y^{\frac{1}{2}} \\ \underline{x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2} + y^{\frac{1}{2}}} \\ x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2} + x^{\frac{1}{2}} y^{\frac{1}{2}} \\ \underline{-x^{\frac{1}{2}} - \frac{1}{2} + x^{\frac{1}{2}} y^{\frac{1}{2}}} \\ \phantom{x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2}} + x^{\frac{1}{2}} y^{\frac{1}{2}} - x^{\frac{1}{2}} y^{\frac{1}{2}} \\ \underline{\phantom{x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2}} + x^{\frac{1}{2}} y^{\frac{1}{2}} - x^{\frac{1}{2}} y^{\frac{1}{2}} + y^{\frac{1}{2}}} \\ x^{\frac{1}{2}} - 2x^{\frac{1}{2}} y^{\frac{1}{2}} - \frac{1}{2} + y^{\frac{1}{2}} - 2x^{\frac{1}{2}} y^{\frac{1}{2}} - 2x^{\frac{1}{2}} y^{\frac{1}{2}} + y^{\frac{1}{2}} \end{array}$$

$$23 \quad \text{Quotient} = x^{\frac{n}{2}} + x^{\frac{n}{2}} a^{\frac{n}{2}} + a^{\frac{n}{2}}$$

$$\begin{array}{r} \left(x^{\frac{n}{2}} - a^{\frac{n}{2}} \right) \overline{) x^{\frac{n}{2}} - a^{\frac{n}{2}}} \\ \underline{x^{\frac{n}{2}} - x^{\frac{n}{2}} a^{\frac{n}{2}}} \\ x^{\frac{n}{2}} a^{\frac{n}{2}} - a^{\frac{n}{2}} \\ \underline{x^{\frac{n}{2}} a^{\frac{n}{2}} - x^{\frac{n}{2}} a^{\frac{n}{2}}} \\ x^{\frac{n}{2}} a^{\frac{n}{2}} - a^{\frac{n}{2}} \\ \underline{x^{\frac{n}{2}} a^{\frac{n}{2}} - a^{\frac{n}{2}}} \\ 0 \end{array}$$

24.

$$\begin{array}{r}
 x^1 - 2x^{\frac{1}{2}} + x^0 \\
 \hline
 x^{\frac{1}{2}} - 2x^0 - x^0 \\
 \hline
 x^0 - 2x^0 - x^0 \\
 \hline
 -2x^0 + 1x^0 \qquad -2x^{\frac{1}{2}} \\
 \hline
 \qquad -x^0 \qquad -2x^{\frac{1}{2}} - x^0 \\
 \hline
 x^0 - 1x^0 - 1x^0 - 2x^{\frac{1}{2}} - 1x^{\frac{1}{2}} - x^0
 \end{array}$$

25

Quo ent = $a^2 x^{-2} - c^1 x^{-1} - a^{-1} x^1 - a^{-2} x^2$
 $c^1 x^{-1} - 1 - a^{-1} x^1$) $ax^{-1} - 2 - a^{-1} x$

$$\begin{array}{r}
 ax^{-1} - a^2 x^{-2} - a^1 x^{-1} \\
 \hline
 a^2 x^{-2} - c^1 x^{-1} - 1 + 2 - 1^1 x \\
 \hline
 a^2 x^{-2} - x^1 x^{-1} + 1 \\
 \hline
 1 \qquad \qquad -a^{-1} x \\
 \hline
 1 - c^{-1} x^1 - a^{-2} x^2 \\
 \hline
 a^{-1} x^1 - a^{-2} x^2 + a^{-1} x \\
 \hline
 c^{-1} x^1 - a^{-2} x^2 - a^{-1} c
 \end{array}$$

26

$$\begin{aligned}
 & \left(\frac{a-c}{\frac{1}{a}-\frac{1}{b}} - \frac{a^2-b^2}{c-c} \right)^{-1} \\
 &= \left| \frac{(a-b)(a^1-b^1) - (a^2-b^2)}{a-c} \right|^{-1} \\
 &= \left[\frac{(a^1-b^1) \{ (a^1-b^1)^2 - (c+a^1 b^1 + b) \}}{(a^1-b^1)(c^1-b^1)} \right]^{-1} \\
 &= \left(\frac{a^1 b^1}{a^1 - b^1} \right)^{-1} = \frac{a^1 - b^1}{a^1 b^1} = \frac{1}{b^1} - \frac{1}{a^1} = \frac{1}{a} - \frac{1}{b}
 \end{aligned}$$

27 The given expression

$$= \frac{(x^1 + 3,^1)(x^2 - 3x^{\frac{1}{2}} + 0,^2) - (x^1 - 3,^1)(x^2 - 3x^{\frac{1}{2}} - 9,^2)}{x - 2,^1}$$

$$\begin{aligned}
 &= \frac{(x+27y) + 6x^{\frac{1}{3}}y^{\frac{1}{3}}(x^{\frac{1}{3}}+3y^{\frac{1}{3}}) + (x-27y) - 6x^{\frac{1}{3}}y^{\frac{1}{3}}(x^{\frac{1}{3}}-3y^{\frac{1}{3}})}{x-27y} \\
 &= \frac{2x + 36x^{\frac{1}{3}}y^{\frac{1}{3}}}{x-27y}
 \end{aligned}$$

28 The numerator

$$= a(a^{\frac{1}{2}} - x^{\frac{1}{2}}) + x(a^{\frac{1}{2}} - x^{\frac{1}{2}}) = (a^{\frac{1}{2}} - x^{\frac{1}{2}})(a+x)$$

The denominator

$$\begin{aligned}
 &= a^2(a^{\frac{1}{2}} - x^{\frac{1}{2}}) + 3ax(a^{\frac{1}{2}} - x^{\frac{1}{2}}) + x^2(a^{\frac{1}{2}} - x^{\frac{1}{2}}) \\
 &= (a^{\frac{1}{2}} - x^{\frac{1}{2}})(a^2 + 3ax + x^2)
 \end{aligned}$$

Thus the given expression

$$\begin{aligned}
 &= \frac{(a^{\frac{1}{2}} - x^{\frac{1}{2}})(a+x)}{(a^{\frac{1}{2}} - x^{\frac{1}{2}})(a^2 + 3ax + x^2)} = \frac{a+x}{a^2 + 3ax + x^2}
 \end{aligned}$$

29 The given expression

$$\begin{aligned}
 &= \frac{a^2 + b^2 - a^{-2} - b^{-2} + (ab - a^{-1}b - ab^{-1} + a^{-1}b^{-1})(ab - a^{-1}b^{-1})}{a^2b^2 - a^{-2}b^{-2}} \\
 &= \frac{a^2 + b^2 - a^{-2} - b^{-2} + (a^2b^2 - a^{-2}b^{-2}) - (a^{-1}b + ab^{-1})(ab - a^{-1}b^{-1})}{a^2b^2 - a^{-2}b^{-2}} \\
 &= \frac{a^2 + b^2 - a^{-2} - b^{-2} + (a^2b^2 - a^{-2}b^{-2}) - a^2 - b^2 + b^{-2} + a^{-2}}{a^2b^2 - a^{-2}b^{-2}} \\
 &= \frac{a^2b^2 - a^{-2}b^{-2}}{a^2b^2 - a^{-2}b^{-2}} = 1
 \end{aligned}$$

30 The given expression

$$\begin{aligned}
 &= \frac{(x^{\frac{1}{2}} - y^{\frac{1}{2}})(x^{\frac{1}{2}} + y^{\frac{1}{2}})}{x^{\frac{1}{2}}(x^{\frac{1}{2}} + y^{\frac{1}{2}})} \times \frac{x^{\frac{1}{2}}y^{\frac{1}{2}}(x^{\frac{1}{2}} + y^{\frac{1}{2}})}{x^{\frac{1}{2}} + y^{\frac{1}{2}}} \\
 &= \frac{(x^{\frac{1}{2}} - y^{\frac{1}{2}})y^{\frac{1}{2}}}{x^{\frac{1}{2}}} = (x^{\frac{1}{2}}y^{\frac{1}{2}} - y^{\frac{1}{2}})x^{-\frac{1}{2}} = x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{-\frac{1}{2}}y^{\frac{1}{2}}
 \end{aligned}$$

31 The 1st term

$$\begin{aligned}
 &= 1 + ab^{-1} + ac^{-1} + a^{-1}b + 1 + bc^{-1} + a^{-1}c + b^{-1}c + 1 \\
 &= 3 + ab^{-1} + ac^{-1} + a^{-1}b + a^{-1}c + bc^{-1} + b^{-1}c
 \end{aligned}$$

The 2nd. term

$$= a^{-1}b^{-1}c^{-1}(a^2b + a^2c + ab^2 + ac^2 + b^2c + bc^2 + 2abc)$$

$$= ac^{-1} + ab^{-1} + bc^{-1} + b^{-1}c + a^{-1}c + 2$$

the given expression = 1

$$32 \quad 2^{x+7} = 4^{x+2}$$

$$\text{or,} \quad 2^{x+7} = 2^{2(x+2)}$$

$$\text{or,} \quad x+7 = 2x+4, \quad x=3$$

$$33 \quad (\sqrt[3]{3})^{x+5} = (\sqrt[3]{3})^{2x+5}$$

$$\text{or,} \quad \frac{x+5}{3} = \frac{2x+5}{3}$$

$$\text{or,} \quad \frac{x+5}{2} = \frac{2x+5}{3}$$

$$\text{or,} \quad 3x-15 = 4x+10, \quad x=5$$

$$34 \quad (\sqrt[5]{4})^{4x+7} = (\sqrt[5]{64})^{2x+7}$$

$$\text{or,} \quad (4)^{\frac{4x+7}{5}} = (64)^{\frac{2x+7}{11}}$$

$$\text{or,} \quad (4)^{\frac{4x+7}{5}} = \{(4)^2\}^{\frac{2x+7}{11}}$$

$$\text{or,} \quad (4)^{\frac{4x+7}{5}} = (4)^{\frac{3(2x+7)}{11}}$$

$$\text{or,} \quad \frac{4x+7}{5} = \frac{6x+21}{11}$$

$$\text{or,} \quad 44x+77 = 60x+105$$

$$\text{or,} \quad 14x=28, \quad x=2$$

$$35 \quad (\sqrt[3]{25})^{2x+1} = (\sqrt[3]{125})^{x+6}$$

$$\text{or} \quad (25)^{\frac{2x+1}{3}} = (125)^{\frac{x+6}{5}}$$

$$\text{or,} \quad (5^2)^{\frac{2x+1}{3}} = \{(5)^3\}^{\frac{x+6}{5}}$$

$$\text{or,} \quad (5)^{\frac{2(2x+1)}{3}} = (5)^{\frac{3(x+6)}{5}}$$

$$\begin{aligned} \text{or,} \quad & \frac{2(2x+1)}{3} = \frac{3(x+6)}{5} \\ \text{or,} \quad & 20x + 10 = 9x + 54 \\ \text{or,} \quad & 11x = 44, \quad x = 4 \end{aligned}$$

36 From (1), we have

$$2^{2x-1} = 2^{2(y-1)} \quad \text{or,} \quad 3x-1 = 2y-2 \quad \text{or,} \quad 3x = 2y-1$$

Also we have $3x - y = 1$ (2) $2y - 1 - y = 1$ $y = 2$

Hence, from (2), $3x = 1 + 2 = 3$, $x = 1$

Thus $x = 1, y = 2$

37 From (1), we have $(3)^{2(2x-3)} = 3^{\frac{2y-x}{2}}$

$$\text{or,} \quad 4x - 6 = \frac{2y - x}{2}$$

$$\text{or,} \quad 8x - 12 = 2y - x$$

$$\text{or,} \quad 9x - 12 = 2y \quad (3)$$

From (2), we have $2^{2x} = 2^{2y}$ or, $3x = 2y$

Hence, from (3), $9x - 12 = 3x$ or, $6x = 12$, $x = 2$

Hence $2y = 6$, $y = 3$

Thus $x = 2, y = 3$

38 From (1), we have $4^{2y-1} = 4^{2(x+y)}$

$$\text{or,} \quad 3y - 1 = 2(x + y)$$

$$\text{or,} \quad y = 2x + 1 \quad (3)$$

From (2), we have $3^{x+y} = 3^{2(2x+1)}$

$$\text{or,} \quad x + 3y = 4x + 6$$

$$\text{or,} \quad 3y = 3x + 6$$

$$\text{or,} \quad y = x + 2 \quad (4)$$

Hence, from (3) and (4), $2x + 1 = x + 2$, $x = 1$

from (1) $y = 1 + 2 = 3$

Thus $x = 1, y = 3$

39 From (1), we have $2^{x+y+z} = 2^{3(x+z-y)}$

$$\text{or,} \quad x + y + z = 3x + 3z - 3y$$

$$\text{or,} \quad 2x - 4y + 2z = 0$$

$$\text{or,} \quad x - 2y + z = 0 \quad (4)$$

From (2), we have $5^{x+2} = 5^{3(x+1)}$

$$\text{or, } 3j + 2 = 2x + 2z$$

$$\text{or, } 2x - 3j + 2z = 2 \quad (5)$$

also, $2x - 4j + 2z = 0 \quad \dots \text{ from (4)}$

$$j = 2$$

From (3), we have

$$3^{2x-2} = 3^{2(3x+1)}$$

$$\text{or, } 2x + 2z + j = 6x + 2j$$

$$\text{or, } 4x - 2z + j = 0$$

$$4x - 2j = -2$$

$$\text{or, } 2x - z = -1$$

also, $x + z = 4 \quad (4)$

$$3x = 3, \quad x = 1,$$

$$\text{and } z = 4 - 1 = 3$$

Thus $x = 1, j = 2, z = 3$

40 From (1), we have $\frac{x+z}{2} = \frac{y+z}{3}$

$$\text{or } \frac{x+j}{2} = \frac{j+z}{3}$$

$$\text{or, } 3x + 3j = 2j + 2z - 2$$

$$\text{or, } 3x + j - 2z = -2 \quad (4)$$

From (2), we have $\frac{x+z-2}{3} = \frac{y+z}{5}$

$$\text{or, } \frac{x+z-2}{3} = \frac{j+z}{5}$$

$$\text{or, } 5x + 5z - 10 = 3j + 3z$$

$$\text{or } 5x - 3j + 2z = 10 \quad (5)$$

From (3), we have $\frac{y}{4} = \frac{x+j+1}{7}$

$$\text{or, } \frac{j}{4} = \frac{x+j+1}{7}$$

$$\text{or, } 7j = 4x + 4j + 4$$

$$\text{or, } 4x - 3j = -4 \quad (6)$$

Adding (4) and (5) we have $8x - 2j = 8$

$$\text{or, } 4x - j = 4 \quad (7)$$

From (6) and (7), we have

$$2j' = 8, \quad j' = 4$$

Hence, $4x - 4 = 4$ or, $x = 2$

$$\text{From (4)} \quad 6 + 4 - 2z = -2, \quad z = 6$$

Thus $x = 2, j = 4$, and $z = 6$

Exercise (81)

$$1 \quad 3\sqrt{5} = \sqrt{9} \sqrt{5} = \sqrt{9 \times 5} = \sqrt{45}$$

$$2 \quad 2\sqrt{3} = \sqrt[3]{8} \times \sqrt[3]{3} = \sqrt[3]{24}$$

$$3 \quad 2\sqrt[3]{6} = \sqrt[3]{16} \sqrt[3]{6} = \sqrt[3]{96}$$

$$4 \quad 4\sqrt[3]{5} = \sqrt[3]{125} \sqrt[3]{5} = \sqrt[3]{1250}$$

$$5 \quad a^2\sqrt{b} = \sqrt[4]{a^8} \sqrt[4]{b} = \sqrt[4]{a^8 b}$$

$$6 \quad x^2\sqrt[3]{y} = \sqrt[6]{x^4} \sqrt[6]{y} = \sqrt[6]{x^4 y}$$

$$7 \quad a^4\sqrt[3]{b^2} = \sqrt[6]{a^{16}} \sqrt[6]{b^2} = \sqrt[6]{a^{16} b^2}$$

Exercise (82)

$$1 \quad \sqrt{18} = \sqrt{9 \times 2} = \sqrt{3^2 \times 2} = 3\sqrt{2}$$

$$2 \quad \sqrt{80} = \sqrt{16 \times 5} = \sqrt{4^2 \times 5} = 4\sqrt{5}$$

$$3 \quad \sqrt[3]{250} = \sqrt[3]{125 \times 2} = \sqrt[3]{5^3 \times 2} = 5\sqrt[3]{2}$$

$$4 \quad \sqrt[3]{128} = \sqrt[3]{32 \times 4} = \sqrt[3]{2^5 \times 2} = 2\sqrt[3]{4}$$

$$5 \quad \sqrt[3]{405} = \sqrt[3]{81 \times 5} = \sqrt[3]{3^4 \times 5} = 3\sqrt[3]{5}$$

$$6 \quad \sqrt[3]{1372} = \sqrt[3]{343 \times 4} = \sqrt[3]{7^3 \times 4} = 7\sqrt[3]{4}$$

$$7 \quad \sqrt[3]{1875} = \sqrt[3]{625 \times 3} = \sqrt[3]{5^4 \times 3} = 5\sqrt[3]{3}$$

$$8 \quad \sqrt[3]{a^3 b} = \sqrt[3]{(a^3)^1 b} = a \sqrt[3]{b}$$

$$9 \quad \sqrt[4]{16a} = 2 \sqrt[4]{a}$$

$$10 \quad \sqrt[3]{-2560} = \sqrt[3]{-512 \times 5} = \sqrt[3]{(-8)^3 \times 5} = -8\sqrt[3]{5}$$

$$11 \quad \sqrt[3]{-192a^3b^4} = \sqrt[3]{(-64) \times 3a^3b^4} = \sqrt[3]{(-4)^3 \times 3a^3b^3 \times b}$$

$$= -4ab \sqrt[3]{3b}$$

$$12 \quad \sqrt[3]{500a^3x^4} = \sqrt[3]{125 \times 4a^3x^4} = \sqrt[3]{(5)^3 \times 4(a^3)^1 x^4}$$

$$= 5a^3x \sqrt[3]{4ax}$$

Exercise (83).

- 1 $\sqrt{12} + \sqrt[3]{75} = \sqrt{4 \times 3} + \sqrt[3]{25 \times 3} = 2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$
- 2 $\sqrt{18} + \sqrt{32} = \sqrt{9 \times 2} + \sqrt{16 \times 2} = 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$
- 3 $\sqrt{20} + \sqrt{180} = \sqrt{4 \times 5} + \sqrt{36 \times 5} = 2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$
- 4 $\sqrt{98} - \sqrt{50} = \sqrt{49 \times 2} - \sqrt{25 \times 2} = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$
- 5 $\sqrt[3]{128} - \sqrt[3]{54} = \sqrt[3]{64 \times 2} - \sqrt[3]{27 \times 2} = \sqrt[3]{4^3 \times 2} - \sqrt[3]{3^3 \times 2}$
 $= 4\sqrt[3]{2} - 3\sqrt[3]{2} = \sqrt[3]{2}$
- 6 $\sqrt[4]{80} + \sqrt[4]{405} = \sqrt[4]{16 \times 5} + \sqrt[4]{81 \times 5} = 2\sqrt[4]{5} + 3\sqrt[4]{5} = 5\sqrt[4]{5}$
- 7 $\sqrt[4]{768} - \sqrt[4]{243} = \sqrt[4]{256 \times 3} - \sqrt[4]{81 \times 3} = \sqrt[4]{4^4 \times 3} - \sqrt[4]{3^4 \times 3}$
 $= 4\sqrt[4]{3} - 3\sqrt[4]{3} = \sqrt[4]{3}$
- 8 $2\sqrt{27} - \sqrt{75} + \sqrt{12} = 2\sqrt{9 \times 3} - \sqrt{25 \times 3} + \sqrt{4 \times 3}$
 $= 6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$
- 9 $2\sqrt{405} - 3\sqrt{125} + \sqrt{45} = 2\sqrt{81 \times 5} - 3\sqrt{25 \times 5} + \sqrt{9 \times 5}$
 $= 18\sqrt{5} - 15\sqrt{5} + 3\sqrt{5} = 21\sqrt{5} - 15\sqrt{5} = 6\sqrt{5}$
- 10 $4\sqrt[3]{192} - 4\sqrt[3]{375} + 2\sqrt[3]{24} = 4\sqrt[3]{64 \times 3} - 4\sqrt[3]{125 \times 3} + 2\sqrt[3]{8 \times 3}$
 $= 16\sqrt[3]{3} - 20\sqrt[3]{3} + 4\sqrt[3]{3} = 0$
- 11 $3\sqrt[3]{40} + 2\sqrt[3]{625} - 4\sqrt[3]{320} = 3\sqrt[3]{8 \times 5} + 2\sqrt[3]{125 \times 5} - 4\sqrt[3]{64 \times 5}$
 $= 6\sqrt[3]{5} + 10\sqrt[3]{5} - 16\sqrt[3]{5} = 0$
- 12 $5\sqrt{-54} - 2\sqrt{-16} + 4\sqrt[3]{686}$
 $= 5\sqrt{-27 \times 2} - 2\sqrt{-8 \times 2} + 4\sqrt[3]{343 \times 2}$
 $= 5\sqrt[3]{(-3)^3 \times 2} - 2\sqrt[3]{(-2)^3 \times 2} + 4\sqrt[3]{(7)^3 \times 2}$
 $= -15\sqrt[3]{2} + 4\sqrt[3]{2} + 28\sqrt[3]{2} = 17\sqrt[3]{2}$
- 13 $\sqrt{45x^3} + \sqrt{80x^3} + \sqrt{5xy^3}$
 $= \sqrt{9 \times 5 \times x^2 \times x} + \sqrt{16 \times 5 \times x^2 \times x} + \sqrt{5xy^3}$
 $= 3x\sqrt{5x} + 4x\sqrt{5x} + y\sqrt{5x}$
 $= 7x\sqrt{5x} + y\sqrt{5x} = (7x + y)\sqrt{5x}$
- 14 The given expression
 $= x^2 \sqrt[3]{a} + y^2 \sqrt[3]{(-2y)^3 a} - z \sqrt[3]{(-3z)^3 a}$
 $= x^2 \sqrt[3]{a} - 2y^2 \sqrt[3]{a} + 3z^2 \sqrt[3]{a} = (x^2 - 2y^2 + 3z^2) \sqrt[3]{a}$

15 The given expression

$$\begin{aligned} &= 2^4 \sqrt[4]{2^4 a^4 2^4} + 3^4 \sqrt[4]{4^4 a^4 2^4} - 4a^4 \sqrt[4]{3^4 2^4} \\ &= 2 \cdot 2a^4 \sqrt[4]{2^4} + 3 \cdot 4a^4 \sqrt[4]{2^4} - 4a^4 \sqrt[4]{3^4 2^4} \\ &= 4a^4 \sqrt[4]{2^4} + 12a^4 \sqrt[4]{2^4} - 12a^4 \sqrt[4]{2^4} = 4a^4 \sqrt[4]{2^4} \end{aligned}$$

Exercise (84)

1 We have $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$,

$$\text{and } \sqrt[4]{2} = 2^{\frac{1}{4}} = 2^{\frac{3}{12}} = \sqrt[12]{2^3} = \sqrt[12]{8}$$

Thus the required surds are $\sqrt[12]{27}$ and $\sqrt[12]{8}$

2 We have $\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\frac{2}{8}} = \sqrt[8]{4^2} = \sqrt[8]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[8]{16}$,

$$\text{and } \sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{2}{8}} = \sqrt[8]{5^2} = \sqrt[8]{25}$$

Thus the required surds are $\sqrt[8]{16}$ and $\sqrt[8]{25}$

3 We have $\sqrt[4]{2} = 2^{\frac{1}{4}} = 2^{\frac{3}{12}} = \sqrt[12]{2^3} = \sqrt[12]{8}$,

$$\text{and } \sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$$

Thus the required surds are $\sqrt[12]{8}$ and $\sqrt[12]{27}$

4 We have $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$,

$$\text{and } \sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$$

Thus the required surds are $\sqrt[12]{27}$ and $\sqrt[12]{125}$

5 We have $\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\frac{3}{12}} = \sqrt[12]{4^3} = \sqrt[12]{64}$,

$$\text{and } \sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\frac{3}{12}} = \sqrt[12]{6^3} = \sqrt[12]{216}$$

Thus the required surds are $\sqrt[12]{64}$ and $\sqrt[12]{216}$

6 We have $\sqrt[4]{2} = 2^{\frac{1}{4}} = 2^{\frac{3}{12}} = \sqrt[12]{2^3} = \sqrt[12]{8}$,

$$\text{and } \sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27} \quad \sqrt[4]{3} \text{ is the greater}$$

7 We have $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$,

$$\text{and } \sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\frac{3}{12}} = \sqrt[12]{4^3} = \sqrt[12]{64} \quad \sqrt[4]{3} \text{ is the greater}$$

8 We have $\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\frac{3}{12}} = \sqrt[12]{6^3} = \sqrt[12]{216}$

$$\text{and } \sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{3}{12}} = \sqrt[12]{10^3} = \sqrt[12]{1000} \quad \sqrt[4]{6} \text{ is the greater}$$

9 We have $\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\frac{1}{2} \cdot \frac{1}{2}} = \sqrt[2]{\sqrt{6}} = \sqrt[2]{216}$,

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{1}{2} \cdot \frac{1}{2}} = \sqrt[2]{2^2} = \sqrt[2]{64},$$

and $\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\frac{2}{3} \cdot \frac{1}{2}} = \sqrt[3]{4^2} = \sqrt[3]{256}$

the required arrangement is $\sqrt[3]{4}$, $\sqrt[4]{6}$ and $\sqrt{2}$

10 We have $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{4} \cdot \frac{1}{3}} = \sqrt[3]{3^3} = \sqrt[3]{1729}$,

$$\sqrt[5]{10} = 10^{\frac{1}{5}} = 10^{\frac{4}{5} \cdot \frac{1}{4}} = \sqrt[4]{10^4} = \sqrt[4]{10000},$$

and $\sqrt[12]{25} = 25^{\frac{1}{12}} = 25^{\frac{2}{12} \cdot \frac{1}{2}} = \sqrt[6]{25^2} = \sqrt[6]{625}$

the required arrangement is $\sqrt[5]{10}$, $\sqrt[4]{3}$ and $\sqrt[12]{25}$

Exercise (85)

1 $\sqrt{5} \times \sqrt[3]{10} = \sqrt[6]{50} = \sqrt[6]{25 \times 2} = 5 \sqrt[6]{2}$

2 $\sqrt[3]{8} \times \sqrt{6} = \sqrt[6]{48} = \sqrt[6]{16 \times 3} = 4 \sqrt[6]{3}$

3 $\sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$

4 $\sqrt{15} \times \sqrt{6} = \sqrt{90} = \sqrt{9 \times 10} = 3 \sqrt{10}$

5 $\sqrt{20} \times \sqrt{45} = \sqrt{4 \times 5} \times \sqrt{9 \times 5} = \sqrt{4 \times 9 \times 5 \times 5} = 2 \times 3 \times 5 = 30$

6 $\sqrt[3]{5} \times \sqrt[3]{25} = \sqrt[3]{5 \times 25} = \sqrt[3]{5 \times 5 \times 5} = 5$

7 $\sqrt[3]{6ax} \times \sqrt[3]{27a^2x^3} = \sqrt[3]{6ax \times 27a^2x^3} = 3x \sqrt[3]{6ax \times a^2} = 3ax \sqrt[3]{6x}$

8 $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[6]{4} \times \sqrt[6]{216} = \sqrt[6]{864}$

9 $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[6]{2 \times 2 \times 2} \times \sqrt[6]{6 \times 6} = \sqrt[6]{288}$

10 $\sqrt[3]{4} \times \sqrt[3]{8} = \sqrt[6]{4 \times 4} \times \sqrt[6]{8 \times 8 \times 8} = \sqrt[6]{4 \times 4 \times 4 \times 4 \times 4 \times 2}$
 $= 4 \sqrt[6]{2}$

11 $\sqrt[3]{9} \times \sqrt[3]{27} = \sqrt[6]{9 \times 9} \times \sqrt[6]{27 \times 27 \times 27} = \sqrt[6]{9 \times 9 \times 9 \times 9 \times 9 \times 3}$
 $= 9 \sqrt[6]{3}$

12 $\sqrt[6]{2} \times \sqrt[6]{3} = \sqrt[18]{2 \times 2 \times 2} \times \sqrt[18]{3 \times 3} = \sqrt[18]{8 \times 9} = \sqrt[18]{72}$

13 $\sqrt[4]{3} \times \sqrt[8]{3} = \sqrt[8]{9} \times \sqrt[8]{3} = \sqrt[8]{27}$

14 $\sqrt[3]{2} \times \sqrt[6]{2} = \sqrt[12]{8} \times \sqrt[12]{4} = \sqrt[12]{32}$

15 $\sqrt[4]{4} \times \sqrt[6]{4} = \sqrt[12]{4 \times 4 \times 4} \times \sqrt[12]{4 \times 4} = \sqrt[12]{64 \times 16} = \sqrt[12]{1024}$

16 $5 \sqrt[3]{8} \times 2 \sqrt[3]{6} = 5 \sqrt[6]{4 \times 2} \times 2 \sqrt[6]{6} = 10 \sqrt[6]{2 \times 2 \times 6} = 20 \sqrt[6]{12}$
 $= 20 \sqrt[6]{4 \times 3} = 40 \sqrt[6]{3}$

$$17 \quad 8\sqrt{12} \times 3\sqrt{24} = 8\sqrt{12} \times 3\sqrt{12 \times 2} = 8 \times 3 \times 12 \times 2 = 288\sqrt{2}$$

$$18 \quad 4\sqrt[3]{72} \times 5\sqrt[3]{576} = 4\sqrt[3]{8 \times 9} \times 5\sqrt[3]{64 \times 9} = 4 \times 2\sqrt[3]{9} \times 5 \times 4\sqrt[3]{9} \\ = 160\sqrt[3]{3 \times 3 \times 3 \times 3} = 480\sqrt[3]{3}$$

$$19 \quad 7\sqrt[3]{8a^3x^2} \times 5\sqrt[3]{27b^3x^3} = 7 \times 2a\sqrt[3]{x^2} \times 5 \times 3b\sqrt[3]{x^3} = 210ab\sqrt[3]{x^5} \\ = 210abx\sqrt[3]{x^2}$$

$$20 \quad 8\sqrt{10} - 4\sqrt{15} = \frac{8 \times \sqrt{5 \times 2}}{4 \times \sqrt{5 \times 3}} = \frac{3\sqrt{2}}{\sqrt{3}}$$

$$21 \quad 3\sqrt{12} - 6\sqrt{27} = \frac{3 \times 2\sqrt{3}}{6 \times 3\sqrt{3}} = \frac{1}{3}$$

$$22 \quad \sqrt[3]{36} - \sqrt[3]{48} = \sqrt[3]{\frac{36}{48}} = \sqrt[3]{\frac{3}{4}}$$

$$23 \quad \sqrt[18]{8} - \sqrt[18]{6} = \sqrt[18]{8 \times 8} - \sqrt[18]{6 \times 6 \times 6} = \sqrt[18]{\frac{8 \times 8}{6 \times 6 \times 6}} = \sqrt[18]{\frac{8}{27}} = \sqrt[18]{\left(\frac{2}{3}\right)^3} \\ = \sqrt[6]{\frac{2}{3}}$$

$$24 \quad \sqrt{2} - \sqrt{6} = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{1}{1.732} = 577$$

$$25 \quad \sqrt{72} - \sqrt{40} = \sqrt{\frac{72}{40}} = \sqrt{\frac{9}{5}} = \frac{3\sqrt{5}}{5} = \frac{6\sqrt{5}}{10} = \frac{6 \times 2.236}{10} = 1.341$$

$$26 \quad \sqrt{276} - \sqrt{22} = \sqrt{\frac{276}{22}} = \sqrt{\frac{25}{2}} = \frac{5\sqrt{2}}{2} = \frac{5 \times 1.414}{2} = 3.536$$

$$27 \quad 10\sqrt{108} - \sqrt{15} = 10\sqrt{\frac{108}{15}} = \frac{10\sqrt{36}}{\sqrt{5}} = \frac{10 \times 6}{\sqrt{5}} = \frac{10 \times 6 \times \sqrt{5}}{5} \\ = 2 \times 6 \times \sqrt{5} = 12\sqrt{5} = 12 \times 2.236 = 26.832$$

Exercise (86).

$$1 \quad (\sqrt{a} + \sqrt{b})(\sqrt{ab}) = \sqrt{a}\sqrt{ab} + \sqrt{b}\sqrt{ab} = a\sqrt{b} + b\sqrt{a}$$

$$2 \quad (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

$$3 \quad (\sqrt{a} - 5)2\sqrt{a} = 3\sqrt{a}2\sqrt{a} - 5 \times 2\sqrt{a} = 6a - 10\sqrt{a}$$

$$4 \quad (4\sqrt{x} + 3\sqrt{y})(4\sqrt{x} - 3\sqrt{y}) = (4\sqrt{x})^2 - (3\sqrt{y})^2 = 16x - 9y$$

$$5 \quad (2\sqrt{x-5} + 4)(3\sqrt{x-5} - 6) = 2\sqrt{x-5}3\sqrt{x-5} + 43\sqrt{x-5} \\ = 6(x-5) - 24 = 6x - 30 - 24 = 6x - 54$$

$$6 \quad (3\sqrt{5} - 4\sqrt{2})(2\sqrt{5} + 3\sqrt{2}) \\ = 3\sqrt{5}2\sqrt{5} - 4\sqrt{2}2\sqrt{5} + 3\sqrt{5}3\sqrt{2} - 4\sqrt{2}3\sqrt{2} \\ = 65 + \sqrt{10} - 122 = 30 + \sqrt{10} - 24 = 6 + \sqrt{10}$$

7. $\{(\sqrt{2}+2\sqrt{3})+\sqrt{7}\}\{(\sqrt{2}+2\sqrt{3})-\sqrt{7}\}=(\sqrt{2}+2\sqrt{3})^2-(\sqrt{7})^2$
 $=2+4\sqrt{6}+4\cdot 3-7=2+12-7+4\sqrt{6}=7+4\sqrt{6}$
8. $\{(3-\sqrt{5})+\sqrt{8}\}\{(3-\sqrt{5})-\sqrt{8}\}=(3-\sqrt{5})^2-(\sqrt{8})^2$
 $=9-6\sqrt{5}+5-8=6-6\sqrt{5}$
9. $\{\sqrt{11}+(\sqrt{6}-\sqrt{3})\}\{\sqrt{11}-\sqrt{6}-\sqrt{3}\}=(\sqrt{11})^2\times(\sqrt{6}-\sqrt{3})^2$
 $=11-(6+3-2\sqrt{6}\sqrt{3})=11-6-3+6\sqrt{2}=2+6\sqrt{2}$
10. $(\sqrt[3]{4}+\sqrt[3]{9}+\sqrt[3]{48})(\sqrt[3]{2}+\sqrt[3]{3})$
 $=\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}+2\cdot 2^{\frac{1}{3}}3^{\frac{1}{3}}\right)\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$
 $=\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)^2\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)=\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)^3$
 $=2+3+3\cdot 2^{\frac{1}{3}}3^{\frac{1}{3}}\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$
 $=5+3\sqrt[3]{2\cdot 2\cdot 3}+3\sqrt[3]{2\cdot 3\cdot 3}=5+3\sqrt[3]{12}+3\sqrt[3]{18}$
11. $(\sqrt{x+a}-\sqrt{x-a})^2=(x+a)+(x-a)-2\sqrt{x^2-a^2}$
 $=2x-2\sqrt{x^2-a^2}$
12. $(2\sqrt{8}+5\sqrt{6})^2=48+2\cdot 2\sqrt{8}\cdot 5\sqrt{6}+25\cdot 6$
 $=32+20\sqrt{8\cdot 6}+150=182+20\cdot 4\sqrt{3}=182+80\sqrt{3}$
13. $(2\sqrt{5}+3\sqrt{7})^2=4\cdot 5+2\cdot 2\sqrt{5}\cdot 3\sqrt{7}+9\cdot 7=20+12\sqrt{35}+63$
 $=83+12\sqrt{35}$
14. $(\sqrt{a^2+2b^2}-\sqrt{a^2-2b^2})^2$
 $=a^2+2b^2-2\sqrt{a^2+2b^2}\sqrt{a^2-2b^2}+a^2-2b^2$
 $=2a^2-2\sqrt{a^4-4b^4}$
15. $(2\sqrt{x^2+y^2}+5\sqrt{x^2-y^2})^2=4(x^2+y^2)+2\cdot 2\cdot 5\sqrt{x^2-y^2}+25(x^2-y^2)$
 $=29x^2-21y^2+20\sqrt{x^4-y^4}$

Exercise (87).

- 1 The given expression

$$\begin{aligned} &= \frac{(5\sqrt{3}+\sqrt{7})(4\sqrt{3}-2\sqrt{7})}{(4\sqrt{3}+2\sqrt{7})(4\sqrt{3}-2\sqrt{7})} = \frac{20\cdot 3+4\sqrt{21}-10\sqrt{21}-2\cdot 7}{16\cdot 3-4\cdot 7} \\ &= \frac{60-6\sqrt{21}-14}{48-28} = \frac{46-6\sqrt{21}}{20} = \frac{23-6\sqrt{21}}{10} \end{aligned}$$

2 The given expression

$$\begin{aligned}
 &= \frac{(3\sqrt{2}+2\sqrt{3})^2}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} = \frac{9 \cdot 2 + 4 \cdot 3 + 12\sqrt{6}}{9 \cdot 2 - 4 \cdot 3} \\
 &= \frac{18 + 12 + 12\sqrt{6}}{18 - 12} = \frac{30 + 12\sqrt{6}}{6} = 5 + 2\sqrt{6}
 \end{aligned}$$

3 The given expression

$$\begin{aligned}
 &= \frac{(4+3\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} = \frac{12 + 9\sqrt{2} + 8\sqrt{2} + 6 \cdot 2}{9 - 4 \cdot 2} \\
 &= \frac{24 + 17\sqrt{2}}{9 - 8} = 24 + 17\sqrt{2}
 \end{aligned}$$

4. The given expression

$$\begin{aligned}
 &= \frac{(3\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{3 \cdot 5 + \sqrt{15} + 3\sqrt{15} + 3}{5 - 3} \\
 &= \frac{15 + 4\sqrt{15} + 3}{2} = \frac{18 + 4\sqrt{15}}{2} = 9 + 2\sqrt{15}
 \end{aligned}$$

5 The given expression

$$\begin{aligned}
 &= \frac{(\sqrt{a+x} + \sqrt{a-x})^2}{(\sqrt{a+x} - \sqrt{a-x})(\sqrt{a+x} + \sqrt{a-x})} \\
 &= \frac{(a+x) + (a-x) + 2\sqrt{a^2-x^2}}{(a+x) - (a-x)} \\
 &= \frac{2a + 2\sqrt{a^2-x^2}}{2x} = \frac{a + \sqrt{a^2-x^2}}{x}
 \end{aligned}$$

6 The given expression

$$\begin{aligned}
 &= \frac{(\sqrt{x^2+1} - \sqrt{x^2-1})^2}{(\sqrt{x^2+1} + \sqrt{x^2-1})(\sqrt{x^2+1} - \sqrt{x^2-1})} \\
 &= \frac{(x^2+1) + (x^2-1) - 2\sqrt{x^2+1}\sqrt{x^2-1}}{(x^2+1) - (x^2-1)} = \frac{2x^2 - 2\sqrt{x^4-1}}{2} = x^2 - \sqrt{x^4-1}
 \end{aligned}$$

7 The given expression

$$\begin{aligned}
 &= \frac{(1+\sqrt{2})-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}} = \frac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-3} \\
 &= \frac{1+\sqrt{2}-\sqrt{3}}{1+2+2\sqrt{2}-3} = \frac{1+\sqrt{2}-\sqrt{3}}{2\sqrt{2}} = \frac{(1+\sqrt{2}-\sqrt{3})\sqrt{2}}{2\sqrt{2}\sqrt{2}} \\
 &= \frac{\sqrt{2}+2-\sqrt{6}}{2 \cdot 2} = \frac{2+\sqrt{2}-\sqrt{6}}{4}
 \end{aligned}$$

$$8 \quad \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{2+1+2\sqrt{2}}{2-1} = 3+2(1414) \\ = 3+2828 = 5828$$

$$9 \quad \frac{\sqrt{3}}{2-\sqrt{3}} = \frac{\sqrt{3}(2+\sqrt{3})}{4-3} = 2\sqrt{3}+3 = 2(1732)+3 = 3464+3 \\ = 6464$$

$$10 \quad \frac{8-5\sqrt{2}}{3-2\sqrt{2}} = \frac{(8-5\sqrt{2})(3+2\sqrt{2})}{9-42} = \frac{24-15\sqrt{2}+16\sqrt{2}-102}{9-8} \\ = 24-20+\sqrt{2} = 4+\sqrt{2} = 4+1414 = 5414$$

$$11 \quad \frac{3}{\sqrt{5}-\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2} = \sqrt{5}+\sqrt{2} = 2236+1414 = 365$$

$$12 \quad \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{(3+\sqrt{5})^2}{9-5} = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4} = \frac{7+3(2236)}{2} \\ = \frac{7+6708}{2} = \frac{13708}{2} = 6854$$

$$13 \quad \frac{\sqrt{5}+\sqrt{3}}{4+\sqrt{15}} = \frac{(\sqrt{5}+\sqrt{3})(4-\sqrt{15})}{16-15} \\ = 4\sqrt{5}+4\sqrt{3}-\sqrt{5}\sqrt{15}-\sqrt{3}\sqrt{15} \\ = 4\sqrt{5}+4\sqrt{3}-5\sqrt{3}-3\sqrt{5} = \sqrt{5}-\sqrt{3} = 2236-1732 = 504$$

$$14 \quad \frac{1}{x+\sqrt{x^2-1}} + \frac{1}{x-\sqrt{x^2-1}} = \frac{x-\sqrt{x^2-1}+x+\sqrt{x^2-1}}{x^2-(x^2-1)} = 2x$$

15 The given expression

$$= \frac{15}{\sqrt{5}2+2\sqrt{5}+2\sqrt{5}2-\sqrt{5}-4\sqrt{5}} \\ = \frac{15}{\sqrt{5}(\sqrt{2}+2+2\sqrt{2}-1-4)} = \frac{15}{\sqrt{5}(3\sqrt{2}-3)} = \frac{15}{3\sqrt{5}(\sqrt{2}-1)} \\ = \frac{\sqrt{5}}{\sqrt{2}-1} = \frac{\sqrt{5}(\sqrt{2}+1)}{2-1} = \sqrt{10}+\sqrt{5}$$

16 The given expression

$$= \frac{\sqrt{2}(2\sqrt{3}+2-3-\sqrt{3})}{(3\sqrt{3}-5)(\sqrt{2}-1)(\sqrt{2}+1)\sqrt{2}} = \frac{(\sqrt{3}-1)}{(3\sqrt{3}-5)(2-1)} = \frac{\sqrt{3}-1}{3\sqrt{3}-5} \\ = \frac{(\sqrt{3}-1)(3\sqrt{3}+5)}{93-25} = \frac{33-3\sqrt{3}+5\sqrt{3}-5}{2} = \frac{9-5+2\sqrt{3}}{2} \\ = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

17. The given expression

$$\begin{aligned}
 &= \frac{4(\sqrt{3} + \sqrt{5} + \sqrt{2})}{(\sqrt{3} + \sqrt{5})^2 - (\sqrt{2})^2} = \frac{4(\sqrt{3} + \sqrt{5} + \sqrt{2})}{3 + 5 + 2\sqrt{15} - 2} = \frac{4(\sqrt{3} + \sqrt{5} + \sqrt{2})}{6 + 2\sqrt{15}} \\
 &= \frac{2(\sqrt{3} + \sqrt{5} + \sqrt{2})}{3 + \sqrt{15}} = \frac{2(\sqrt{3} + \sqrt{5} + \sqrt{2})(3 - \sqrt{15})}{9 - 15} \\
 &= \frac{2(3\sqrt{3} + 3\sqrt{5} + 3\sqrt{2} - 3\sqrt{5} - 5\sqrt{3} - \sqrt{30})}{-6} \\
 &= \frac{1}{3}(2\sqrt{3} - 3\sqrt{2} + \sqrt{30})
 \end{aligned}$$

18. The given expression

$$\begin{aligned}
 &= \frac{1}{(3+2\sqrt{2})^3} + \frac{1}{(3-2\sqrt{2})^3} = \frac{(3-2\sqrt{2})^3 + (3+2\sqrt{2})^3}{(9-8)^3} \\
 &= 27 - 54\sqrt{2} + 72 - 8 + 27 + 54\sqrt{2} + 72 + 8 = 54 + 144 = 198
 \end{aligned}$$

19. The given expression

$$= \frac{(x + \sqrt{x^2 - 1})^2 - (x - \sqrt{x^2 - 1})^2}{x^2 - (x^2 - 1)} = 4x\sqrt{x^2 - 1}$$

20. The given expression

$$\begin{aligned}
 &= \frac{(\sqrt{x^2 + 1} + \sqrt{x^2 - 1})^2 + (\sqrt{x^2 + 1} - \sqrt{x^2 - 1})^2}{(x^2 + 1) - (x^2 - 1)} \\
 &= \frac{2\{(x^2 + 1) + (x^2 - 1)\}}{2} = 2x^2
 \end{aligned}$$

21

$$\begin{aligned}
 \frac{1}{\sqrt[3]{3} + \sqrt[3]{2}} &= \frac{3^{\frac{2}{3}} - 3^{\frac{1}{3}} 2^{\frac{1}{3}} + 2^{\frac{2}{3}}}{(3^{\frac{1}{3}} + 2^{\frac{1}{3}})(3^{\frac{2}{3}} - 3^{\frac{1}{3}} 2^{\frac{1}{3}} + 2^{\frac{2}{3}})} \\
 &= \frac{9^{\frac{1}{3}} - 6^{\frac{1}{3}} + 4^{\frac{1}{3}}}{3 + 2} = \frac{\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4}}{5}
 \end{aligned}$$

22

$$\begin{aligned}
 \frac{1}{\sqrt[3]{4} - \sqrt[3]{3}} &= \frac{4^{\frac{2}{3}} + 4^{\frac{1}{3}} 3^{\frac{1}{3}} + 3^{\frac{2}{3}}}{(4^{\frac{1}{3}} - 3^{\frac{1}{3}})(4^{\frac{2}{3}} + 4^{\frac{1}{3}} 3^{\frac{1}{3}} + 3^{\frac{2}{3}})} \\
 &= \frac{16^{\frac{1}{3}} + 12^{\frac{1}{3}} + 9^{\frac{1}{3}}}{4 - 3} = \sqrt[3]{16} + \sqrt[3]{12} + \sqrt[3]{9} = 2\sqrt[3]{2} + \sqrt[3]{12} + \sqrt[3]{9}
 \end{aligned}$$

Exercise (88)

1 Let $\sqrt{4-2\sqrt{3}} = \sqrt{x} - \sqrt{y}$

Then, squaring both sides, $4-2\sqrt{3} = x+y-2\sqrt{xy}$

Hence, $x+y=4$ and $xy=3$

These relations are satisfied by the numbers 3 and 1

Thus the required root $= \sqrt{3}-1$

2 Let $\sqrt{7+4\sqrt{3}} = \sqrt{r} + \sqrt{y}$

Then, $7+4\sqrt{3} = r+y+2\sqrt{xy}$

Hence, $x+y=7$ and $2\sqrt{xy}=4\sqrt{3}$ or $xy=12$

The numbers satisfying these relations are 4 and 3

Thus the required root $= \sqrt{4} + \sqrt{3} = 2 + \sqrt{3}$

3. Let $\sqrt{11-6\sqrt{2}} = \sqrt{x} - \sqrt{y}$

Then, $11-6\sqrt{2} = r+y-2\sqrt{xy}$

Hence, $x+y=11$ and $2\sqrt{xy}=6\sqrt{2}$, or $xy=18$

The numbers satisfying these relations are 9 and 2

Thus the required root $= \sqrt{9} - \sqrt{2} = 3 - \sqrt{2}$

4 Let $\sqrt{8+2\sqrt{15}} = \sqrt{x} + \sqrt{y}$

Then, $8+2\sqrt{15} = r+y+2\sqrt{xy}$

Hence, $x+y=8$ and $xy=15$

The numbers satisfying these relations are 5 and 3

Thus the required root $= \sqrt{5} + \sqrt{3}$

5 Let $\sqrt{14-6\sqrt{5}} = \sqrt{x} - \sqrt{y}$

Then, $14-6\sqrt{5} = x+y-2\sqrt{xy}$

Hence, $x+y=14$ and $2\sqrt{xy}=6\sqrt{5}$, or $xy=45$.

The numbers satisfying these relations are 9 and 5

Thus the required root $= \sqrt{9} - \sqrt{5} = 3 - \sqrt{5}$

6 Let $\sqrt{28+10\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Then $28+10\sqrt{3} = x+y+2\sqrt{xy}$

Hence, $x+y=28$ and $2\sqrt{xy}=10\sqrt{3}$, or $xy=75$

The numbers satisfying these relations are 25 and 3

Thus the required root $= \sqrt{25} + \sqrt{3} = 5 + \sqrt{3}$

- 7 Let $\sqrt{21-8\sqrt{5}} = \sqrt{r} - \sqrt{y}$
 Then, $21-8\sqrt{5} = x+y-2\sqrt{xy}$
 Hence, $r+y=21$ and $2\sqrt{xy}=8\sqrt{5}$, or $xy=80$
 The numbers satisfying these relations are 16 and 5
 Thus the required root $= \sqrt{16} - \sqrt{5} = 4 - \sqrt{5}$
- 8 Let $\sqrt{17+12\sqrt{2}} = \sqrt{x} + \sqrt{y}$
 Then, $17+12\sqrt{2} = r+y+2\sqrt{xy}$
 Hence, $x+y=17$ and $2\sqrt{xy}=12\sqrt{2}$, or $xy=72$
 The numbers satisfying these relations are 9 and 8
 Thus the required root $= \sqrt{9} + \sqrt{8} = 3 + 2\sqrt{2}$
- 9 Let $\sqrt{41+12\sqrt{5}} = \sqrt{x} + \sqrt{y}$
 Then, $41+12\sqrt{5} = r+y+3\sqrt{xy}$
 Hence, $r+y=41$ and $\sqrt{xy}=12\sqrt{5}$, or $xy=180$
 The numbers satisfying these relations are 36 and 5
 Thus the required root $= \sqrt{36} + \sqrt{5} = 6 + \sqrt{5}$
- 10 Let $\sqrt{37-20\sqrt{3}} = \sqrt{r} - \sqrt{y}$
 Then, $37-20\sqrt{3} = x+y-2\sqrt{xy}$
 Hence, $x+y=37$ and $2\sqrt{xy}=20\sqrt{3}$ or $xy=300$
 The numbers satisfying these relations are 25 and 12
 Thus the required root $= \sqrt{25} - \sqrt{12} = 5 - 2\sqrt{3}$
- 11 Let $\sqrt{31+4\sqrt{21}} = \sqrt{r} + \sqrt{y}$
 Then, $31+4\sqrt{21} = x+y+2\sqrt{xy}$
 Hence, $x+y=31$ and $2\sqrt{xy}=4\sqrt{21}$, or $xy=84$
 The numbers satisfying these relations are 28 and 3
 Thus the required root $= \sqrt{28} + \sqrt{3} = 2\sqrt{7} + \sqrt{3}$
- 12 Let $\sqrt{73-12\sqrt{35}} = \sqrt{x} - \sqrt{y}$
 Then, $73-12\sqrt{35} = r+y-2\sqrt{xy}$
 Hence, $x+y=73$ and $2\sqrt{xy}=12\sqrt{35}$, or $xy=1260$
 The numbers satisfying these relations are 45 and 28
 Thus the required root $= \sqrt{45} - \sqrt{28} = 3\sqrt{5} - 2\sqrt{7}$

13 Let $\sqrt{47+4\sqrt{33}} = \sqrt{x} + \sqrt{y}$

Then, $47+4\sqrt{33} = x+y+2\sqrt{xy}$

Hence, $x+y=47$ and $2\sqrt{xy}=4\sqrt{33}$, or $xy=132$

The numbers satisfying these relations are 44 and 3

Thus the required root $= \sqrt{44} + \sqrt{3} = 2\sqrt{11} + \sqrt{3}$

14 Let $\sqrt{4-\sqrt{7}} = \sqrt{x} - \sqrt{y}$

Then, $4-\sqrt{7} = x+y-2\sqrt{xy}$

Hence, $x+y=4$ and $2\sqrt{xy}=\sqrt{7}$, or $xy=\frac{7}{4}$

$x-y = \sqrt{(x+y)^2 - 4xy} = \sqrt{16-7} = \sqrt{9} = 3$

Thus we have $x+y=4$ and $x-y=3$

Hence, $x=\frac{7}{2}$ and $y=\frac{1}{2}$

The required root is therefore $= \sqrt{\frac{7}{2}} - \sqrt{\frac{1}{2}}$

15 Let $\sqrt{6-\sqrt{35}} = \sqrt{x} - \sqrt{y}$

Then, $6-\sqrt{35} = x+y-2\sqrt{xy}$

Therefore, $x+y=6$ and $2\sqrt{xy}=\sqrt{35}$

Hence, $x-y = \sqrt{(x+y)^2 - 4xy} = \sqrt{36-35} = 1$

Thus we have $x+y=6$ and $x-y=1$

Hence, $x=\frac{7}{2}$ and $y=\frac{5}{2}$,

the required root is $\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}$

16 $\sqrt{18} - \sqrt{16} = \sqrt{2}(\sqrt{9} - \sqrt{8}) = \sqrt{2}(3 - \sqrt{8})$

Then $\sqrt{\sqrt{18} - \sqrt{16}} = \frac{1}{2} \sqrt{3 - \sqrt{8}}$

Now let $\sqrt{3 - \sqrt{8}} = \sqrt{x} - \sqrt{y}$

Then, $3 - \sqrt{8} = x+y-2\sqrt{xy}$

Therefore, $x+y=3$ and $2\sqrt{xy}=\sqrt{8}$

Hence $x-y = \sqrt{(x+y)^2 - 4xy} = \sqrt{9-8} = 1$

Thus we have $x+y=3$ and $x-y=1$

Thus $\sqrt{3 - \sqrt{8}} = \sqrt{2} - 1$

$\therefore \sqrt{\sqrt{18} - \sqrt{16}} = \frac{1}{2} \sqrt{2} (\sqrt{2} - 1)$

$$17 \quad \sqrt{32} - \sqrt{24} = \sqrt{8}(\sqrt{4} - \sqrt{3}) = \sqrt{8}(2 - \sqrt{3})$$

$$\text{Then } \sqrt{\sqrt{32} - \sqrt{24}} = \sqrt[4]{8} \sqrt{2 - \sqrt{3}}$$

$$\text{Now let } \sqrt{2 - \sqrt{3}} = \sqrt{x} - \sqrt{y}$$

$$\text{Hence, } 2 - \sqrt{3} = x + y - 2\sqrt{xy}$$

$$\text{Therefore, } x + y = 2 \text{ and } 2\sqrt{xy} = \sqrt{3},$$

$$(x+y) = \sqrt{(x+y)^2 - 4xy} = \sqrt{4-3} = 1$$

$$\text{Thus we have } x+y=2 \text{ and } x-y=1, \quad x= \frac{3}{2} \text{ and } y=\frac{1}{2}$$

$$\text{Hence, } \sqrt{2 - \sqrt{3}} = \sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}(\sqrt{3}-1),$$

$$\text{the required root} = \sqrt[4]{8} \cdot \frac{1}{\sqrt{2}}(\sqrt{3}-1)$$

$$= \sqrt{2} \cdot \frac{1}{\sqrt{2}}(\sqrt{3}-1) = \sqrt{3}-1$$

$$18 \quad \sqrt[4]{27} + \sqrt[4]{24} = \sqrt[4]{3}(\sqrt[4]{9} + \sqrt[4]{8}) = \sqrt[4]{3}(3 + \sqrt{8})$$

$$\text{Then } \sqrt{\sqrt[4]{27} + \sqrt[4]{24}} = \sqrt[4]{3} \sqrt{3 + \sqrt{8}}$$

$$\text{Now let } \sqrt{3 + \sqrt{8}} = \sqrt{x} + \sqrt{y},$$

$$3 + \sqrt{8} = x + y + 2\sqrt{xy}$$

$$\text{Hence, } x + y = 3 \text{ and } 2\sqrt{xy} = \sqrt{8},$$

$$x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{9-8} = 1$$

$$\text{Thus we have } x+y=3 \text{ and } x-y=1$$

$$x=2 \text{ and } y=1$$

$$\text{Thus } \sqrt{3 + \sqrt{8}} = \sqrt{2} + 1$$

$$\text{the required root} = \sqrt[4]{3}(\sqrt{2}+1)$$

$$19 \quad 5\sqrt[4]{5} + \sqrt[4]{120} = \sqrt[4]{5}(5 + \sqrt[4]{24}) = \sqrt[4]{5}(5 + 2\sqrt[4]{6})$$

$$\text{Then } \sqrt{5\sqrt[4]{5} + \sqrt[4]{120}} = \sqrt[4]{5} \sqrt{5 + 2\sqrt[4]{6}}$$

$$\text{Now let } \sqrt{5 + 2\sqrt[4]{6}} = \sqrt{x} + \sqrt{y}$$

$$5 + 2\sqrt[4]{6} = x + y + 2\sqrt{xy}$$

$$\text{Hence, } x+y=5 \text{ and } 2\sqrt{xy}=2\sqrt[4]{6}$$

The numbers satisfying these relations are 3 and 2

Thus
$$\sqrt{5+2\sqrt{6}} = \sqrt{3} + \sqrt{2}$$

the required root
$$= \sqrt[4]{5}(\sqrt{3} + \sqrt{2})$$

20 We have
$$2 + \sqrt{3} = 2 + 2\sqrt{\frac{1}{2}}$$

$$= 2 + 2\sqrt{\frac{1}{2}} = \frac{1}{2} + \frac{1}{2} + \sqrt{\frac{1}{2}} = (\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}})^2,$$

$$\sqrt{2 + \sqrt{3}} = \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \frac{\sqrt{3+1}}{\sqrt{2}}$$

Similarly
$$\sqrt{2 - \sqrt{3}} = \frac{\sqrt{3-1}}{\sqrt{2}}$$

the given expression

$$= \frac{\sqrt{2}(2 + \sqrt{3})}{2 + \sqrt{3} + 1} + \frac{\sqrt{2}(2 - \sqrt{3})}{2 - (\sqrt{3} - 1)} = \frac{\sqrt{2}(2 + \sqrt{3})}{\sqrt{3}(\sqrt{3} + 1)} + \frac{\sqrt{2}(2 - \sqrt{3})}{\sqrt{3}(\sqrt{3} - 1)}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \frac{2\sqrt{3} - 2 + 3 - \sqrt{3} + 2\sqrt{3} + 2 - 3 + \sqrt{3}}{(3-1)}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \frac{2\sqrt{3}}{2} = \sqrt{2}$$

21 We have
$$1+x = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$$

and
$$1-x = 1 - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2}$$

Let
$$\sqrt{2+\sqrt{3}} = \sqrt{a} + \sqrt{b}$$

$$2 + \sqrt{3} = a + b + 2\sqrt{ab}$$

$$a + b = 2 \text{ and } 2\sqrt{ab} = \sqrt{3}$$

$$(a-b)^2 = (a+b)^2 - 4ab = 4 - 3 = 1,$$

$$a - b = \pm 1$$

$$a = \frac{1}{2} \text{ and } b = \frac{3}{2}$$

or,

$$a = \frac{3}{2} \text{ and } b = \frac{1}{2}$$

$$\sqrt{2 + \sqrt{3}} = \sqrt{\frac{3}{2}} + \sqrt{\frac{1}{2}}$$

Similarly $\sqrt{2-\sqrt{3}} = \pm(\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{2}})$

Now $\sqrt{1+i} = \sqrt{\frac{2+\sqrt{3}}{2}} = \frac{\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}}}{\sqrt{2}} = \frac{1}{2}(\sqrt{3}+1)$

and $\sqrt{1-i} = \sqrt{\frac{2-\sqrt{3}}{2}} = \frac{\pm(\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{2}})}{\sqrt{2}} = \pm\frac{1}{2}(\sqrt{3}-1)$

the given expression

$$\begin{aligned}
 &= \frac{\frac{2+\sqrt{3}}{2}}{1+\frac{\sqrt{3}+1}{2}} + \frac{\frac{2-\sqrt{3}}{2}}{1\pm\frac{\sqrt{3}-1}{2}} = \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{2\pm(\sqrt{3}-1)} \\
 &= \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}-1} \text{ or } \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{2-\sqrt{3}+1} \\
 &= \frac{2+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)} + \frac{2-\sqrt{3}}{\sqrt{3}+1} \text{ or } \frac{2+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)} + \frac{2-\sqrt{3}}{\sqrt{3}(\sqrt{3}-1)} \\
 &= \frac{2+\sqrt{3}+2\sqrt{3}-3}{\sqrt{3}(\sqrt{3}+1)} \text{ or } \frac{(2+\sqrt{3})(\sqrt{3}-1)+(2-\sqrt{3})(\sqrt{3}+1)}{\sqrt{3}(3-1)} \\
 &= \frac{3\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)} \text{ or } \frac{2\sqrt{3}+3-2-\sqrt{3}+2\sqrt{3}-3+2-\sqrt{3}}{2\sqrt{3}} \\
 &= \frac{3\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)} \text{ or } \frac{2\sqrt{3}}{2\sqrt{3}} (1 \text{ e } 1)
 \end{aligned}$$

22 The given expression

$$\begin{aligned}
 &= \frac{(\sqrt{a^2+x^2} + \sqrt{a^2-x^2})^2}{(a+x)-(a-x)} = \frac{(a+x)+(a-x)+2\sqrt{a^2-x^2}}{2x} \\
 &= \frac{2a+2\sqrt{a^2-x^2}}{2x} = \frac{a + \sqrt{a^2 - \frac{4a^2b^2}{(b^2+1)^2}}}{\frac{2ab}{b^2+1}} \\
 &= \frac{a(b^2+1) + a\sqrt{(b^2+1)^2 - 4b^2}}{2ab} = \frac{a(b^2+1) + a\sqrt{(b^2-1)^2}}{2ab} \\
 &= \frac{b^2+1+b^2-1}{2b} = \frac{2b^2}{2b} = b
 \end{aligned}$$

$$\begin{aligned}
 23 \quad \text{Let } \sqrt{a^2 + 2x\sqrt{a^2 - x^2}} &= \sqrt{m} + \sqrt{n} \\
 a^2 + 2x\sqrt{a^2 - x^2} &= m + n + 2\sqrt{mn}, \\
 m + n &= a^2 \text{ and } mn = x^2(a^2 - x^2), \\
 m - n &= \sqrt{(m+n)^2 - 4mn} = \sqrt{a^4 - 4x^2a^2 + 4x^4} \\
 &= a^2 - 2x^2, \\
 m &= a^2 - x^2 \text{ and } n = x^2
 \end{aligned}$$

Thus the required root $= \sqrt{a^2 - x^2} + x$

$$\begin{aligned}
 24 \quad \text{Let } \sqrt{2a + 2\sqrt{a^2 - b^2}} &= \sqrt{m} + \sqrt{n} \\
 2a + 2\sqrt{a^2 - b^2} &= m + n + 2\sqrt{mn}, \\
 m + n &= 2a \text{ and } mn = a^2 - b^2, \\
 m - n &= \sqrt{4a^2 - 4a^2 + 4b^2} = 2b, \\
 m &= a + b \text{ and } n = a - b
 \end{aligned}$$

Thus the required root $= \sqrt{a+b} + \sqrt{a-b}$

$$\begin{aligned}
 25 \quad \text{Let } \sqrt{a+x+\sqrt{2ax+x^2}} &= \sqrt{m} + \sqrt{n} \\
 a+x+\sqrt{2ax+x^2} &= m+n+2\sqrt{mn}, \\
 m+n &= a+x \text{ and } 2\sqrt{mn} = \sqrt{2ax+x^2}, \\
 m-n &= \sqrt{(m+n)^2 - 4mn} = \sqrt{(a+x)^2 - 2ax - x^2} = a, \\
 m &= \frac{2a+x}{2} \text{ and } n = \frac{x}{2},
 \end{aligned}$$

Thus the required root $= \sqrt{\frac{2a+x}{2}} + \sqrt{\frac{x}{2}} = \sqrt{a+\frac{1}{2}x} + \sqrt{\frac{1}{2}x}$

26 The given expression

$$\begin{aligned}
 &= 2x - 1 + 2\sqrt{(x-3)(x+2)} \\
 &= (x+2) + (x-3) + 2\sqrt{(x+2)(x-3)} = \{\sqrt{x+2} + \sqrt{x-3}\}^2 \\
 \text{the required root} &= \sqrt{x+2} + \sqrt{x-3}
 \end{aligned}$$

27 The given expression $= (x+y) + z + 2\sqrt{(x+y)z}$

$$= (\sqrt{x+y} + \sqrt{z})^2,$$

the required root $= \sqrt{x+y} + \sqrt{z}$

Exercise (89).

- 1 Squaring both sides, we have $r+7=1+2\sqrt{x+r}$
Hence, $2\sqrt{x}=6$ or, $\sqrt{x}=3$, $x=9$
- 2 Squaring both sides, we have $3x+16=3x+4\sqrt{3x+4}$
Hence, $4\sqrt{3x}=12$ or, $\sqrt{3x}=3$ or, $3x=9$ $x=3$
- 3 Squaring both sides, we have $x+9=1+x+2\sqrt{x}$
Hence, $2\sqrt{x}=8$ or, $\sqrt{x}=4$, $x=16$
- 4 Squaring both sides we have $3x+16-8\sqrt{3x}=3x+4$
Hence, $8\sqrt{3x}=12$ or, $2\sqrt{3x}=3$ or, $12x=9$, $x=\frac{3}{4}$
- 5 Squaring both sides, we have $5r+10=5x+4\sqrt{5r+4}$
Hence, $4\sqrt{5x}=6$ or, $2\sqrt{5x}=3$ or, $20x=9$, $x=\frac{9}{20}$
- 6 By transposition, we have $\sqrt{x-16}=8-\sqrt{x}$
Hence, squaring both sides $x-16=64-16\sqrt{x}+x$
or, $16\sqrt{x}=80$ or, $\sqrt{x}=5$, $x=25$
- 7 By transposition, we have $\sqrt{2x+9}=9-\sqrt{2x}$
Hence squaring both sides, $2x+9=81-18\sqrt{2x}+2x$
or, $18\sqrt{2x}=72$ or, $\sqrt{2x}=4$ or, $2x=16$, $x=8$
- 8 By transposition, we have $\sqrt{x+11}=1+\sqrt{x}$
Hence, squaring both sides $x+11=1+2\sqrt{x}+x$
or, $2\sqrt{x}=10$ or, $\sqrt{x}=5$, $x=25$
- 9 By transposition, we have $\sqrt{8x+33}=2\sqrt{2x+3}$
Hence, squaring both sides, $8x+33=8x+12\sqrt{2x+3}+9$
or, $12\sqrt{2x+3}=24$ or, $\sqrt{2x+3}=2$ or, $2x+3=4$, $x=2$
- 10 By transposition, we have $\sqrt{2ax+x^2}=a-x$
Hence, squaring both sides, $2ax+x^2=a^2+x^2-2ax$
or, $4ax=a^2$ or, $4x=a$, $x=\frac{a}{4}$
- 11 By transposition, we have $\sqrt{2ax+x^2}=b-a-x$
Hence, squaring both sides,
$$2ax+x^2=b^2+a^2+x^2-2ab-2bx+2ax$$

or, $a^2+b^2-2ab=2bx$, $x=\frac{(b-a)^2}{2b}$

- 12 Squaring both sides, we have

$$x-4+9+6\sqrt{x-4}=7+11$$

$$\text{or, } 6\sqrt{x-4}=6 \text{ or, } x-4=1, \quad x=5$$

- 13 Squaring both sides, we have

$$x-5=36-12\sqrt{x+7}+x+7$$

$$\text{or } 12\sqrt{x+7}=48 \text{ or, } \sqrt{x+7}=4 \text{ or } x+7=16 \quad x=9$$

- 14 By transposition, we have
- $\sqrt{x+9}=1+\sqrt{1+2}$

Hence, squaring both sides

$$x+9=1+x+2+2\sqrt{x+2}$$

$$\text{or, } 2\sqrt{x+2}=6 \text{ or, } \sqrt{x+2}=3 \text{ or, } x+2=9, \quad x=7$$

- 15 By transposition, we have
- $\sqrt{3x+1}=2+\sqrt{3x-11}$

Hence, squaring both sides, $3x+1=4+4\sqrt{3x-11}+3x-11$

$$\text{or, } 4\sqrt{3x-11}=8 \text{ or, } \sqrt{3x-11}=2$$

$$\text{or } 3x-11=4 \text{ or } 3x=15, \quad x=5$$

- 16 By transposition, we have
- $\sqrt{5x+6}=10-\sqrt{5x-14}$

Hence, squaring both sides, $5x+6=100-20\sqrt{5x-14}+5x-14$

$$\text{or, } 20\sqrt{5x-14}=80 \text{ or, } \sqrt{5x-14}=4$$

$$\text{or, } 5x-14=16 \text{ or, } 5x=30, \quad x=6$$

- 17 By transposition, we have

$$\sqrt{7x-12}=8-\sqrt{7x-4}$$

Hence, squaring both sides,

$$7x-12=64-16\sqrt{7x+4}+7x+4$$

$$\text{or, } 16\sqrt{7x+4}=80 \text{ or, } \sqrt{7x+4}=5$$

$$\text{or, } 7x+4=25 \text{ or, } 7x=21 \quad x=3$$

- 18 By transposition, we have

$$\sqrt{x^2-3x+5}=1+\sqrt{x^2-x+1}$$

Hence, squaring both sides,

$$x^2-3x+5=1+x^2-x+1+2\sqrt{x^2-x+1}$$

$$\text{or, } -2x+3=2\sqrt{x^2-x+1}$$

$$\text{or, } 4x^2-12x+9=4x^2-4x+4$$

$$\text{or, } 8x=5, \quad x=\frac{5}{8}$$

19

$$\frac{r-1}{\sqrt{r+1}} = 4 + \frac{\sqrt{x}-1}{2}$$

or,

$$\sqrt{r}-1 = 4 + \frac{\sqrt{r}-1}{2}$$

or,

$$\frac{1}{2}(\sqrt{r}-1) = 4$$

or,

$$\sqrt{r}-1 = 8 \quad \text{or,} \quad \sqrt{r} = 9, \quad r = 81$$

20

$$\frac{ax-1}{\sqrt{ax+1}} = 4 + \frac{\sqrt{ax}-1}{2}$$

or,

$$\sqrt{ax}-1 = 4 + \frac{1}{2}(\sqrt{ax}-1)$$

or,

$$\frac{1}{2}(\sqrt{ax}-1) = 4$$

or,

$$\sqrt{ax}-1 = 8 \quad \text{or,} \quad \sqrt{ax} = 9 \quad \text{or,} \quad ax = 81, \quad x = \frac{81}{a}$$

21

$$\frac{ax-b^2}{\sqrt{ax+b}} = c + \frac{\sqrt{ax}-b}{c}$$

or,

$$\sqrt{ax}-b = c + \frac{1}{c}(\sqrt{ax}-b)$$

or,

$$\left(1 - \frac{1}{c}\right)(\sqrt{ax}-b) = c$$

or,

$$(c-1)(\sqrt{ax}-b) = c^2$$

or,

$$\sqrt{ax}-b = \frac{c^2}{c-1}$$

or

$$\sqrt{ax} = \frac{c^2}{c-1} + b$$

or,

$$ax = \left(\frac{c^2}{c-1} + b\right)^2,$$

$$x = \frac{1}{a} \left(\frac{c^2}{c-1} + b\right)^2$$

22

$$\frac{200+120\sqrt{5x}}{91-5} = (3\sqrt{x}-\sqrt{5})^2$$

or,

$$\frac{40\sqrt{5}(3\sqrt{x}+\sqrt{5})}{(3\sqrt{x})^2-(\sqrt{5})^2} = (3\sqrt{x}-\sqrt{5})^2$$

or,

$$\frac{40\sqrt{5}}{3\sqrt{x}-\sqrt{5}} = (3\sqrt{x}-\sqrt{5})^2$$

or,

$$40\sqrt{5} = (3\sqrt{x}-\sqrt{5})^3$$

$$\begin{aligned}
 \text{or,} & \quad (2\sqrt{5})^2 = (3\sqrt{x} - \sqrt{5})^2 \\
 \text{or,} & \quad 2\sqrt{5} = 3\sqrt{x} - \sqrt{5} \\
 \text{or,} & \quad 3\sqrt{5} = 3\sqrt{x}, \quad x = 5
 \end{aligned}$$

23 Squaring both sides, we have

$$\begin{aligned}
 4a+x+a+x-2\sqrt{(a+x)(4a+x)} &= 41-8a \\
 \text{or,} & \quad -2\sqrt{(a+x)(4a+x)} = 2x-13a \\
 \text{or,} & \quad 4(4a^2+5ax+x^2) = 4x^2-52ax+169a^2 \\
 \text{or,} & \quad 72ax = 153a^2 \\
 \text{or,} & \quad 8x = 17a, \quad x = \frac{17a}{8}
 \end{aligned}$$

$$\begin{aligned}
 24 \quad \sqrt{1} + \sqrt{a+x} &= \frac{3a}{\sqrt{a+x}} \\
 \text{or,} & \quad \sqrt{ax+x^2}+a+x = 3a \\
 \text{or,} & \quad \sqrt{ax+x^2} = 2a-x \\
 \text{or,} & \quad ax+x^2 = 4a^2-4ax+x^2 \\
 \text{or,} & \quad 5ax = 4a^2, \quad x = \frac{4a}{5}
 \end{aligned}$$

$$\begin{aligned}
 25 \quad \sqrt{x} + \sqrt{x+13} &= \frac{91}{\sqrt{x+13}} \\
 \text{or,} & \quad \sqrt{x^2+13x}+x+13 = 91 \\
 \text{or,} & \quad \sqrt{x^2+13x} = 78-x \\
 \text{or,} & \quad x^2+13x = 6084+x^2-156x \\
 \text{or,} & \quad 169x = 6084, \quad x = 36
 \end{aligned}$$

$$\begin{aligned}
 26 \quad \sqrt{x+a} + \sqrt{x-a} &= \frac{b}{\sqrt{x+a}} \\
 \text{or,} & \quad 1+a+\sqrt{x^2-a^2} = b \\
 \text{or,} & \quad \sqrt{x^2-a^2} = b-a-1 \\
 \text{or,} & \quad x^2-a^2 = b^2+a^2+1^2-2ab-2bx+2ax \\
 \text{or,} & \quad 2x(b-a) = 2a^2+b^2-2ab, \\
 & \quad x = \frac{2a^2-2ab+b^2}{2(b-a)}
 \end{aligned}$$

$$\begin{aligned}
 27 \quad & \frac{3\sqrt{x-4}}{\sqrt{x+2}} = \frac{15-3\sqrt{r}}{\sqrt{r+40}} \\
 \text{or,} \quad & 3 - \frac{10}{\sqrt{x+2}} = 3 - \frac{105}{\sqrt{r+40}} \\
 \text{or,} \quad & \frac{2}{\sqrt{x-2}} = \frac{21}{\sqrt{r+40}} \\
 \text{or} \quad & 2\sqrt{x+80} = 21\sqrt{r+42} \\
 \text{or,} \quad & 19\sqrt{r} = 38 \\
 \text{or,} \quad & \sqrt{r} = 2, \quad x = 4
 \end{aligned}$$

$$\begin{aligned}
 28 \quad & \sqrt{x} - \sqrt{x - \sqrt{1-x}} = 1 \\
 \text{or,} \quad & \sqrt{x - \sqrt{1-x}} = 1 - \sqrt{x} \\
 \text{or,} \quad & x - \sqrt{1-x} = 1 + x - 2\sqrt{x} \\
 \text{or,} \quad & -\sqrt{1-x} = 1 - 2\sqrt{x} \\
 \text{or,} \quad & 1 - x = 1 + 4x - 4\sqrt{x} \\
 \text{or,} \quad & 5x = 4\sqrt{x} \quad \text{or,} \quad 5\sqrt{x} = 4 \quad \text{or,} \quad \sqrt{x} = \frac{4}{5}, \quad x = \frac{16}{25}
 \end{aligned}$$

29 By transposition, we have

$$\sqrt{8 - \sqrt{x^2 + 8x}} = 2\sqrt{2} - \sqrt{x}$$

Hence, squaring both sides,

$$\begin{aligned}
 & 8 - \sqrt{x^2 + 8x} = 8 + x - 4\sqrt{2x} \\
 \text{or,} \quad & -\sqrt{x^2 + 8x} = \sqrt{x} - 4\sqrt{2} \\
 \text{or,} \quad & x - 8 = x + 32 - 8\sqrt{2x} \\
 \text{or,} \quad & 8\sqrt{2x} = 24 \\
 \text{or} \quad & \sqrt{2x} = 3 \quad \text{or,} \quad 2x = 9, \quad x = \frac{9}{2} = 4\frac{1}{2}
 \end{aligned}$$

$$30 \quad \sqrt{1-x} + \sqrt{1-x + \sqrt{1+x}} = \sqrt{1+x}$$

$$\text{or,} \quad \sqrt{1-x + \sqrt{1+x}} = \sqrt{1+x} - \sqrt{1-x}$$

Squaring both sides, we have

$$\begin{aligned}
 1-x+\sqrt{1+x} &= 1+1+1-1-2\sqrt{1-x^2} \\
 \text{or, } \sqrt{1+x} &= (1+x)-2\sqrt{1-x^2} \\
 \text{or, } 1 &= \sqrt{1+x}-2\sqrt{1-x} \\
 \text{or, } 1 &= 1+x+4-4x-4\sqrt{1-x^2} \\
 \text{or, } 3x-4 &= 4\sqrt{1-x^2} \\
 \text{or, } 9x^2-24x+16 &= 16-16x^2 \\
 \text{or, } 25x^2 &= 24x, & x = \frac{24}{25}
 \end{aligned}$$

$$31 \quad \frac{\sqrt{a+x}}{a} + \frac{\sqrt{a+1}}{x} = \frac{\sqrt{x}}{b}$$

$$\text{or, } \sqrt{a+x}(a+1) = \frac{ax\sqrt{x}}{b}$$

Hence, squaring both sides, $(a+1)^2 = \frac{a^2 x^3}{b^2}$

$$\text{or, } a+1 = \frac{a}{b} \left(\frac{a^2}{b^2} \right)^{\frac{1}{3}}$$

$$\text{or, } \frac{a^{\frac{1}{3}}}{b^{\frac{1}{3}}} - 1 = \frac{a}{b}, \quad a = \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{a^{\frac{1}{3}} - b^{\frac{1}{3}}}$$

$$32 \quad \sqrt[5]{x+8} = \sqrt[10]{1^2+64x+36}$$

$$\text{or, } x+8 = \sqrt{x^2+64x+36}$$

$$\text{or, } x^2+16x+64 = 1^2+64x+36$$

$$\text{or, } 48x = 28, \quad x = \frac{7}{12} = \frac{1}{\frac{12}{7}}$$

$$33 \quad (1+x)^{\frac{1}{3}} + (1-x)^{\frac{1}{3}} = 2^{\frac{1}{3}}$$

Cubing both sides, we have

$$(1+x) + (1-x) + 3(1-x^2)^{\frac{1}{3}} 2^{\frac{1}{3}} = 2$$

$$\text{or, } 1-x^2=0 \text{ or } x^2=1 \quad x = \pm 1$$

$$34 \quad (a+x)^2 + (a-x)^2 = 3(a^2-x^2)^{\frac{1}{3}}$$

Cubing both sides, we have

$$(a+x)^2 + (a-x)^2 + 3(a^2-x^2)^{\frac{2}{3}} 3(a^2-x^2)^{\frac{1}{3}} = 27(a^2-x^2)$$

$$\text{or, } 2(a^2+x^2) + 9(a^2-x^2) = 27(a^2-x^2)$$

or, $20a^2 = 16a^2$

or, $5a^2 = 4a^2$, $a = \frac{2a}{\sqrt{5}}$

35 $\left(\frac{1}{a} + \frac{a}{b}\right)^{\frac{1}{4}} + 9\left(\frac{1}{a} - \frac{a}{b}\right)^{\frac{1}{4}} = 6\left(\frac{1}{a^2} - \frac{a^2}{b^2}\right)^{\frac{1}{4}}$

or, $\left\{\left(\frac{1}{a} + \frac{a}{b}\right)^{\frac{1}{4}}\right\}^4 + \left\{3\left(\frac{1}{a} - \frac{a}{b}\right)^{\frac{1}{4}}\right\}^4 - 2\left(\frac{1}{a} + \frac{a}{b}\right)^{\frac{1}{4}} \cdot 3\left(\frac{1}{a} - \frac{a}{b}\right)^{\frac{1}{4}} = 0$

or, $\left(\frac{1}{a} + \frac{a}{b}\right)^{\frac{1}{4}} - 3\left(\frac{1}{a} - \frac{a}{b}\right)^{\frac{1}{4}} = 0$

or, $\left(\frac{1}{a} + \frac{a}{b}\right)^{\frac{1}{4}} = 3\left(\frac{1}{a} - \frac{a}{b}\right)^{\frac{1}{4}}$

$\frac{1}{a} + \frac{a}{b} = 81\left(\frac{1}{a} - \frac{a}{b}\right)$

or, $\frac{80a}{a} = \frac{82a}{b}$, $a = \frac{82a^2}{80b} = \frac{41a^2}{40b}$

36 $\frac{x-47}{\sqrt{x+2}-7} + \frac{x-19}{\sqrt{x-3}-4} = \frac{4x-124}{\sqrt{4x-5}-11}$

or, $\frac{(x+2)-49}{\sqrt{x+2}-7} + \frac{(x-3)-16}{\sqrt{x-3}-4} = \frac{(4x-3)-121}{\sqrt{4x-3}-11}$

or, $\sqrt{x+2}+7 + \sqrt{x-3}+4 = \sqrt{4x-3}+11$

or, $\sqrt{x+2} + \sqrt{x-3} = \sqrt{4x-3}$,

$x+2+x-3+2\sqrt{x^2-x-6} = 4x-3$

or, $\sqrt{x^2-x-6} = x-1$

or, $x^2-x-6 = x^2-2x+1$, $x=7$

37 $\frac{2x-49}{\sqrt{2x+15}-8} + \frac{18x+22}{\sqrt{18x+31}+3} = \frac{8x+191}{2\sqrt{2x+54}-5}$

or, $\frac{(2x+15)-64}{\sqrt{2x+15}-8} + \frac{(18x+31)-9}{\sqrt{18x+31}+3} = \frac{4(2x+54)-25}{2\sqrt{2x+54}-5}$

or, $\sqrt{2x+15}+8 + \sqrt{18x+31}-3 = 2\sqrt{2x+54}+5$

or, $\sqrt{2x+15} + \sqrt{18x+31} = 2\sqrt{2x+54}$

Hence, squaring both sides,

$$2x + 15 + 18x + 31 + 2\sqrt{36x^2 + 332x + 465} = 81 + 216$$

$$\text{or, } 12x - 170 = -2\sqrt{36x^2 + 332x + 465}$$

$$\text{or, } 6x - 85 = -\sqrt{36x^2 + 332x + 465}$$

$$36x^2 - 1020x + 7225 = 36x^2 + 332x + 465$$

$$\text{or, } 1352x = 6760, \quad x = 5$$

38 By transposition, we have

$$x + a = \sqrt{a^2 + x\sqrt{b^2 + x^2}}$$

$$x^2 + 2ax + a^2 = a^2 + x\sqrt{b^2 + x^2}$$

$$\text{or, } x + 2a = \sqrt{b^2 + x^2}$$

$$x^2 + 4ax + 4a^2 = b^2 + x^2$$

$$\text{or, } 4ax = b^2 - 4a^2, \quad \frac{b^2 - 4a^2}{4a}$$

$$39 \quad \sqrt{x^2 + 9} + \sqrt{x^2 - 9} = 4 + \sqrt{34}$$

$$\text{or, } \sqrt{x^2 + 9} - 4 = \sqrt{34} - \sqrt{x^2 - 9}$$

$$x^2 + 9 + 16 - 8\sqrt{x^2 + 9} = 34 + x^2 - 9 - 2\sqrt{34x^2 - 306}$$

$$\text{or, } 4\sqrt{x^2 + 9} = \sqrt{34x^2 - 306}$$

$$16x^2 + 144 = 34x^2 - 306$$

$$\text{or, } 18x^2 = 450 \quad \text{or, } x^2 = 25, \quad x = 5$$

$$40 \quad \sqrt{3x^2 + 16} - \sqrt{3x^2 - 16} = 8 - 4\sqrt{2}$$

$$\text{or, } \sqrt{3x^2 + 16} + 4\sqrt{2} = \sqrt{3x^2 - 16} + 8$$

$$3x^2 + 16 + 32 + 8\sqrt{6x^2 + 32} = 3x^2 - 16 + 64 + 16\sqrt{3x^2 - 16}$$

$$\text{or, } \sqrt{6x^2 + 32} = 2\sqrt{3x^2 - 16}$$

$$6x^2 + 32 = 12x^2 - 64$$

$$\text{or, } 6x^2 = 96 \quad \text{or, } x^2 = 16, \quad x = 4$$

Exercise (90)

Note —The expressions must first be arranged according to either descending or ascending powers of any of the algebraical quantities and then the operation performed

$$1 \quad \begin{array}{r} 4x^2z^2 + 12xyz + 9y^2 \\ 4x^2z^2 \end{array} \left(\begin{array}{r} 2xz + 3y \\ 4xz + 3y \end{array} \right) \begin{array}{r} 12xyz + 9y^2 \\ 12xyz + 9y^2 \end{array}$$

Thus the required root = $2xz + 3y$

$$2 \quad \begin{array}{r} x^4 - 4x^3 + 10x^2 - 12x + 9 \\ x^4 \end{array} \left(\begin{array}{r} x^2 - 2x + 3 \\ 2x^2 - 2x \end{array} \right) \begin{array}{r} -4x^3 + 10x^2 - 12x + 9 \\ -4x^3 + 4x^2 \end{array}$$

$$\begin{array}{r} 6x^2 - 12x + 9 \\ 6x^2 - 4x + 3 \end{array} \left(\begin{array}{r} 6x^2 - 12x + 9 \\ 6x^2 - 12x + 9 \end{array} \right)$$

Thus the required root = $x^2 - 2x + 3$

$$3 \quad \begin{array}{r} x^6 - 2x^4 + 2x^3 + x^2 - 2x + 1 \\ x^6 \end{array} \left(\begin{array}{r} x^2 - x + 1 \\ 2x^3 - x \end{array} \right) \begin{array}{r} -2x^4 + 2x^3 + x^2 - 2x + 1 \\ -2x^4 \end{array}$$

$$\begin{array}{r} 2x^3 - 2x + 1 \\ 2x^3 - 2x + 1 \end{array} \left(\begin{array}{r} 2x^3 - 2x + 1 \\ 2x^3 - 2x + 1 \end{array} \right)$$

Thus the required root = $x^2 - x + 1$

$$4 \quad \begin{array}{r} 4x^4 - 12x^3 + 25x^2 - 24x + 16 \\ 4x^4 \end{array} \left(\begin{array}{r} 2x^3 - 3x + 4 \\ 4x^3 - 3x \end{array} \right) \begin{array}{r} -12x^3 + 25x^2 - 24x + 16 \\ -12x^3 + 9x^2 \end{array}$$

$$\begin{array}{r} 16x^2 - 24x + 16 \\ 16x^2 - 6x + 4 \end{array} \left(\begin{array}{r} 16x^2 - 24x + 16 \\ 16x^2 - 24x + 16 \end{array} \right)$$

Thus the required root = $2x^2 - 3x + 4$

$$5 \quad \begin{array}{r} 4x^4 + 8ax^3 + 4a^2x^2 + 16b^2x + 16ab^2x + 16b^4 \\ 4x^4 \end{array} \left(\begin{array}{r} 2x^2 + 2ax + 4b^2 \\ 4x^3 + 2ax \end{array} \right) \begin{array}{r} 8ax^3 + 4a^2x^2 + 16b^2x + 16ab^2x + 16b^4 \\ 8ax^3 + 4a^2x^2 \end{array}$$

$$\begin{array}{r} 16b^2x + 16ab^2x + 16b^4 \\ 16x^2 + 4ax + 4b^2 \end{array} \left(\begin{array}{r} 16b^2x + 16ab^2x + 16b^4 \\ 16b^2x + 16ab^2x + 16b^4 \end{array} \right)$$

Thus the required root = $2x^2 + 2ax + 4b^2$

$$\begin{array}{r}
 6 \quad \frac{9x^4 - 2x^3y + 16x^2y^2 - 2xy^3 + 9y^4}{9x^4} \left(3x^2 - \frac{xy}{3} + 3y^2 \right) \\
 \frac{6x^2 - \frac{2y}{3}}{3} \left) \frac{-2x^3y + 16x^2y^2 - 2xy^3 + 9y^4}{-2x^3y + \frac{16}{3}x^2y^2} \\
 \frac{6x^2 - \frac{2xy}{3} + 3y^2}{3} \left) \frac{18x^4y^3 - 2xy^3 + 9y^4}{18x^2y^2 - 2xy^3 + 9y^4}
 \end{array}$$

Thus the required root $= 3x^2 - \frac{xy}{3} + 3y^2$

$$\begin{array}{r}
 7 \quad \frac{x^4 - 2x^3 + \frac{3x^2}{2} - \frac{x}{2} + \frac{1}{16}}{2x^2 - x} \left(\frac{x^4}{-2x^3 + \frac{3x^2}{2} - \frac{x}{2} + \frac{1}{16}} \right. \\
 \left. \frac{-2x^3 + x^2}{2x^2 - 2x + \frac{1}{4}} \right) \frac{\frac{x^2}{2} - \frac{x}{2} + \frac{1}{16}}{\frac{x^2}{2} - \frac{x}{2} + \frac{1}{16}}
 \end{array}$$

Thus the required root $= x^2 - x + \frac{1}{4}$

$$\begin{array}{r}
 8 \quad \frac{49x^4 - \frac{14x^3}{5} + \frac{1051x^2}{25} - \frac{6x}{5} + 9}{49x^4} \left(7x^2 - \frac{x}{5} + 3 \right) \\
 \frac{14x^2 - \frac{x}{5}}{5} \left) \frac{-\frac{14x^3}{5} + \frac{1051x^2}{25} - \frac{6x}{5} + 9}{-\frac{14x^3}{5} + \frac{x^2}{25}} \\
 \frac{14x^2 - \frac{2x}{5} + 3}{5} \left) \frac{42x^2 - \frac{6x}{5} + 9}{42x^2 - \frac{6x}{5} + 9}
 \end{array}$$

Thus the required root $= 7x^2 - \frac{x}{5} + 3$

9

$$\begin{array}{r}
 x^4 - x^3 + \frac{x^2}{4} + 4x - 2 + \frac{4}{x^2} \left(x^3 - \frac{x}{2} + \frac{2}{x} \right) \\
 \hline
 2x^3 - \frac{x}{2} \left(-x^3 + \frac{x^2}{4} + 4x - 2 + \frac{4}{x^2} \right) \\
 \hline
 -x^3 + \frac{x^2}{4} \\
 \hline
 2x^3 - x + \frac{2}{x} \left(4x - 2 + \frac{4}{x^2} \right) \\
 \hline
 4x - 2 + \frac{4}{x^2}
 \end{array}$$

Thus the required root $= x^3 = \frac{x}{2} + \frac{2}{x}$

10

$$\begin{array}{r}
 \frac{a^4}{4} + \frac{a^3}{x} + \frac{a^2}{x^3} - ax - 2 + \frac{x^2}{a^2} \left(\frac{a^3}{2} + \frac{a}{x} - \frac{2}{a} \right) \\
 \hline
 \frac{a^4}{4} \\
 \hline
 a^3 + \frac{a}{x} \left(\frac{a^3}{2} + \frac{a^2}{x^3} - ax - 2 + \frac{x^2}{a^2} \right) \\
 \hline
 \frac{a^3}{x} + \frac{a^2}{x^3} \\
 \hline
 a^3 + \frac{2a}{x} - \frac{x}{a} \left(-ax - 2 + \frac{x^2}{a^2} \right) \\
 \hline
 -ax - 2 + \frac{a^2}{x^2} \\
 \hline
 -ax - 2 + \frac{a^2}{x^2}
 \end{array}$$

Thus the required root $= \frac{a^2}{2} + \frac{a}{x} - \frac{x}{a}$

11

$$\begin{array}{r}
 \frac{a^2}{4b^2} - \frac{a}{b} \quad 1 + \frac{4b}{a} + \frac{4b^2}{a^2} \left(\frac{b}{2b} - 1 - \frac{2b}{a} \right) \\
 \hline
 \frac{a^2}{4b^2} \\
 \hline
 \frac{a}{b} - 1 \left(-\frac{a}{b} - 1 \right) \\
 \hline
 -\frac{a}{b} - 1
 \end{array}$$

$$\frac{\frac{a}{b} - 2 - \frac{2b}{a}}{-2 + \frac{4b}{a} + \frac{4b^2}{a^2}}$$

Thus the required root $= \frac{a}{2b} - 1 - \frac{2b}{a}$

$$12 \quad \frac{\frac{9x^2}{x^2} - \frac{6a}{5x} + \frac{101}{25} - \frac{4x}{15a} + \frac{4x^2}{9a^3} \left(\frac{3a}{x} - \frac{1}{5} + \frac{2x}{3a} \right. \\ \left. \frac{9a^3}{x^2} \right) - \frac{6a}{5x} + \frac{101}{25} \\ - \frac{6a}{5x} + \frac{1}{25} \\ \frac{6a}{x} - \frac{2}{5} + \frac{2x}{3a} \Big) 4 - \frac{4x}{15a} + \frac{4x^2}{9a^3} \\ 4 - \frac{4x}{15a} + \frac{4x^2}{9a^3}$$

Thus the required root $= \frac{3a}{x} - \frac{1}{5} + \frac{2x}{3a}$

$$13 \quad \frac{4x^3 - 8x^2y^2 + 4xy^6 + y^8}{4x^4} \left(2x^3 - 2xy^3 - y^4 \right. \\ \left. 4x^2 - 2xy^2 \right) - \frac{8x^2y^2 + 4xy^6 + y^8}{-8x^2y^2 + 4x^2y^4} \\ 4x^3 - 4xy^3 - y^4 \Big) - \frac{4x^2y^4 + 4xy^6 + y^8}{-4x^2y^4 + 4xy^6 + y^8}$$

Thus the required root $= 2x^2 - 2xy^2 - y^4$

$$14 \quad \frac{\frac{49x^2}{y^2} - \frac{42x}{y} + 7 + \frac{6y}{7x} + \frac{y^2}{49x^2} \left(\frac{7x}{y} - 3 - \frac{y}{7x} \right. \\ \left. \frac{49x^2}{y^2} \right) - \frac{42x}{y} + 7 + \frac{6y}{7x} + \frac{y^2}{49x^2} \\ - \frac{42x}{y} + 9$$

$$\begin{aligned} & \left(\frac{14x}{y} - 6 - \frac{y}{7x} \right) - 2 + \frac{6y}{7x} + \frac{y^2}{49x^2} \\ & \quad - 2 + \frac{6y}{7x} + \frac{y^2}{49x^2} \end{aligned}$$

Thus the required root $= \frac{7x}{y} - 3 - \frac{y}{7x}$

15

$$\begin{aligned} & \frac{\frac{x^2}{y^2} - \frac{x}{y} - 1\frac{1}{4} + \frac{y}{x} + \frac{y^2}{x^2}}{\frac{x^2}{y^2}} \left(\frac{x}{y} - \frac{1}{2} - \frac{y}{x} \right) \\ & \quad - \frac{\frac{x}{y} - 1\frac{1}{4} + \frac{y}{x} + \frac{y^2}{x^2}}{-\frac{x}{y} + \frac{1}{4}} \\ & \quad - 2 + \frac{y}{x} + \frac{y^2}{x^2} \\ & \quad - 2 + \frac{y}{x} + \frac{y^2}{x^2} \end{aligned}$$

Thus the required root $= \frac{x}{y} - \frac{1}{2} - \frac{y}{x}$

16

$$\begin{aligned} & \frac{\frac{4x^2}{49y^2} - \frac{20x}{7y} + 25\frac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2}}{\frac{4x^2}{49y^2}} \left(\frac{2x}{7y} - 5 + \frac{3y}{4x} \right) \\ & \quad - \frac{\frac{20x}{7y} + 25}{\frac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2}} \\ & \quad - 10 + \frac{3y}{4x} \end{aligned}$$

Thus the required root $= \frac{2x}{7y} - 5 + \frac{3y}{4x}$

$$\begin{array}{r}
 17 \quad \frac{x^3 - 2x^{\frac{3}{2}} + 3x - 2x^{\frac{1}{2}} + 1}{x^2} \left(x - x^{\frac{1}{2}} + 1 \right. \\
 \left. \begin{array}{r} 2x - x^{\frac{1}{2}} \\ -2x^{\frac{3}{2}} + 3x - 2x^{\frac{1}{2}} + 1 \\ -2x^{\frac{1}{2}} + 1 \end{array} \right) \\
 \hline
 2x - 2x^{\frac{1}{2}} + 1 \quad \left. \begin{array}{r} 2x - 2x^{\frac{1}{2}} + 1 \\ 2x - 2x^{\frac{1}{2}} + 1 \end{array} \right)
 \end{array}$$

Thus the required root $= x - x^{\frac{1}{2}} + 1$

$$\begin{array}{r}
 18 \quad \frac{x^{\frac{5}{3}} - 4x^{\frac{4}{3}} + 2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}}}{x^{\frac{5}{3}}} \left(x^{\frac{5}{6}} - 2x^{\frac{1}{2}} - x^{\frac{1}{6}} \right. \\
 \left. \begin{array}{r} 2x^{\frac{5}{6}} - 2x^{\frac{1}{2}} \\ -4x^{\frac{4}{3}} + 2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \\ -4x^{\frac{4}{3}} + 4x \end{array} \right) \\
 \hline
 2x^{\frac{5}{6}} - 4x^{\frac{1}{2}} - x^{\frac{1}{6}} \quad \left. \begin{array}{r} -2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \\ -2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \end{array} \right)
 \end{array}$$

Thus the required root $= x^{\frac{5}{6}} - 2x^{\frac{1}{2}} - x^{\frac{1}{6}}$

$$\begin{array}{r}
 19 \quad \frac{a^2x^{-3} + 2ax^{-1} + 3 + 2a^{-1}x + a^{-2}x^3}{a^2x^{-3}} \left(ax^{-1} + 1 + a^{-1}x \right. \\
 \left. \begin{array}{r} 2ax^{-1} + 1 \\ 2ax^{-1} + 3 + 2a^{-1}x + a^{-2}x^3 \\ 2ax^{-1} + 1 \end{array} \right) \\
 \hline
 2ax^{-1} + 2 + a^{-1}x \quad \left. \begin{array}{r} 2 + 2a^{-1}x + a^{-2}x^3 \\ 2 + 2a^{-1}x + a^{-2}x^3 \end{array} \right)
 \end{array}$$

Thus the required root $= ax^{-1} + 1 + a^{-1}x$

$$\begin{array}{r}
 20 \quad \frac{x^{\frac{3}{2}} - x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}} + 2x^{\frac{1}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y}{x^{\frac{3}{2}}} \left[\left(x^{\frac{1}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{4}} \right. \right. \\
 \left. \begin{array}{r} 2x^{\frac{1}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} \\ -2x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}} \\ -2x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}} \end{array} \right) \\
 \hline
 2x^{\frac{1}{4}} - 2x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{4}} \quad \left. \begin{array}{r} 2x^{\frac{1}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y \\ 2x^{\frac{1}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y \end{array} \right)
 \end{array}$$

Thus the required root $= x^{\frac{1}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{4}}$

$$\left[\left(\frac{3x^{\frac{1}{2}}}{2} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y \right) \right]$$

21

$$\begin{aligned} & \frac{9x^{\frac{1}{2}}}{4} - 5x^{\frac{1}{2}}y^{\frac{1}{2}} + \frac{179x^{\frac{1}{2}}y}{45} - \frac{4x^{\frac{1}{2}}y^{\frac{1}{2}}}{3} + \frac{4xy^{\frac{1}{2}}}{25} \\ & \frac{\frac{9x^{\frac{1}{2}}}{4}}{\left(\frac{3x^{\frac{1}{2}}}{2} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y \right)} = \frac{-5x^{\frac{1}{2}}y^{\frac{1}{2}} + \frac{179x^{\frac{1}{2}}y}{45} - \frac{4x^{\frac{1}{2}}y^{\frac{1}{2}}}{3} + \frac{4xy^{\frac{1}{2}}}{25}}{-5x^{\frac{1}{2}}y^{\frac{1}{2}} + \frac{25xy^{\frac{1}{2}}}{9}} \\ & \frac{\left(\frac{3x^{\frac{1}{2}}}{2} - \frac{10}{3}xy^{\frac{1}{2}} + \frac{2}{3}x^{\frac{1}{2}}y \right)}{\left(\frac{3x^{\frac{1}{2}}}{2} - \frac{10}{3}xy^{\frac{1}{2}} + \frac{2}{3}x^{\frac{1}{2}}y \right)} = \frac{\frac{6xy^{\frac{1}{2}}}{5} - \frac{4x^{\frac{1}{2}}y^{\frac{1}{2}}}{3} + \frac{4xy^{\frac{1}{2}}}{25}}{\frac{6xy^{\frac{1}{2}}}{5} - \frac{4x^{\frac{1}{2}}y^{\frac{1}{2}}}{3} + \frac{4xy^{\frac{1}{2}}}{25}} \end{aligned}$$

Thus the required root = $\frac{3x^{\frac{1}{2}}}{2} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y$

22

$$\frac{\frac{a^{2m} - 4a^{m+n} + 4a^{2n}}{a^{2m}}}{\frac{2a^{2m} - 2a^{2n}}{a^{2m}}} = \frac{-4a^{m+n} + 4a^{2n}}{-4a^{m+n} + 4a^{2n}} \left(\frac{a^m - 2a^n}{a^m - 2a^n} \right)$$

Thus the required root = $a^m - 2a^n$

$$\left[\left(a^{2m+1} + 3a^{2n} - 5c^{2m-2} \right) \right]$$

23

$$\begin{aligned} & \frac{a^{4m+2} + 6a^{2m+1} + 9a^{2m} - 10a^{2m+1}c^{m-2} - 30a^{2m}c^{m-3} + 25c^{2m-4}}{a^{4m+2}} \\ & \frac{2a^{2m+1} + 3a^{2m}}{6a^{2m+1} + 9a^{2m}} \left(\frac{6a^{2m+1} + 9a^{2m} - 10a^{2m+1}c^{m-2} - 30a^{2m}c^{m-3}}{6a^{2m+1} + 9a^{2m}} \right) \\ & \frac{2a^{2m+1} + 6a^{2m} - 5c^{2m-1}}{6a^{2m+1} + 9a^{2m}} \left(\frac{-10a^{2m+1}c^{m-2} - 30a^{2m}c^{m-3} + 25c^{2m-4}}{-10a^{2m+1}c^{m-2} - 30a^{2m}c^{m-3} + 25c^{2m-4}} \right) \end{aligned}$$

Thus the required root = $a^{2m+1} + 3a^{2n} - 5c^{2m-2}$

EXERCISE (91)

- 1 $25x^2y^2 - 40xy + 16 = (5xy)^2 - 2 \cdot 5xy \cdot 4 + (4)^2 = (5xy - 4)^2$,
the required root $= 5xy - 4$
- 2 $49a^2x^4 - 42ab^2x^2 + 9b^4 = (7ax^2)^2 - 2(7ax^2)(3b^2) + (3b^2)^2$
 $= (7ax^2 - 3b^2)^2$,
the required root $= 7ax^2 - 3b^2$
- 3 $49a^6b^8 + 126a^7b^7 + 81a^8b^6$
 $= (7a^3b^4)^2 + 2(7a^3b^4)(9a^4b^3) + (9a^4b^3)^2 = (7a^3b^4 + 9a^4b^3)^2$,
the required root $= 7a^3b^4 + 9a^4b^3$.
- 4 $\frac{x^8y^4}{4} - \frac{x^7y^7}{5} + \frac{x^6y^{10}}{25}$
 $= \left(\frac{x^4y^2}{2}\right)^2 - 2\left(\frac{x^4y^2}{2}\right)\left(\frac{x^3y^5}{5}\right) + \left(\frac{x^3y^5}{5}\right)^2 = \left(\frac{x^4y^2}{2} - \frac{x^3y^5}{5}\right)^2$,
the required root $= \frac{x^4y^2}{2} - \frac{x^3y^5}{5}$
- 5 $\frac{25a^2b^2}{4} + \frac{c^4}{9} - \frac{5abc^2}{3}$
 $= \left(\frac{5ab}{2}\right)^2 - 2\left(\frac{5ab}{2}\right)\left(\frac{c^2}{3}\right) + \left(\frac{c^2}{3}\right)^2 = \left(\frac{5ab}{2} - \frac{c^2}{3}\right)^2$,
the required root $= \frac{5ab}{2} - \frac{c^2}{3}$
- 6 The given expression
 $= (a^2 + 2ab + b^2) + (2ac + 2bc) + c^2 = (a+b)^2 + 2c(a+b) + c^2$
 $= \{(a+b) + c\}^2$,
the required root $= a + b + c$
- 7 The given expression
 $= a^2 - 2ab + b^2 + 2ac - 2bc + c^2 = (a-b)^2 + 2c(a-b) + c^2$
 $= \{(a-b) + c\}^2$,
the required root $= a - b + c$
- 8 The given expression
 $= (2a)^2 + b^2 - 2(2a)b + 6bc - 12ac + 9c^2$
 $= (2a-b)^2 - 2(2a-b)(3c) + (3c)^2 = \{(2a-b) - 3c\}^2$,
the required root $= 2a - b - 3c$

9 The given expression

$$\begin{aligned} &= a^4 + 2a^2(2b^2) + (2b^2)^2 - 6a^2c^2 - 12b^2c^2 + 9c^4 \\ &= (a^2 + 2b^2)^2 - 2(a^2 + 2b^2)(3c^2) + (3c^2)^2 = (a^2 + 2b^2 - 3c^2)^2, \\ &\text{the required root} = a^2 + 2b^2 - 3c^2 \end{aligned}$$

10 The given expression

$$\begin{aligned} &= (2a^2)^2 - 2(2a^2)(3b^2) + (3b^2)^2 + 20a^2c^2 - 30b^2c^2 + (5c^2)^2 \\ &= (2a^2 - 3b^2)^2 + 2(2a^2 - 3b^2)(5c^2) + (5c^2)^2 = (2a^2 - 3b^2 + 5c^2)^2, \\ &\text{the required root} = 2a^2 - 3b^2 + 5c^2 \end{aligned}$$

11 The given expression

$$\begin{aligned} &= x^2 - bx + \left(\frac{b}{2}\right)^2 + \frac{2ax}{3} - \frac{ab}{3} + \frac{a^2}{9} \\ &= \left(x - \frac{b}{2}\right)^2 + 2\left(x - \frac{b}{2}\right)\frac{a}{3} + \left(\frac{a}{3}\right)^2 = \left(x - \frac{b}{2} + \frac{a}{3}\right)^2, \\ &\text{the required root} = x - \frac{b}{2} + \frac{a}{3} \end{aligned}$$

12 The given expression

$$\begin{aligned} &= \left(x - \frac{1}{2}\right)^2 + 4 - 4\left(x - \frac{1}{2}\right) - \left(x - \frac{1}{2}\right)^2 - 2\left(x - \frac{1}{2}\right)2 + (2)^2 \\ &= \left(x - \frac{1}{2} - 2\right)^2, \\ &\text{the required root} = x - 2 - \frac{1}{2} \end{aligned}$$

13 The given expression

$$\begin{aligned} &= x^4 + 2 + \frac{1}{x^4} + 2\left(x^2 + \frac{1}{x^2}\right) + 1 = \left(x^2 + \frac{1}{x^2}\right)^2 + 2\left(x^2 + \frac{1}{x^2}\right) + 1 \\ &= \left\{\left(x^2 + \frac{1}{x^2}\right) + 1\right\}^2, \\ &\text{the required root} = x^2 + 1 + \frac{1}{x^2} \end{aligned}$$

14 The given expression

$$\begin{aligned} &= \frac{a^2}{b^2} + 2 + \frac{b^2}{a^2} + \frac{2a}{b} + \frac{2b}{a} + 1 \\ &= \left(\frac{a}{b} + \frac{b}{a}\right)^2 + 2\left(\frac{a}{b} + \frac{b}{a}\right) + 1 = \left(\frac{a}{b} + \frac{b}{a} + 1\right)^2, \\ &\text{the required root} = \frac{a}{b} + 1 + \frac{b}{a} \end{aligned}$$

15 The given expression

$$\begin{aligned}
 &= \frac{x^2}{y^2} + 2 + \frac{1}{x^2} - \left(\frac{x}{y} + \frac{1}{x} \right) \sqrt{2 + \frac{1}{x^2}} \\
 &= \left(\frac{x}{y} + \frac{1}{x} \right)^2 - 2 \left(\frac{x}{y} + \frac{1}{x} \right) \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} \right)^2 = \left(\frac{x}{y} + \frac{1}{x} - \frac{1}{\sqrt{2}} \right)^2, \\
 &\text{the required root} = \frac{x}{y} - \frac{1}{\sqrt{2}} + \frac{1}{x}
 \end{aligned}$$

16 The given expression

$$\begin{aligned}
 &= \left(\frac{3x}{a} \right)^2 - 2 \left(\frac{3x}{a} \right) + 1 - \frac{2a}{3x} + 2 + \frac{a^2}{9x^2} \\
 &= \left(\frac{3x}{a} - 1 \right)^2 + 2 \left(\frac{3x}{a} - 1 \right) \frac{a}{3x} + \left(\frac{a}{3x} \right)^2 = \left(\frac{3x}{a} - 1 + \frac{a}{3x} \right)^2, \\
 &\text{the required root} = \frac{3x}{a} - 1 + \frac{a}{3x}
 \end{aligned}$$

17 The given expression

$$\begin{aligned}
 &= x^2 + \frac{1}{x^2} + 2 + 4 \left(x + \frac{1}{x} \right) + 4 \\
 &= \left(x + \frac{1}{x} \right)^2 + 2 \left(x + \frac{1}{x} \right) + 2 + (2)^2 = \left(x + \frac{1}{x} + 2 \right)^2, \\
 &\text{the required root} = x + 2 + \frac{1}{x}
 \end{aligned}$$

18 The given expression

$$\begin{aligned}
 &= (a\sqrt{2})^2 - 2(a\sqrt{2})(a - \sqrt{2}) + (a - \sqrt{2})^2 \\
 &= (a\sqrt{2} - a + \sqrt{2})^2, \\
 &\text{the required root} = a\sqrt{2} - a + \sqrt{2}
 \end{aligned}$$

19 The given expression

$$\begin{aligned}
 &= a^2 - 2a(b - c + d) + b^2 - 2b(c - d) + c^2 + d^2 - 2cd \\
 &= a^2 - 2a(b - c + d) + b^2 - 2b(c - d) + (c - d)^2 \\
 &= a^2 - 2a(b - c + d) + \{b - (c - d)\}^2 \\
 &= a^2 - 2a(b - c + d) + (b - c + d)^2 = \{a - (b - c + d)\}^2 \\
 &= (a - b + c - d)^2 \\
 &\text{the required root} = a - b + c - d
 \end{aligned}$$

20 The given expression

$$\begin{aligned}
 &= \{(a-b)^2\}^2 - 2(a^2 + b^2)(a-b)^2 + (a^2 + b^2)^2 + a^4 + b^4 - 2a^2b^2 \\
 &= \{(a-b)^2 - (a^2 + b^2)\}^2 + (a^2 - b^2)^2 \\
 &= (a^2 - 2ab + b^2 - a^2 - b^2)^2 + (a^2 - b^2)^2 = (-2ab)^2 + (a^2 - b^2)^2 \\
 &= a^4 + b^4 - 2a^2b^2 + 4a^2b^2 = a^4 + b^4 + 2a^2b^2 = (a^2 + b^2)^2, \\
 &\text{the required root} = a^2 + b^2
 \end{aligned}$$

21 The given expression

$$\begin{aligned}
 &= a^4 + b^4 - 2a^2b^2 + c^4 + d^4 - 2c^2d^2 + 2a^2c^2 - 2a^2d^2 - 2b^2(c^2 - d^2) \\
 &= (a^2 - b^2)^2 + (c^2 - d^2)^2 + 2a^2(c^2 - d^2) - 2b^2(c^2 - d^2) \\
 &= (a^2 - b^2)^2 + (c^2 - d^2)^2 + 2(a^2 - b^2)(c^2 - d^2) = \{(a^2 - b^2) + (c^2 - d^2)\}^2 \\
 &= (a^2 - b^2 + c^2 - d^2)^2, \\
 &\text{the required root} = a^2 - b^2 + c^2 - d^2
 \end{aligned}$$

22 The given expression

$$\begin{aligned}
 &= (a^n)' + 2(a^2)a + a^2 - (a^2 + a) + \frac{1}{2} \\
 &= (a^2 + a)^2 - 2(a^2 + a) \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = (a^2 + a - \frac{1}{2})^2, \\
 &\text{the required root} = a^2 + a - \frac{1}{2}
 \end{aligned}$$

23 The given expression

$$\begin{aligned}
 &= 2\{a^2(b+c)^2 + b^2(c+a)^2 + c^2(a+b)^2 + 2abc(a+b+c)\} \\
 &= 2\{a^2(b+c)^2 + a^2(b^2 + c^2 + 2bc) + 2a(b^2c + c^2b) \\
 &\quad + 2abc(b+c) + 2b^2c^2\} \\
 &= 2\{a^2(b+c)^2 + a^2(b+c)^2 + 2abc(b+c) + 2abc(b+c) + 2b^2c^2\} \\
 &= 4\{a^2(b+c)^2 + 2abc(b+c) + b^2c^2\} = 4\{a(b+c) + bc\}^2, \\
 &\text{the required root} = 2a(b+c) + 2bc
 \end{aligned}$$

Exercise (92)

1

$$\begin{array}{r}
 x^3 + 27x^2 + 243x + 729 \quad (x+9) \\
 \underline{x^3} \\
 3 \times (x)^2 = 3x^2 \\
 \underline{3x^2 \times 9 = 27x^2} \\
 (9)^2 = 81 \\
 \underline{3x^2 + 27x + 81} \\
 271^2 + 243x + 729
 \end{array}$$

Thus the required root = $x + 9$

2

I hus the required root = $3x - 8$

3

Thus the required root = $4a - 3b$

$$[(x^2 - 3x + 2)$$

4

Thus the required root = $x^2 - 3x + 2$

$$[(2x^2 + x - 3)$$

5

Thus the required 100t = $2x + x - 3$

$$6. \quad \frac{1-9t^2+33t^4-63t^6+66t^8-36t^{10}+8t^{12}}{1-9t^2+27t^4-27t^6} \left(1-3t^2+2t^4 \right)$$

$$\frac{3 \times 1^2 = 3}{3 \times 1 \times (-3t^2) = -9t^2} \quad \frac{9t^4}{3-9t^2+9t^4}$$

$$\frac{3(1-3t^2)^2 = 3-18t^2+27t^4}{3(1-3t^2) \times 2t^4 = 6t^4-18t^6} \quad \frac{4t^8}{3-18t^2+33t^4-18t^6+4t^8}$$

$$\frac{-9t^2+27t^4-27t^6}{6t^4-18t^6+33t^8-36t^{10}+8t^{12}}$$

$$\frac{6t^4-36t^6+66t^8-36t^{10}+8t^{12}}{1-3t^2+2t^4}$$

Thus the required root = $1-3t^2+2t^4$

$$\frac{8t^6-36t^8+66t^{10}-63t^{12}+33t^{14}-9t^{16}+t^{18}}{8t^6} \left(2t^2-3t^4+t^6 \right)$$

7

$$\frac{3 \times (2t^2)^2 = 12t^4}{3 \times (2t^2) \times (-3t^4) = -18t^6} \quad \frac{9t^8}{12t^4-18t^6+9t^8}$$

$$\frac{3(2t^2-3t^4)^2 = 12t^4-36t^6+27t^8}{-3(2t^2-3t^4) \times t^6 = 6t^8t^2-9t^8} \quad \frac{t^4}{12t^4-36t^6+33t^8-9t^8+t^4}$$

Thus the required root = $2t^2-3t^4+t^6$

Exercise (93).

- 1 We have $4 \quad 5=32 \quad 40$ and $7 \quad 8=35 \quad 40$,
 $7 \quad 8 > 4 \quad 5$
 - 2 We have $7 \quad 10=49 \quad 70$ and $11 \quad 14=55 \quad 70$,
 $11 \quad 14 > 7 \quad 10$
 - 3 We have $9 \quad 5=72 \quad 40$ and $13 \quad 8=65 \quad 40$,
 $9 \quad 5 > 13 \quad 8$
 - 4 We have $22 \quad 27=110 \quad 135$ and $32 \quad 45=96 \quad 135$,
 $22 \quad 27 > 32 \quad 45$
 - 5 We have $28 \quad 39=140 \quad 195$, and $49 \quad 65=147 \quad 195$,
 $49 \quad 65 > 28 \quad 39$
 - 6 The required ratio $= \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{a}{d} = a \quad d$
 - 7 The required ratio $= \frac{3}{5} \times \frac{7}{9} \times \frac{15}{28} = \frac{1}{4} = 1 \quad 4$
 - 8 The required ratio $= \frac{a+x}{a-x} \times \frac{a^2+x^2}{(a+x)^2} \times \frac{a^2-x^2}{a^4-x^4}$
 $= \frac{(a^2+x^2)(a^2-x^2)}{(a-x)(a+x)(a^2+x^2)} = 1 = 1 \quad 1$
 - 9 The required ratio $= \frac{16}{5} \times \left(\frac{5}{4}\right)^3 \times \left(\frac{9}{4}\right)^{\frac{1}{2}}$
 $= \frac{16}{5} \times \frac{5 \times 5 \times 5}{4 \times 4 \times 4} \times \frac{3}{2} = \frac{75}{8} = 75 \quad 8$
 - 10 The required ratio $= \frac{25}{18} \times \left(\frac{81}{49}\right)^{\frac{1}{2}} \times \left(\frac{2}{3}\right)^3 \times \left(\frac{7}{5}\right)^2$
 $= \frac{25}{18} \times \frac{9}{7} \times \frac{8}{27} \times \frac{49}{25} = \frac{28}{27} = 28 \quad 27$
 - 11 We have $\frac{2x+5y}{3x+5y} = \frac{9}{10}$, $20x+50y=27x+45y$
or, $7x=5y$ or, $\frac{x}{y} = \frac{5}{7}$, $x \quad y = 5 \quad 7$
 - 12 We have $\frac{x}{y} = \frac{3}{4}$ $x = \frac{3}{4}y$
- Hence, $\frac{5x+9y}{16x+5y} = \frac{15y+9y}{12y+5y} = \frac{15+36}{4 \times 17} = \frac{51}{4 \times 17} = \frac{3}{4} = 3 \quad 4$

13 Let x and y be the required numbers

$$\text{we get } \frac{x}{y} = \frac{7}{8} \quad (1)$$

$$\text{and } x + y = 135 \quad (2)$$

From (1), $x = \frac{7}{8}y$

From (2), $y(\frac{7}{8} + 1) = 135$ or, $\frac{15}{8}y = 135$, $y = 72$

Hence $x = 135 - y = 135 - 72 = 63$

Thus the numbers are 72 and 63

14 Let x and y be the required numbers of which x is the greater

$$\text{we get } \frac{x}{y} = \frac{5}{3} \quad (1)$$

$$\text{and } x - y = 34 \quad (2)$$

From (1), $x = \frac{5}{3}y$

From (2) $y(\frac{5}{3} - 1) = 34$ or, $\frac{2}{3}y = 34$, $y = 51$

Hence, from (2), $x = 85$

Thus the numbers are 85 and 51

15 Let x and y be the required numbers, of which y is the greater

$$\text{we get } \frac{x}{y} = \frac{4}{5} \quad (1)$$

$$\text{and } \frac{x+7}{y+7} = \frac{5}{6} \quad (2)$$

From (1) we get $x = \frac{4}{5}y$

From (2) we get $6x + 42 = 5y + 35$ or, $24y + 42 = 5y$

or, $25y - 24y = 35$, $y = 35$ and $\therefore x = 28$

Thus the required numbers are 28 and 35

16 Let x and y be the required numbers, of which y is the greater

$$\therefore \text{ we get } \frac{x}{y} = \frac{7}{9} \quad (1)$$

$$\text{and } \frac{x-10}{y-10} = \frac{8}{11} \quad (2)$$

$$\text{From (1) we get } 9x - 7y = 0 \quad (3)$$

From (2) we get $11x - 110 = 8y - 80$

$$\text{or, } 11x - 8y - 33 = 0 \quad (4)$$

From (3) and (4), we get by cross multiplication

$$\frac{x}{210} = \frac{y}{270} = \frac{1}{-72+77} = \frac{1}{5}$$

$$x = 42 \text{ and } y = 54$$

Thus the required numbers are 42 and 54

17 We have $\frac{23+x}{19+x} = 2$

$$\text{or, } 23+x = 38+2x, \quad x = -15$$

18 Let x be the required number

$$\text{Then we have } \frac{25+x}{37+x} = \frac{5}{6}$$

$$\text{or, } 150+6x = 185+5x, \quad x = 35$$

19 Let x be the required number

$$\text{Then we have } \frac{29+x}{38+x} = \frac{4}{7}$$

$$\text{or, } 203+7x = 152+4x \text{ or, } 3x = -51, \quad x = -71$$

20 Let x be the required quantity

$$\text{Then we have } \frac{a+x}{b+x} = \frac{c}{d}$$

$$\text{or, } ad+dx = bc+cx \text{ or, } \cancel{ax} - dx = ad-bc,$$

$$x = \frac{ad-bc}{c-d}$$

21 We have $\frac{a^2-x^2}{a^2+x^2} = \frac{a-x}{a+x}$

$$= (a-x) \left\{ \frac{(a+x)^2 - (a^2+x^2)}{(a^2+x^2)(a+x)} \right\} = \frac{(a-x) 2ax}{(a^2+x^2)(a+x)}$$

Now $(a-x)$ is positive, a being greater than x ,

the difference of the ratios is positive and hence the former is greater than the latter

22 We have $\frac{a^2+b^2}{a+b} - \frac{a^2-b^2}{a-b}$

$$= \frac{a^2+b^2}{a+b} \cdot \frac{a-b}{a+b} = \frac{a^2+b^2-a^2-b^2-2ab}{a+b} = \frac{-2ab}{a+b},$$

which is negative, a and b both being positive

$$\text{Hence } \frac{a^2+b^2}{a+b} < \frac{a^2-b^2}{a-b}$$

$$\begin{aligned}
 23 \quad (226)^3 - (225)^3 &= \left(\frac{226}{225}\right)^3 = \left\{1 + \frac{1}{225}\right\}^3 = 1 + \frac{3}{225} + \text{etc} \\
 &= 1 + \frac{1}{75} \quad (\text{neglecting the other terms which are very small}) \\
 &= \frac{76}{75}
 \end{aligned}$$

$$\begin{aligned}
 24 \quad \sqrt{3546} - \sqrt{3542} &= \left(\frac{3546}{3542}\right)^{\frac{1}{2}} = \left(\frac{1773}{1771}\right)^{\frac{1}{2}} = \frac{(1773 \times 1771)^{\frac{1}{2}}}{1771} \\
 &= \frac{\{(1772+1)(1772-1)\}^{\frac{1}{2}}}{1771} = \frac{\{(1772)^2 - 1\}^{\frac{1}{2}}}{1771} = \frac{1772}{1771} \quad (\text{approximately})
 \end{aligned}$$

$$25 \quad \text{The ratio of A's expenditure to his income} = \frac{8\frac{1}{2}}{15} \quad (1)$$

$$B's \quad \quad \quad = \frac{11\frac{1}{2}}{20} \quad (2)$$

$$C's \quad \quad \quad = \frac{15\frac{5}{8}}{25} \quad (3)$$

The ratios are $\frac{35}{4 \times 15}$, $\frac{45}{4 \times 20}$ and $\frac{125}{8 \times 25}$

or $\frac{7}{12}$, $\frac{9}{16}$ and $\frac{5}{8}$ respectively

These ratios are again equivalent to

$\frac{28}{48}$, $\frac{27}{48}$ and $\frac{30}{48}$ respectively

The second ratio being the least B is the most frugal

EXERCISE (94)

- 1 Let x be the required quantity

$$\text{We get } 9 - 6 = 6 - x \text{ or, } 9x = 6^2 = 36, \quad x = 4$$

Thus the required quantity is 4

- 2 Let x be the required quantity

$$\text{Then we have } 8 - 12 = 12 - x \text{ or, } 8x = 144, \quad x = 18$$

Thus the required quantity is 18

- 3 Let x be the required quantity

$$\text{Then we have } 6 - 15 = 15 - x$$

$$\text{or, } 6x = (15)^2 = 225, \quad x = 37\frac{1}{2}$$

Thus the required quantity is $37\frac{1}{2}$

- 4 Let x be the required quantity
Then we have $16 \quad 24 = 24 \quad x$
or, $16x = (24)^2 = 576$, $x = 36$
Thus 36 is the required quantity
- 5 Let x be the required quantity
Then we have $6 \quad 8 = 15 \quad x$ or, $6x = 120$, $x = 20$
Thus 20 is the required quantity
- 6 Let x be required quantity
Then we have $14 \quad 24 = 35 \quad x$ or, $14x = 24 \times 35$, $x = 60$
Thus 60 is the required quantity
- 7 Let x be the required quantity
Then we have $0014 \quad 14 = 02 \quad x$
or, $0014x = 14 \times 02$ or, $14x = 14 \times 20$, $x = 20$
Thus 20 is the required quantity
- 8 Let x be the required quantity
Then we have $4 \quad x = x \quad 9$ or, $x^2 = 36$, $x = 6$
Thus 6 is the required quantity
- 9 Let x be the required number
Then we have $7 \quad x = x \quad 28$ or, $x^2 = 7 \times 28$ $\therefore x = 14$
Thus 14 is the required number
- 10 Let x be the required number
Then we have $6 \quad x = x \quad 54$ or, $x^2 = 6 \times 54$, $x = 18$
Thus 18 is the required number

Exercise (95)

- 1 From (1), by componendo and dividendo,
we get $\frac{x}{y} = \frac{6}{4} = \frac{3}{2}$ or, $2x = 3y$ (3)
From (2), we get $4x = 36$ $x = 9$,
from (3) $3y = 18$ $y = 6$
Thus $x = 9$ and $y = 6$
- 2 From (1), by Componendo and dividendo,
we have $\frac{3x}{-5y} = \frac{5}{-3}$ or, $9x = 25y$ (3)
 $36x - 100y = 0$,

and from (2) $36x - 81y = 171$

Hence, $-19y = -171$, $y = 9$,

Hence, from (3), $x = 25$

Thus $x = 25$ and $y = 9$

3 From (1) by componendo and dividendo,

we have $\frac{5x}{-7y} = \frac{8}{-6} = \frac{4}{-3}$ or, $15x = 28y$ (3)

$$15x - 28y = 0$$

and from (2), $15x - 25y = 90$

$$-3y = -90, \quad y = 30$$

from (3), $15x = 28 \times 30$, $x = 56$

Thus $x = 56$ and $y = 30$

4 $16\left(\frac{a-x}{a+x}\right)^3 = \frac{a+1}{a-x}$

or, $16\left(\frac{a-x}{a+x}\right)^4 = 1$

or, $\left(\frac{a-x}{a+x}\right)^4 = \frac{1}{16} = \left(\frac{1}{2}\right)^4$

or, $\frac{a-x}{a+x} = \frac{1}{2}$, or, $2a - 2x = a + 1$

or, $a = 3x$, $x = \frac{a}{3}$

5 $\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$

By componendo and dividendo, we get

$$\frac{2x}{\sqrt{4x^2 - 1}} = \frac{5}{3}, \quad \text{or,} \quad \frac{4x^2}{4x^2 - 1} = \frac{25}{9}$$

or, $36x^2 = 100x^2 - 25$, or, $64x^2 = 25$

or, $8x = 5$, $x = \frac{5}{8}$

6 By componendo and dividendo $\frac{1}{-\sqrt{1-x}} = \frac{4}{-2} = -2$

or, $1 = 2\sqrt{1-x}$ or, $1 = 4(1-x)$ or, $4x = 3$,
 $x = \frac{3}{4}$

7 By componendo and dividendo, $\frac{\sqrt{36x+1}}{\sqrt{36x}} = \frac{10}{8} = \frac{5}{4}$

Hence, squaring both sides, $\frac{36x+1}{36x} = \frac{25}{16}$

or, $16 \ 36x + 16 = 25 \ 36x$

or, $9 \ 36x = 16$, $x = \frac{16}{9 \ 36} = \frac{4}{81}$

8 By componendo and dividendo, $\frac{1+x^2}{x} = \frac{125-x}{125x-1}$

or, $125x + 125x^3 - 1 - x^3 = 125x - x^3$

or, $125x^3 - 1 = 0$, or, $(5x)^3 = 1$, or, $5x = 1$, $x = \frac{1}{5}$

9 By componendo and dividendo, $\frac{\sqrt{5}}{\sqrt{5-x}} = \frac{6-\frac{3}{2}}{4} = \frac{3}{2}$

Hence, squaring both sides,

$\frac{5}{5-x} = \frac{9}{4}$, or, $20 = 45 - 9x$, or, $9x = 25$,

$x = \frac{25}{9} = 2\frac{7}{9}$

10 By componendo and dividendo, $\frac{a+x}{\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$

Hence, squaring both sides,

$$\frac{(a+x)^2}{a^2-x^2} = \frac{(b+x)^2}{(b-x)^2}$$

or, $\frac{a+x}{a-x} = \frac{(b+x)^2}{(b-x)^2}$

Again, by componendo and dividendo

$$\frac{a}{x} = \frac{(b+x)^2 + (b-x)^2}{(b+x)^2 - (b-x)^2} = \frac{b^2 + x^2}{2bx}$$

or, $2ab = b^2 + x^2$, or, $x^2 = 2ab - b^2$

$$x = \sqrt{2ab - b^2}$$

11 By componendo and dividendo,

$$\frac{a^{\frac{1}{2}}}{-\{a - (a^2 - ax)^{\frac{1}{2}}\}^{\frac{1}{2}}} = \frac{b+1}{b-1}$$

Hence, squaring both sides,

$$\frac{a}{a - (a^2 - ax)^{\frac{1}{2}}} = \frac{(b+1)^2}{(b-1)^2}$$

or, $a(b-1)^2 = a(b+1)^2 - (b+1)^2(a^2 - ax)^{\frac{1}{2}}$

$$\text{or, } a\{(b+1)^2 - (b-1)^2\} = (b+1)^2(a^2 - ax)^2$$

$$\text{or, } (b+1)^2(a^2 - ax)^2 = 4ab$$

$$\text{or, } (b+1)^4(a^2 - ax) = 16a^2b^2$$

$$\text{or, } a - x = \frac{16ab^2}{(b+1)^4}$$

$$x = a - \frac{16ab^2}{(b+1)^4} = a \left\{ 1 - \frac{16b^2}{(b+1)^4} \right\}$$

12 We have

$$\frac{a+3b+2c+6d}{a+3b-2c-6d} = \frac{a-3b+2c-6d}{a-3b+2c-6d}$$

Hence, by componendo and dividendo,

$$\frac{a+3b}{2c+6d} = \frac{a-3b}{2c-6d}$$

$$\text{or, } \frac{a+3b}{a-3b} = \frac{2c+6d}{2c-6d} = \frac{c+3d}{c-3d} \quad (\text{by alternendo})$$

Again, by componendo and dividendo,

$$\frac{a}{3b} = \frac{c}{3d}, \quad \frac{a}{b} = \frac{c}{d} \quad \text{or, } a \quad b \quad c \quad d$$

13 We have

$$\frac{2a+b+4c+2d}{2a+b-4c-2d} = \frac{2a-b+4c-2d}{2a-b-4c+2d}$$

By componendo and dividendo,

$$\frac{2a+b}{4c+2d} = \frac{2a-b}{4c-2d}$$

$$\text{or, } \frac{2a+b}{2a-b} = \frac{4c+2d}{4c-2d} = \frac{2c+d}{2c-d} \quad (\text{alternendo})$$

Again, by componendo and dividendo,

$$\frac{2a}{b} = \frac{2c}{d} \quad \text{or, } a \quad b \quad c \quad d$$

14 We have

$$x = \frac{\sqrt{2a+3b} - \sqrt{2a-3b}}{\sqrt{2a+3b} + \sqrt{2a-3b}}$$

By componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}, \quad \frac{(x+1)^2}{(x-1)^2} = \frac{2a+3b}{2a-3b}$$

Again, by componendo and dividendo.

$$\frac{(x+1)^2 - (x-1)^2}{(x+1)^2 + (x-1)^2} = \frac{2a}{3b}$$

$$\text{or, } \frac{2x^2 - 2}{2x} = \frac{2a}{3b} \quad \text{or} \quad \frac{x^2 - 1}{x} = \frac{2a}{3b}$$

$$\text{or } 3^2 x^2 - 3b = 2ax \quad \therefore 3^2 x^2 - 2ax - 3b = 0$$

15 We have $x = \frac{2\sqrt{21}}{\sqrt{2}-\sqrt{3}} = \frac{2\sqrt{3}\sqrt{8}}{\sqrt{2}-\sqrt{3}} \quad \frac{x}{\sqrt{8}} = \frac{2\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

Hence, by componendo and dividendo

$$\frac{x - \sqrt{8}}{x + \sqrt{8}} = \frac{3\sqrt{3} - 2}{3\sqrt{3} + 2}$$

$$\text{Again } x = \frac{2\sqrt{12}\sqrt{12}}{\sqrt{2}-\sqrt{3}} \quad \text{or} \quad \frac{x}{\sqrt{12}} = \frac{2\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

Hence, by componendo and dividendo $\frac{x - \sqrt{12}}{x + \sqrt{12}} = \frac{3\sqrt{2} - \sqrt{3}}{3\sqrt{2} + \sqrt{3}};$
the given expression

5 We have $a^2 + c^2 + e^2 = x^2(b^2 + d^2 + f^2)$,
the given expression $= (x^2)^{\frac{1}{2}} = x$ = each of the given ratios

6 We have $a^3 - 2c^3 + 3e^3 = x^3(b^3 - 2d^3 + 3f^3)$,
the given expression $= (x^3)^{\frac{1}{3}} = x$ = each of the given ratios

7 We have $\sqrt[3]{a^3 + c^3 + e^3} = \sqrt[3]{x^3(b^3 + d^3 + f^3)} = x\sqrt[3]{b^3 + d^3 + f^3}$,
the given expression $= x$ = each of the given expressions

In each of the following 3 examples, let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = k$,

$$a = bk, c = dk, e = fk \text{ and } g = hk$$

8 We have $a^{-1} + c^{-1} + e^{-1} + g^{-1} = k^{-1}(b^{-1} + d^{-1} + f^{-1} + h^{-1})$,
the given expression $= (k^{-1})^{-1} = k$ = each of the given ratios

9 We have $a^4 - 2c^4 + 3e^4 - 4g^4 = k^4(b^4 - 2d^4 + 3f^4 - 4h^4)$,
the given expression $= \sqrt[4]{k^4} = k$ = each of the given ratios

10 We have $3a^{-2} - 7c^{-2} - 8e^{-2} + 15g^{-2}$
 $= k^{-2}(3b^{-2} - 7d^{-2} - 8f^{-2} + 15h^{-2})$
the given expression $= \sqrt[2]{(k^{-2})^{-1}} = \sqrt[2]{k^2} = k$
= each of the given ratios

Exercise (97)

1 We have $\frac{a}{b} = \frac{c}{d}$ (1)
 $\frac{a}{c} = \frac{b}{d}$ (2) (by alternendo)

Both $\frac{a}{b}$ and $\frac{a}{c}$ are ratios of greater inequality for a is greater than both b and c . Hence $\frac{c}{d}$ and $\frac{b}{d}$ also are ratios of greater inequality, b and c are each $> d$

2 Because $\frac{a}{b} = \frac{c}{d}$
 $\frac{a-b}{b} = \frac{c-d}{d}$
or, $\frac{a-b}{c-d} = \frac{b}{d}$

$$\text{or, } \frac{a+c}{b+d} = \frac{c}{d} = \frac{a}{b}$$

$$\therefore c, \quad a \cdot b = a+c \cdot b+d$$

$$7 \quad \frac{a}{b} = \frac{c}{d}$$

$$\text{or, } \frac{a}{c} = \frac{b}{d}$$

$$\text{or, } \frac{a^2}{c^2} = \frac{b^2}{d^2}$$

$$\text{or, } \frac{a^2+c^2}{c^2} = \frac{b^2+d^2}{d^2}$$

$$\text{or, } \frac{a^2+c^2}{b^2+d^2} = \frac{c^2}{d^2} = \frac{a^2}{b^2}$$

$$\therefore c, \quad a^2 \cdot b^2 = a^2+c^2 \cdot b^2+d^2$$

$$8 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose) ,}$$

$$a=bk \text{ and } c=dk ,$$

$$\frac{a^2+c^2}{b^2+d^2} = \frac{b^2k^2+d^2k^2}{b^2+d^2} = k^2$$

$$= k \cdot k = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$9 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$a=bk \text{ and } c=dk,$$

$$\frac{(a-c)^2}{(b-d)^2} = \frac{(bk-dk)^2}{(b-d)^2} = 1^2 = \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \quad \sim$$

$$10 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$a=bk \text{ and } c=dk$$

$$(a+c)^3 = k^3(b+d)^3$$

$$\frac{(a+c)^3}{(b+d)^3} = k^3 \quad (1)$$

$$\text{Also, we have } a(a-c)^2 = bk(bk-dk)^2 = k^3\{b(b-d)^2\}$$

$$\frac{a(a-c)^2}{b(b-d)^2} = k^3 \quad (2)$$

Hence, from (1) and (2),

$$(a+c)^3 \cdot (b+d)^3 = a(a-c)^2 \cdot b(b-d)^2$$

$$11 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$a = bk \text{ and } c = dk,$$

$$\frac{a^2 + b^2}{a^2 - b^2} = \frac{b^2 k^2 + b^2}{b^2 k^2 - b^2} = \frac{k^2 + 1}{k^2 - 1} \quad (1)$$

Also, we have

$$\frac{ac + bd}{ac - bd} = \frac{k^2 bd + bd}{k^2 bd - bd} = \frac{k^2 + 1}{k^2 - 1} \quad (2)$$

Hence, from (1) and (2),

$$a^2 + b^2 \quad a^2 - b^2 = ac + bd \quad ac - bd$$

$$12 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$\therefore a = bk \text{ and } c = dk,$$

$$\frac{a(a+c)}{c^2} = \frac{bk(bk+dk)}{d^2 k^2} = \frac{b(b+d)}{d^2}$$

$$13 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$a = bk \text{ and } c = dk,$$

$$\frac{c}{d} = \frac{dk}{d} = k \quad (1)$$

$$\text{Also, we have } \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}} = \frac{\sqrt{k^2 b^2 + k^2 d^2}}{\sqrt{b^2 + d^2}} = k \quad (2)$$

Hence, from (1) and (2),

$$c \quad d = \sqrt{a^2 + c^2} \quad \sqrt{b^2 + d^2}$$

$$14 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),}$$

$$a = bk \text{ and } c = dk,$$

$$\frac{a+b}{c+d} = \frac{b(k+1)}{d(k+1)} = \frac{b}{d} \quad (1)$$

$$\text{Also, } \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}} = \frac{\sqrt{b^2(k^2 + 1)}}{\sqrt{d^2(k^2 + 1)}} = \sqrt{\frac{b^2}{d^2}} = \frac{b}{d} \quad (2)$$

Hence, from (1) and (2),

$$a+b \quad c+d = \sqrt{a^2 + b^2} \quad \sqrt{c^2 + d^2}$$

$$\begin{aligned}
 15 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),} \\
 a = bk \text{ and } c = dk, \\
 \frac{a+b}{c+d} = \frac{bk+b}{dk+d} = \frac{b}{d} \quad (1)
 \end{aligned}$$

$$\text{Also, } \frac{\sqrt{3a^2+5b^2}}{\sqrt{3c^2+5d^2}} = \frac{\sqrt{3b^2k^2+5b^2}}{\sqrt{3d^2k^2+5d^2}} = \frac{b\sqrt{3k^2+5}}{d\sqrt{3k^2+5}} = \frac{b}{d} \quad (2)$$

Hence, from (1) and (2),

$$a+b : c+d = \sqrt{3a^2+5b^2} : \sqrt{3c^2+5d^2}$$

$$\begin{aligned}
 16 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),} \\
 a = bk \text{ and } c = dk, \\
 \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{b^2k^2+kb^2+b^2}{b^2k^2-b^2k+b^2} = \frac{k^2+k+1}{k^2-k+1} \quad (1)
 \end{aligned}$$

$$\text{Also, } \frac{c^2+cd+d^2}{c^2-cd+d^2} = \frac{d^2k^2+kd^2+d^2}{d^2k^2-d^2k+d^2} = \frac{k^2+k+1}{k^2-k+1} \quad (2)$$

Hence, from (1) and (2),

$$a^2+ab+b^2 : a^2-ab+b^2 = c^2+cd+d^2 : c^2-cd+d^2$$

$$\begin{aligned}
 17 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),} \\
 a = bk \text{ and } c = dk, \\
 \frac{a^2+ac+c^2}{a^2-ac+c^2} = \frac{b^2k^2+bdk^2+d^2k^2}{b^2k^2-bdk^2+d^2k^2} = \frac{b^2+bd+d^2}{b^2-bd+d^2}
 \end{aligned}$$

$$\begin{aligned}
 18 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),} \\
 a = bk \text{ and } c = dk, \\
 \frac{ma+nb}{mc+nd} = \frac{mkb+nb}{mkd+nd} = \frac{b(mk+n)}{d(mk+n)} = \frac{b}{d} \quad (1)
 \end{aligned}$$

$$\text{Also, } \frac{b^2c}{d^2a} = \frac{b^2dk}{d^2bk} = \frac{b}{d} \quad (2)$$

Hence, from (1) and (2),

$$ma+nb : mc+nd = b^2c : d^2a$$

$$\begin{aligned}
 19 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k, \quad a = bk, c = dk \text{ and } e = fk, \\
 \frac{ac}{bd} = \frac{bdk^2}{bd} = k^2 \quad (1)
 \end{aligned}$$

$$\text{Also, } \frac{2a^2 + 3c^2 + 5e^2}{2b^2 - 3d^2 + 5f^2} = \frac{2b^2k^2 - 3d^2f^2 - 5f^2k^2}{2b^2 - 3d^2 - 5f^2} = k^2. \quad (2)$$

Hence, from (1) and (2)

$$ac + bd = 2a^2 - 3c^2 - 5e^2 \quad 2b^2 - 3d^2 - 5f^2$$

$$20 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose),}$$

$$a = bk, c = dk \text{ and } e = fk,$$

$$\frac{a^2 - c^2 + e^2}{b^2 + d^2 - f^2} = \frac{b^2k^2 - d^2k^2 + f^2k^2}{b^2 + d^2 - f^2} = k^2 \quad (1)$$

$$\text{Also, } \frac{ce}{af} = \frac{k^2af}{df} = k^2 \quad \dots (2)$$

Hence, from (1) and (2),

$$a^2 - c^2 - e^2 \quad b^2 + d^2 - f^2 = ce : df$$

$$21 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose)}$$

$$\dots \quad a = bk, c = dk \text{ and } e = fk,$$

$$\frac{pa - qc + re}{pb - qd + rf} = \frac{pkb + qkd - rkf}{pb - qd + rf} = k \quad (1)$$

$$\text{Also, } \frac{\sqrt[3]{ace}}{\sqrt[3]{baf}} = \frac{\sqrt[3]{k^3odf}}{\sqrt[3]{baf}} = k \quad \dots (2)$$

Hence, from (1) and (2),

$$pa + qc - re \quad pb + qd - rf = \sqrt[3]{ace} \quad \sqrt[3]{bdf}$$

$$22 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose),}$$

$$a = bk, c = dk \text{ and } e = fk$$

$$\frac{a^2}{b^2} = \frac{b^2k^2}{b^2} = k^2 \quad (1)$$

$$\text{Also } \frac{ac + ce - ae}{bd - df + bf} = \frac{k^2bd - k^2df - k^2bf}{bd - df + bf} = k^2 \quad (2)$$

Hence, from (1) and (2),

$$a^2 : b^2 = ac + ce - ae \quad bd + df + bf.$$

$$23 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose),}$$

$$a = bk, c = dk \text{ and } e = fk,$$

$$\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2} = \frac{k^2b^2 + k^2d^2 + k^2f^2}{b^2 + d^2 + f^2} = k^2 \quad (1)$$

$$\text{Also, } \frac{acc}{bdf} = \frac{kb \cdot kd \cdot kf}{bdf} = k^2 \quad (2)$$

Hence, from (1) and (2),

$$a^2 + c^2 + e^2 = b^2 + d^2 + f^2 = acc = bdf$$

$$\begin{aligned} 24 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = l \text{ (suppose) ,} \\ a = bl, c = dl \text{ and } e = fl, \\ \sqrt{\frac{a^2c^2 + c^2e^2 + a^2e^2}{b^2d^2 + d^2f^2 + b^2f^2}} = \frac{\sqrt{l^2b^2k^2d^2 + k^2d^2l^2f^2 + l^2b^2k^2f^2}}{\sqrt{b^2d^2 + d^2f^2 + b^2f^2}}, \\ = \sqrt{l^2k^2} = k^2 \end{aligned} \quad (1)$$

$$\text{Also, } \frac{acc}{bdf} = \frac{kb \cdot kd \cdot kf}{bdf} = k^2 \quad (2)$$

Hence, from (1) and (2),

$$\sqrt{a^2c^2 + c^2e^2 + a^2e^2} = \sqrt{b^2d^2 + d^2f^2 + b^2f^2} = acc = bdf$$

$$\begin{aligned} 25 \quad \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = k \text{ (suppose) ,} \\ \frac{a}{c} = \frac{a}{b} \cdot \frac{b}{c} = \frac{b}{d} = k^2 \\ \frac{c}{e} = \frac{c}{d} \cdot \frac{d}{e} = \frac{d}{e} = k^2 \end{aligned} \quad (1)$$

$$\text{Also, } \frac{a^4}{b^4} = l^4 \quad (2)$$

Hence from (1) and (2)

$$a = c = a^4 = b^4$$

$$\begin{aligned} 26 \quad \frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = l \text{ (suppose) ,} \\ x = l(b+c-a), y = l(c+a-b) \text{ and } z = l(a+b-c), \\ x(b-c) + y(c-a) + z(a-b) \\ = l\{b(b-c) + c(b+c-a) + l(c-a)(c+a-b) + l(a-b)(a+b-c)\} \\ = l\{b^2 - c^2 - a(b-c) + c^2 - a^2 - b(c-a) + a^2 - b^2 - c(a-b)\} \\ = -l\{a(b-c) + b(c-a) + c(a-b)\} \\ = 0 \end{aligned}$$

$$\begin{aligned} 27 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose) } \quad a = bk \text{ and } c = dk, \\ \frac{a^2 + c^2}{b^2 + d^2} = \frac{b^2k^2 + d^2k^2}{b^2 + d^2} = k^2 \end{aligned} \quad (1)$$

$$\text{Also, } \frac{\sqrt{b^4 + c^4}}{\sqrt{b^4 + d^4}} = \frac{\sqrt{b^2 k^4 + d^2 k^4}}{\sqrt{b^2 + d^2}} = k^2 \quad (2)$$

Hence, from (1) and (2),

$$a^2 + c^2 \quad b^2 + d^2 = \sqrt{a^2 + c^2} \quad \sqrt{b^2 + d^2}$$

$$28 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose)}, \quad a = bk, \quad c = dk \text{ and } e = fk,$$

$$\begin{aligned} & 27(a + b)(c + d)(e + f) \\ &= 27(bk + b)(dk + d)(fk + f) = 27bdf(k + 1)^3 \end{aligned} \quad (1)$$

$$\begin{aligned} \text{Also, } bdf \left(\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f} \right)^3 \\ &= bdf \left(\frac{a}{b} + 1 + \frac{c}{d} + 1 + \frac{e}{f} + 1 \right)^3 \\ &= bdf (3k + 3)^3 = 27 bdf (k + 1)^3 \end{aligned} \quad (2)$$

Hence, from (1) and (2),

$$27(a + b)(c + d)(e + f) = bdf \left(\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f} \right)^3$$

$$29 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose)}, \quad a = bk \text{ and } c = dk,$$

$$\frac{ad + bc}{2bd} = \frac{kbd + kbd}{2bd} = k \quad (1)$$

$$\text{Also, } \frac{a^2 + c^2}{ab + cd} = \frac{k^2 b^2 + k^2 d^2}{kb^2 + kd^2} = k \quad (2)$$

Hence, from (1) and (2),

$$ad + bc \quad 2bd = a^2 + c^2 \quad ab + cd$$

$$30 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose)}, \quad a = bk \text{ and } c = dk,$$

$$\begin{aligned} \frac{a^2 + b^2}{ab + ad - bc} &= \frac{b^2 k^2 + b^2}{kb^2 + b^2 - kbd} = \frac{b^2(k^2 + 1)}{b^2 k} \\ &= \frac{k^2 + 1}{k} \end{aligned} \quad (1)$$

$$\text{Also, } \frac{c^2 + d^2}{cd - ad + bc} = \frac{k^2 d^2 + d^2}{kd^2 - kbd + kbd} = \frac{d^2(k^2 + 1)}{d^2 k} = \frac{k^2 + 1}{k} \quad (2)$$

Hence, from (1) and (2),

$$a^2 + b^2 \quad ab + ad - bc = c^2 + d^2 \quad cd - ad + bc$$

$$31 \quad \frac{a}{b} = \frac{b}{c} = k \text{ (suppose)} \quad a = b/k \text{ and } b = kc, \quad a = c/k^2$$

$$\frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{1^2 c^2 + 1^2 c^2 + k^2 c^2}{1^2 c^2 + 1^2 c^2 + c^2} = \frac{1^2 + k^2 + k^2}{1^2 + 1 + 1} = \frac{k^2(k^2 + 1 + 1)}{k^2 + k + 1}$$

$$= k^2 \quad (1)$$

$$\text{Also, } \frac{a}{c} = \frac{c/k^2}{c} = k^2 \quad (2)$$

$$\text{Hence, from (1) and (2), } a^2 + ab + b^2 = b^2 + bc + c^2 = a^2 + c^2$$

$$32 \quad \frac{1}{b} = \frac{b}{c} = l \text{ (suppose)} \quad a = b/l \text{ and } b = lc, \quad a = l^2 c$$

$$a - 2b + c = l^2 c - 2lc + c = c(l^2 - 2l + 1) = c(l - 1)^2 \quad (1)$$

$$\text{Also } \frac{(a - b)^2}{a} = \frac{(l^2 c - lc)^2}{l^2 c} = \frac{l^2 c^2 (l - 1)^2}{l^2 c} = c(l - 1)^2 \quad (2)$$

$$\text{Also } \frac{(b - c)^2}{c} = \frac{(lc - c)^2}{c} = \frac{c^2 (l - 1)^2}{c} = c(l - 1)^2 \quad (3)$$

$$\text{Hence from (1), (2), and (3)}$$

$$a - 2b + c = \frac{(a - b)^2}{a} = \frac{(b - c)^2}{c}$$

$$33 \quad \frac{a}{b} = \frac{b}{c} = l \text{ (suppose)} \quad a = bl \text{ and } b = cl$$

$$a = l^2 c$$

$$a^2 b^2 c^2 \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right) = \frac{b^2 c^2}{a} + \frac{a^2 c^2}{b} + \frac{a^2 b^2}{c}$$

$$\frac{l^2 c^4}{l^2 c} + \frac{l^4 c^4}{lc} + \frac{l^4 c^2 l^2 c^2}{c} = c^3 + l^2 c^3 + l^6 c^3$$

$$= c^3 + (lc)^3 + (l^2 c)^3 = a^3 + b^3 + c^3$$

$$34 \quad \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k \text{ (suppose)}, \quad a = bk, b = cl \text{ and } c = dk$$

$$a = l^2 c = k^2 d \text{ and } b = k^2 d$$

$$(b + c)(b + d) = (k^2 d + l d)(l^2 d + d)$$

$$= l d^2 (k + 1)(l^2 + 1), \quad (1)$$

$$\text{Also, } (c + a)(c + d) = (dl + l^2 d)(al + d)$$

$$= k d^2 (k^2 + 1)(k + 1) \quad (2)$$

$$\text{Hence, from (1) and (2), } (b + c)(b + d) = (c + a)(c + d)$$

$$35 \quad \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = r \text{ (suppose)}$$

$$a = rk \quad b = ck \text{ and } c = dk$$

$$\therefore a = r^2c = r^2a \text{ and } b = k^2d,$$

$$\begin{aligned} & (a+b)(b+c) - (a+c)(b+a) \\ &= (rk^2d + d)(r^2d + rk^2d) - (r^2d + rk^2d)(rk^2d + d) \\ &= rk^2d^2(r^2 + 1)(k^2 - 1) - rk^2d^2(r^2 - 1)k^2 \\ &= rk^2d^2(k^2 - k^2 - k^2 - 1 - r^4 - 2k^2 - 1) \\ &= rk^2d^2(r^2 - k^2 - 2r - 1) \\ &= k^2d^2(r^2 - 2r - 1) \\ &= c^2(k-1)^2 = (a-b-c)^2 = b-c^2 \end{aligned}$$

$$36 \quad \text{As in the last sum } c = rk^2d \quad b = r^2d \text{ and } a = k^2a$$

$$\begin{aligned} \therefore \left(\frac{a-b}{c} - \frac{a-c}{b} \right)^2 &= \left(\frac{a-c}{c} - \frac{d-c}{b} \right)^2 \\ &= \left(\frac{rk^2d - r^2d}{rk^2d} - \frac{r^2d - r^2d}{rk^2d} \right)^2 = \left(\frac{c - r^2a}{rk^2d} - \frac{c - r^2a}{rk^2d} \right)^2 \\ &= \left(k^2 - k - \frac{k^2 - 1}{k} \right)^2 = \left(\frac{1 - k^2}{k} - \frac{1 - k}{k^2} \right)^2 \\ &= \left(\frac{k^2 - k^2 - k^2 - 1}{k} \right)^2 = \left(\frac{k^2 - k^2 - 1 - k}{k^2} \right)^2 \\ &= \left(\frac{k^2 - 1}{k} \right)^2 = \left(\frac{k^2 - 1}{k^2} \right)^2 \\ &= (k^2 - 1)^2 \left(\frac{1}{k^2} - \frac{1}{k^4} \right) \quad \dots \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Also } (a-b)^2 \left(\frac{1}{c^2} - \frac{1}{b^2} \right) &= (rk^2d - r^2d)^2 \left(\frac{1}{r^2k^4d^2} - \frac{1}{r^4d^2} \right) \\ &= d^2(k^2 - 1)^2 \left(\frac{1}{k^2} - \frac{1}{k^4} \right) \times \frac{1}{d^2} = (k^2 - 1)^2 \left(\frac{1}{k^2} - \frac{1}{k^4} \right) \quad (2) \end{aligned}$$

Hence, from (1) and (2), the expressions are equal.

$$37. \quad \text{We have as in the last sum } a = k^2c \quad b = r^2d \text{ and } c = rd$$

$$\therefore \frac{a}{b} = \frac{k^2d}{r^2a} = k \quad \dots \quad (1)$$

$$\text{also, } \frac{\frac{1}{k^2d} - \frac{1}{rd} - \frac{1}{d}}{\frac{1}{r^2c} - \frac{1}{r^2d} - \frac{1}{rc}} = \frac{\frac{1}{k^2} - \frac{1}{r} - 1}{k \left(\frac{1}{k^2} - \frac{1}{r} - 1 \right)} = k \quad (2)$$

Hence, from (1) and (2), the expressions are equal

38 We have $a = k^2d$, $b = k^2d$ and $c = kd$

$$\frac{a}{d} = \frac{k^2d}{d} = k^2 \quad (1)$$

$$\text{Also } \frac{a^2 + b^2 + c^2}{b^2 + c^2 + d^2} = \frac{k^4d^2 + k^4d^2 + k^2d^2}{k^4d^2 + k^2d^2 + d^2} = \frac{k^2(k^4 + k^2 + 1)}{k^2 + k^2 + 1} = k^2 \quad (2)$$

Hence, from (1) and (2) the expressions are equal

39 $\frac{a}{b} = \frac{c}{d} = k$ (suppose), $a = bk$ and $c = dk$

$$\frac{a^2 + ab}{c^2 + cd} = \frac{b^2k^2 + b^2k}{d^2k^2 + d^2k} = \frac{b^2(k^2 + k)}{d^2(k^2 + k)} = \frac{b^2}{d^2} \quad (1)$$

$$\text{Also, } \frac{b^2 - 2ab}{d^2 - 2cd} = \frac{b^2 - 2b^2k}{d^2 - 2d^2k} = \frac{b^2(1 - 2k)}{d^2(1 - 2k)} = \frac{b^2}{d^2} \quad (2)$$

Hence from (1) and (2), $a^2 + ab$ $c^2 + cd = b^2 - 2ab$ $d^2 - 2cd$

40 $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ (suppose) $a = bk$, $c = dk$ and $e = fk$,

$$\begin{aligned} (a^2 + b^2)(ce + df)^2 &= (b^2k^2 + b^2)(dfk^2 + df)^2 \\ &= b^2k^2 + 1)(f^2 + 1)d^2f^2 = d^2f^2b^2(k^2 + 1)^2 \end{aligned} \quad (1)$$

$$(c^2 + d^2)(ae + bf)^2 = (d^2k^2 + d^2)(bfk^2 + bf)^2 = b^2f^2d^2(k^2 + 1)^2 \quad (2)$$

$$\text{and } (e^2 + f^2)(ac + bd)^2 = f^2f^2 + f^2)(fdk^2 + bd)^2 = b^2f^2d^2(k^2 + 1)^2 \quad (3)$$

Hence, from (1), (2) and (3), the expressions are equal

Exercise (98)

1 From the 1st equation we have $a^2x^2 = b^2$
and from the second equation, $c^2x^2 = a^2$,

$$\frac{a^2}{c^2} = \frac{b^2}{a^2}$$

Thus $a^4d^2 = b^4c^2$ is the required Eliminant

2 From the 1st equation we have

$$ax^2 = b \quad \text{or, } x^2 = \frac{b}{a}, \quad \text{or, } x^6 = \frac{b^3}{a^3},$$

and from the 2nd equation,

$$\begin{aligned} cx^2 &= d, \quad \text{or, } x^2 = \frac{d}{c}, \quad \text{or, } x^6 = \frac{d^3}{c^3}, \\ \frac{b^3}{a^3} &= \frac{d^3}{c^3} \end{aligned}$$

Thus $b^3c^2 = a^3d^2$ is the required Eliminant

- 3 From the 1st equation, we have

$$mx^3 = n \quad \text{or,} \quad x^3 = \frac{n}{m} \quad \text{or,} \quad x^{12} = \frac{n^4}{m^4},$$

and from the 2nd equation,

$$px^4 = q \quad \text{or,} \quad x^4 = \frac{q}{p} \quad \text{or,} \quad x^{12} = \frac{q^3}{p^3}, \quad \frac{n^4}{m^4} = \frac{q^3}{p^3}$$

Thus $n^4 p^3 = m^4 q^3$ is the required Eliminant

- 4 We have by cross multiplication,

$$\frac{x^2}{bd-c} = \frac{x}{-ad} = \frac{1}{a}, \quad \frac{x^2}{a(bd-c)} = \frac{x^2}{a^2 d^2}, \quad bd-c = ad^2$$

Thus $ad^3 - bd + c = 0$ is the required Eliminant

- 5 By cross multiplication, we have

$$\frac{x^2}{mb-an} = \frac{x}{-lb} = \frac{1}{al} \quad \frac{x^2}{al(mb-an)} = \frac{x^2}{l^2 b^2},$$

$$a(mb-an) = lb^2$$

Thus $lb^2 - amb + a^2 n = 0$ is the required Eliminant

- 6 By cross multiplication, we have

$$\frac{x^3}{bn-cm} = \frac{x}{cl-an} = \frac{1}{am-bl}$$

$$\frac{x^3}{(bn-cm)(am-bl)} = \frac{x^3}{(cl-an)^2}$$

$(bn-cm)(am-bl) = (cl-an)^2$ is the reqd Eliminant

- 7 By adding the two equations, we have

$$2x = 2a \quad \text{or,} \quad x = a \quad \text{and by subtracting} \quad \frac{2}{x} = 2b \quad \text{or,} \quad \frac{1}{x} = b$$

$$x \frac{1}{x} = ab$$

Thus $ab = 1$ is the required Eliminant

- 8 By addition and subtraction of the two equations, we have respectively

$$4x = 10p \quad \text{or,} \quad x = \frac{5p}{2}, \quad \text{and} \quad \frac{6}{x} = 14q \quad \text{or,} \quad \frac{1}{x} = \frac{7q}{3}$$

$$x \frac{1}{x} = \frac{35pq}{6}$$

Thus $35pq = 6$ is the required Eliminant.

- 9 By cross multiplication, we have

$$\frac{x^3}{b_1c_2 - c_1b_2} = \frac{x}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - b_1a_2}$$

$$\left(\frac{x^3}{b_1c_2 - c_1b_2}\right)\left(\frac{1}{a_1b_2 - b_1a_2}\right)^2 = \left(\frac{x}{c_1a_2 - a_1c_2}\right)^3$$

Thus $(b_1c_2 - c_1b_2)^2(a_1b_2 - b_1a_2)^2 = (c_1a_2 - a_1c_2)^3$ is the reqd
Eliminant

- 10 By cross multiplication we have

$$\frac{x^2}{b_1c_2 - b_2c_1} = \frac{x^2}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - b_1a_2}$$

$$\left(\frac{x^2}{b_1c_2 - b_2c_1}\right)^3\left(\frac{1}{a_1b_2 - b_1a_2}\right) = \left(\frac{x^2}{c_1a_2 - a_1c_2}\right)^3$$

Thus $(b_1c_2 - b_2c_1)^3(a_1b_2 - b_1a_2) = (c_1a_2 - a_1c_2)^3$ is the reqd
Eliminant

- 11 By cross multiplication, we have

$$\frac{x^4}{b_1c_2 - c_1b_2} = \frac{x^3}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - b_1a_2}$$

$$\left(\frac{x^4}{b_1c_2 - c_1b_2}\right)^3\left(\frac{1}{a_1b_2 - b_1a_2}\right) = \left(\frac{x^3}{c_1a_2 - a_1c_2}\right)^4$$

Thus $(b_1c_2 - c_1b_2)^3(a_1b_2 - b_1a_2) = (c_1a_2 - a_1c_2)^4$ is the reqd
Eliminant

- 12 Multiplying (2) by a_1 , we have $ax^2 + amx^3 + an = 0$ (3)

From (1) and (3) by subtraction, we get

$$amx^3 + (an - b)x - c = 0 \quad (4)$$

From (2) and (4) by cross multiplication, we have

$$\frac{x^3}{-mc - an^2 + nb} = \frac{x}{amn + c} = \frac{1}{an - b - mx^2}$$

$$\frac{x^3}{an^2 + mc - nb} \cdot \frac{1}{am^2 - an + b} = \frac{x^2}{(amn + c)^2}$$

Thus $(an^2 + mc - nb)(am^2 - an + b) = (amn + c)^2$ is the reqd
Eliminant

- 13 Multiplying the 1st equation by 2, we have

$$ax^3 + bx^2 + cx = 0 \text{ and the 2nd equation by } a,$$

$$ax^3 + 2ax^2 + 3a = 0$$

$$\text{by subtraction, we have } x^2(2a - b) - cx + 3a = 0$$

(3)

From (1) and (3), by cross multiplication, we have

$$\frac{x^2}{3ab+c^2} = \frac{x}{c(2a-b)-3a^2} = \frac{1}{-ac-b} \frac{1}{2a-b},$$

$$\frac{x^2}{3ab+c^2} \frac{1}{(b^2-2ab-ac)} = \frac{x^2}{(3a^2-2ac+bc)^2}.$$

Thus $(3ab+c^2)(b^2-2ab-ac)=(3a^2-2ac+bc)^2$ is the reqd.
Eliminant

14 From (1), we get $ax+by-m=0$

From (2), we get $bx-ay-n=0$

by cross multiplication, $\frac{x}{-bn-am} = \frac{y}{-mb+an} = \frac{1}{-a^2-b^2}$

$$x = \frac{bn+am}{a^2+b^2} \text{ and } y = \frac{bm-an}{a^2+b^2},$$

$$\text{from (3), we have } \left(\frac{bn+am}{a^2+b^2}\right)^2 + \left(\frac{bm-an}{a^2+b^2}\right)^2 = 1$$

$$\text{or, } (bn+am)^2 + (bm-an)^2 = (a^2+b^2)^2$$

$$\text{or, } b^2n^2 + a^2m^2 + 2abmn + b^2m^2 + a^2n^2 - 2abmn = (a^2+b^2)^2$$

$$\text{or, } a^2(m^2+n^2) + b^2(m^2+n^2) = (a^2+b^2)^2$$

$$\text{or, } (a^2+b^2)(m^2+n^2) = (a^2+b^2)^2$$

Thus $a^2+b^2=m^2+n^2$ is the reqd Eliminant

15 From (1), we get $ax-cy+b=0$

From (2), we get $c_1x-a_1y-b_1=0$

by cross multiplication, $\frac{x}{cb_1+ba_1} = \frac{y}{bc_1+ab_1} = \frac{1}{-aa_1+c}$,

$$x = \frac{cb_1+ba_1}{-aa_1+cc_1} \text{ and } y = \frac{bc_1+ab_1}{-aa_1+cc_1},$$

$$\text{from (3), we have } \left(\frac{cb_1+ba_1}{-aa_1+cc_1}\right)^2 + \left(\frac{bc_1+ab_1}{-aa_1+cc_1}\right)^2 = 1,$$

$(cb_1+ba_1)^2 + (bc_1+ab_1)^2 = (cc_1-aa_1)^2$ is the reqd
Eliminant

16 Multiplying (1) by 1, we have $ax^2+by^2=0$

$$\text{and } lx^2+my^2=0$$

(2)

by cross multiplication, $\frac{x^2}{bn} = \frac{xy}{-an} = \frac{y^2}{am-bl}$,

$$\frac{x^2 y^2}{b m (a m - b l)} = \frac{x^2 y^2}{a^2 n^2}, \quad a b m n - b^2 l n = a^2 n^2$$

Thus $a^2 n + b^2 l = a b m$ is the reqd Eliminant

17 From (1) we get $x = ay + az$ or, $x - ay - az = 0$ (4)

From (2) we get $y = bz + br$ or, $br - y + bz = 0$... (5)

from (4) and (5), we get

$$\frac{x}{-ab-a} = \frac{y}{-ab-b} = \frac{z}{-1+ab} = k \text{ (suppose) },$$

$$x = -k(ab+a), y = -k(ab+b) \text{ and } z = k(ab-1),$$

$$\text{from (3), we get } \frac{k(ab-1)}{-k(ab+a) - k(ab+b)} = c,$$

$$\text{or, } \frac{ab-1}{-2ab-a-b} = c, \text{ or, } ab-1 = -2abc-ac-bc$$

Thus $ab+bc+ca+2abc=1$ is the reqd Eliminant

18 From (1), $\frac{y}{-x} = \frac{a+1}{a-1}$ (4)

From (2), $\frac{z}{-x} = \frac{b+1}{b-1}$ (5)

From (3), $\frac{x}{-y} = \frac{c+1}{c-1}$ (6)

$$\text{Multiplying (4) and (5), we have } \frac{y}{x} = \frac{(a+1)(b+1)}{(a-1)(b-1)}$$

$$\text{Also from (3) } \frac{y}{x} = \frac{1-c}{1+c},$$

$$\frac{(a+1)(b+1)}{(a-1)(b-1)} = \frac{1-c}{1+c} \text{ or, } \frac{ab+a+b+1}{ab-a-b+1} = \frac{1-c}{1+c}$$

$$\text{or, } \frac{ab+1}{-(a+b)} = \frac{1}{c} \text{ (by componendo and dividendo)}$$

$$\text{or, } abc+c = -a-b$$

Thus $a+b+c+abc=0$ is the reqd Eliminant

19 We have, from the given equations,

$$\left(\frac{y}{x} + \frac{z}{y}\right)^2 = a^2, \quad \left(\frac{z}{x} + \frac{x}{y}\right)^2 = b^2 \quad \text{and} \quad \left(\frac{x}{y} + \frac{y}{x}\right)^2 = c^2$$

$$\left(\frac{y}{x} + \frac{z}{y}\right)^2 + \left(\frac{z}{x} + \frac{x}{y}\right)^2 + \left(\frac{x}{y} + \frac{y}{x}\right)^2 = a^2 + b^2 + c^2$$

(from Ex 6 Page 184,)

$$4 + \left(\frac{y}{z} + \frac{z}{y}\right)\left(\frac{z}{x} + \frac{x}{z}\right)\left(\frac{x}{y} + \frac{y}{x}\right) = a^2 + b^2 + c^2$$

or, $4 + abc = a^2 + b^2 + c^2$

Thus $a^2 + b^2 + c^2 - abc = 4$ is the reqd Eliminant

- 20 By adding the first three equations,

We have $z^2(y-z) + y^2(z-x) + x^2(x-y) = a + b + c$

or, $-(x-y)(y-z)(z-x) = a + b + c$

$$-z^2y^2z^2(z-y)(y-z)(z-x) = z^2y^2z^2(a+b+c)$$

or, $-abc = a^2(a+b+c)$

Thus $a^2(a+b+c) + abc = 0$ is the reqd Eliminant

- 21 From (1), we have $a - bz - cy = 0$

From (2), we have $az - b + cx = 0$

by cross multiplication,

$$\frac{a}{-xz-y} = \frac{b}{-y'z-x} = \frac{c}{-1+z^2} = \lambda \text{ (suppose) },$$

$a = -\lambda(z+y)$, $b = -\lambda(yz+x)$, and $c = \lambda(z^2-1)$

from (3), we have $-ky(zx+y) - \lambda z(yz+x) = \lambda(z^2-1)$

or, $-xyz - y^2 - x^2 = z^2 - 1$

Thus $z^2 + y^2 + x^2 + 2xyz = 1$ is the reqd Eliminant

Miscellaneous Exercises (4)

I

- 1 The given expression

$$= \sqrt[3]{(1-27+1)(-1+3-3)} - \sqrt[3]{(-1)(-27)(1)}$$

$$= \sqrt[3]{(-25)(-1)} - \sqrt[3]{27}$$

$$= \sqrt[3]{25} - \sqrt[3]{27} = 5 - 3 = 2$$

- 2 The given expression

$$= 3a - 2b + 2c - 2(a-b) + 3(c+a) - 9c + 4(c-a)$$

$$= 3a - 2b + 2c - 2a + 2b + 3c + 3a - 9c + 4c - 4a$$

$$= 0$$

$$\begin{aligned} 3 \quad & 3(a+b)^2 - 2(a^2 - b^2) - a(a+b) \\ &= (a+b)\{3(a+b) - 2(a-b) - a\} \\ &= (a+b)\{3a + 3b - 2a + 2b - a\} = 5b(a+b) \end{aligned}$$

$$\begin{array}{r} 4 \quad \text{Quotient} = 2x^2 - 4xy + 5y^2 \\ x^2 - 3xy + 4y^2 \bigg) \frac{2x^4 - 10x^3y + 25x^2y^2 - 31xy^3 + 20y^4}{2x^4 - 6x^3y + 8x^2y^2} \\ \quad \quad \quad \underline{-4x^3y + 17x^2y^2 - 31xy^3 + 20y^4} \\ \quad \quad \quad \quad \quad \underline{-4x^3y + 12x^2y^2 - 16xy^3} \\ \quad \quad \quad \quad \quad \quad \quad \underline{5x^2y^2 - 15xy^3 + 20y^4} \\ \quad \quad \quad \quad \quad \quad \quad \quad \underline{5x^2y^2 - 15xy^3 + 20y^4} \end{array}$$

$$\begin{aligned} 5 \quad & \text{The given expression} \\ &= \frac{b}{a+b} \left\{ 1 - \frac{a}{(a+b)} - \frac{ab}{(a+b)^2} \right\} = \frac{b}{a+b} \left\{ \frac{(a+b)^2 - a(a+b) - ab}{(a+b)^2} \right\} \\ &= \frac{b}{a+b} \left\{ \frac{a^2 + 2ab + b^2 - a^2 - ab - ab}{(a+b)^2} \right\} = \frac{b}{a+b} \frac{b^2}{(a+b)^2} = \frac{b^3}{(a+b)^3} \end{aligned}$$

$$6 \quad \frac{x+a}{1-a} - \frac{x-b}{1+b} = \frac{2(a+b)}{r}$$

$$\text{or,} \quad 1 + \frac{2a}{x-a} - 1 + \frac{2b}{x+b} = \frac{2(a+b)}{x}$$

$$\text{or,} \quad \frac{a}{x-a} + \frac{b}{x+b} = \frac{a+b}{x}$$

$$\text{or,} \quad a \left(\frac{1}{x-a} - \frac{1}{x} \right) = b \left(\frac{1}{x} - \frac{1}{x+b} \right)$$

$$\text{or} \quad a \left\{ \frac{x-b}{x(x-a)} \right\} = b \left\{ \frac{x+b-x}{x(x+b)} \right\}$$

$$\text{or,} \quad \frac{a^2}{x-a} = \frac{b^2}{x+b}$$

$$\text{or,} \quad a^2x + a^2b = b^2x - ab^2$$

$$\text{or,} \quad x(a^2 - b^2) = -ab(a+b), \quad x = \frac{ab}{b-a}$$

$$\begin{aligned} 7 \quad & x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x} \right)^3 - 3x \left(x + \frac{1}{x} \right) \\ &= (\sqrt{3})^3 - 3\sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0 \end{aligned}$$

$$8 \quad \frac{3\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(3\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= 9 - 2\sqrt{6} + 3\sqrt{6} - 4 = 5 + \sqrt{6}$$

II.

1 The given expression

$$= (2+2)(1-2) + (4-3)(2+3) + (-6+1)(-3-1)$$

$$= 4(-1) + (1)(5) + (-5)(-4)$$

$$= -4 + 5 + 20 = 25 - 4 = 21$$

$$2 \quad \begin{array}{r} 2x^2+3x-1 \bigg) \begin{array}{r} 8x^4-24x^3+3x+1 \\ 8x^4+12x^3 \\ \hline -12x^3-20x^2+3x+1 \\ -12x^3-18x^2+6x \\ \hline -2x^2-3x+1 \\ -2x^2-3x+1 \\ \hline \end{array} \end{array}$$

$$3 \quad \begin{array}{r} x+4 \bigg) \begin{array}{r} x^2+7x+c \\ x^2+4x \\ \hline 3x+c \\ 3x+12 \\ \hline c-12 \end{array} \end{array}$$

But in order that x^2+7x+c may be exactly divisible by $x+4$ there should be no remainder, that is, $c-12$ must be equal to 0, $c=12$

$$4 \quad \frac{1}{2\sqrt{7}-3\sqrt{2}} - \frac{1}{2\sqrt{7}+3\sqrt{2}} = \frac{2\sqrt{7}+3\sqrt{2}-2\sqrt{7}+3\sqrt{2}}{(2\sqrt{7})^2-(3\sqrt{2})^2}$$

$$= \frac{6\sqrt{2}}{28-18} = \frac{6\sqrt{2}}{10} = \frac{3\sqrt{2}}{5}$$

$$5 \quad \begin{array}{r} x^3-7x+6 \bigg) \begin{array}{r} x^4-3x^3-2x^2+12x-8 \\ x^4 \\ \hline -3x^3+5x^2+6x-8 \\ -3x^3+ \quad +21x-18 \\ \hline 5 \mid 5x^2-15x+10 \\ \quad x^2-3x+2 \end{array} \end{array}$$

$$\begin{array}{r} x^2-3x+2 \bigg) \begin{array}{r} x^3 \\ x^3-3x^2+2x \\ \hline 3x^2-9x+6 \\ 3x^2-9x+6 \\ \hline \end{array} \end{array}$$

Thus the reqd H C F is x^2-3x+2

6 The given expression

$$\begin{aligned}
 &= 5 \left(\frac{1}{5} + \frac{7}{x-7} - \frac{3}{x-3} \right) \left(\frac{1}{5} - \frac{7}{x+7} + \frac{3}{x+3} \right) \\
 &= 5 \left\{ \frac{x^2 - 10x + 21 + 35x - 105 - 15x + 105}{5(x-7)(x-3)} \right\} \times \\
 &\quad \left\{ \frac{x^2 + 10x + 21 - 35x - 105 + 15x + 105}{5(x+7)(x+3)} \right\} \\
 &= 5 \left\{ \frac{x^2 + 10x + 21}{5(x^2 - 10x + 21)} \right\} \left\{ \frac{x^2 - 10x + 21}{5(x^2 + 10x + 21)} \right\} = \frac{1}{5}
 \end{aligned}$$

7, $\frac{x+1}{6} + \frac{3x-1}{8} - \frac{5x-7}{12} + 1 = \frac{7x-5}{24}$

or, $4x + 4 + 9x - 3 - 10x + 14 + 24 = 7x - 5$

or, $-4x = -14$, $x = 11$

8 $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3x \frac{1}{x} \left(x - \frac{1}{x}\right) = 1 + 3 = 4$

III

1 The given expression

$$\begin{aligned}
 &= \frac{9(-8-64) + 4(64-27) + 16(27+8)}{-8+12-6} \\
 &= \frac{9(-72) + 4(37) + 16(35)}{-2} = \frac{-648 + 148 + 560}{-2} = \frac{60}{-2} = -30
 \end{aligned}$$

2 $\frac{1+x}{1-x} + \frac{1-x}{1+x} - \frac{1+x^2}{1-x^2} - \frac{1-x^2}{1+x^2}$

$$\begin{aligned}
 &= \frac{(1+x)^2 + (1-x)^2 - (1+x^2) - (1-x^2)}{1-x^2} - \frac{1-x^2}{1+x^2} \\
 &= \frac{2(1+x^2) - (1+x^2) - (1-x^2)}{1-x^2} - \frac{1-x^2}{1+x^2} = \frac{1+x^2}{1-x^2} - \frac{1-x^2}{1+x^2} \\
 &= \frac{(1+x^2)^2 - (1-x^2)^2}{1-x^4} = \frac{4x^2}{1-x^4}
 \end{aligned}$$

3 $a^3 - b^3 + 6bc - 9c^2 = a^3 - (b^3 - 6bc + 9c^2) = a - (b-3c)^2$

$$= \{a + (b-3c)\} \{a - (b-3c)\} = (a+b-3c)(a-b+3c)$$

4 $x^3 + 5ax^2 - 5a^2x - a^3 = (x^3 - a^3) + (5ax^2 - 5a^2x)$

$$= (x-a)(x^2 + ax + a^2 + 5ax) = (x-a)(x^2 + 6ax + a^2)$$

and $5x^3 - 3ax^2 - 5a^2x + 3a^3$

$$= x^2(5x - 3a) - a^2(5x - 3a) = (x+a)(x-a)(5x - 3a),$$

the reqd H.C.F. = $(x-a)$

$$\begin{aligned}
 5 \quad & x^2 - 5x + 6 = (r-3)(x-2) \\
 & 1^2 - 4x + 3 = (r-3)(r-1) \\
 & r^2 - 3x + 2 = (1-2)(1-1) \\
 & \text{the reqd L C M} = (r-1)(x-2)(r-3)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \text{The numerator} \\
 & = x^2(x^3 + 5x^2 + 8x + 4) = 1^2(x^3 + x^2 + 4x^2 + 4x + 4x + 4) \\
 & = r^2\{r^2(1+1) + 4x(r+1) + 4(r+1)\} \\
 & = x^2(1+1)(x^2 + 4x + 4) = r^2(1+1)(1+2)^2, \\
 & \text{and the denominator} \\
 & = x(r^4 + x^3 + 8r + 8) = 1\{1^4(1+1) + 8(1+1)\} \\
 & = x(x+1)(1^3 + 8) = 1(r+1)(1+2)(1^2 - 2x + 4), \\
 & \text{the reqd result} = \frac{r(r+2)}{r^2 - 2x + 4}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad & \frac{1}{x+3} + \frac{1}{r-2} = \frac{2}{x-7} \quad \text{or,} \quad \frac{1}{r+3} - \frac{1}{x-1} = \frac{1}{x-7} - \frac{1}{x-2} \\
 & \text{or,} \quad \frac{r-7-1-3}{(1+3)(1-7)} = \frac{r-2-1+7}{(1-7)(x-2)} \\
 & \text{or,} \quad \frac{-10}{(x+3)(1-7)} = \frac{5}{(1-7)(1-2)} \quad \text{or,} \quad \frac{-2}{1+3} = \frac{1}{x-2} \\
 & \text{or,} \quad -2x+4 = x+3 \quad \text{or,} \quad -3x = -1, \quad x = \frac{1}{3}
 \end{aligned}$$

$$8 \quad \frac{a}{b} = \frac{x}{y} = k \text{ (suppose),} \quad a = bk \text{ and } 1 = yk$$

$$\begin{aligned}
 & \frac{ab}{1y} = \frac{1b^2}{ky^2} = \frac{b^2}{y^2}, \\
 \text{also,} \quad & \frac{a^2 + b^2}{r^2 + y^2} = \frac{kb^2 + b^2}{ky^2 + y^2} = \frac{b^2(k+1)}{y^2(k+1)} = \frac{b^2}{y^2} \\
 \text{Hence, } & \frac{ab}{xy} = \frac{a^2 + b^2}{x^2 + y^2}
 \end{aligned}$$

IV

1 The given expression

$$= \frac{x + \frac{x^2 - y^2}{x^2 + y^2 + y}}{\sqrt{x^2 + y^2 + y}} \times \frac{r - \sqrt{1^2 - y^2}}{\sqrt{x^2 + y^2 - y}} = \frac{1^2 - (x^2 - y^2)}{(1^2 + y^2) - y^2} = \frac{y^2}{1}$$

2 The reqd expression

$$\begin{aligned}
 & = \frac{1^8 + x^4y^4 + y^8}{x^2 - ry + y^3} = \frac{(x^4 - x^2y^2 + y^4)(1^2 + ry + y^2)(1^3 - xy + y^2)}{r^2 - 1y + y^2} \\
 & = (1^4 - 1^2y^2 + y^4)(1^2 + xy + y^2) = x^6 + 1^5y - x^3y^3 + xy^6 + y^6
 \end{aligned}$$

- 3 (i) $x^3 + x^2 - x - 1 = x^2(x+1) - (x+1)$
 $= (x+1)(x^2-1) = (x+1)(x+1)(x-1) = (x+1)^2(x-1)$
- (ii) $a^3b^3 - a^3 - b^3 + 1 = a^3(b^3-1) - (b^3-1)$
 $= (a^3-1)(b^3-1) = (a+1)(a-1)(b+1)(b-1)$
- 4 $(ax+by)^2 + (bx-ay)^2$
 $= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$
 $= a^2(x^2+y^2) + b^2(x^2+y^2) = (a^2+b^2)(x^2+y^2)$
- 5 $81^2 + 27 = (27)^2 + (3)^2 = (27+3)(41^2-61+9)$
 $161^2 + 36 = (41^2+9)^2 - 361^2 = (41^2+9+61)(41^2+9-61)$
and $61^2 - 51 - 6 = 61^2 - 9 \cdot 7 + 42 - 6$
 $= 31(21-3) + 2(21-3) = (21-3)(31+2)$
the reqd L.C.M.
 $= (21+3)(41^2-61+9)(41^2+61+9)(21-3)(31+2)$
 $= (41^2-9)(161^2+361^2+81)(31+2)$
 $= [(41^2)^2 - (9)^2](31+2) = (641^2 - 729)(31+2)$
- 6 $\frac{3x-4}{x} + \frac{2}{4x+3} = 3$ or, $3 - \frac{4}{x} + \frac{2}{4x+3} = 3$
or, $\frac{2}{4x+3} = \frac{4}{x}$ or, $21 = 161 \pm 12$
or, $-141 = 12$ $x = -12$
- 7 We have $bx+ay = 2a^2b^2$
or, $bx+ay - 2a^2b^2 = 0$ (1)
also $by-ax = ab(b^2-a^2)$
or, $-ax+by - ab(b^2-a^2) = 0$ (2)
From (1) and (2), by cross multiplication,

$$\frac{x}{-a^2b(b^2-a^2)+2a^2b^3} = \frac{y}{2a^2b+ab^3(b^2-a^2)} = \frac{1}{b^3+a^3}$$

$$x = \frac{-a^2b^3+a^4b+2a^2b^3}{b^3+a^3} = \frac{a^2b(a^2+b^2)}{b^3+a^3} = a^2b$$
and $y = \frac{2a^2b^3+ab^4-a^2b^3}{b^3+a^3} = \frac{ab^2(a^2+b^2)}{b^3+a^3} = ab^2$
- 8 $\frac{x}{a+b-c} = \frac{1}{a-b+c} = \frac{z}{b+c-a} = 1$ (suppose)

$$x = k(a+b-c), y = k(a-b+c) \text{ and } z = k(b+c-a) \\ x+y+z = k(a+b+c)$$

Thus $\frac{x+y+z}{a+b+c} = k = \text{each of the ratios}$

V

1 The given expression

$$= \frac{(2-5y)(1+5y)}{(x+5y)(x-2y)} \times \frac{(x-2y)(x+2y)}{(1+2y)(1-5y)} = 1$$

2 Quotient $= a^2(b-c) + a(b^3-c^3) + bc(b-c)$

$$\begin{array}{r} a+(b+c) \left\{ \begin{array}{l} a^3(b-c) - a(b^3-c^3) + bc(b^3-c^3) \\ a^3(b-c) + a^3(b-c^3) \\ -a^3(b^3-c^3) - a(b^3-c^3) + bc(b^3-c^3) \\ -a^3(b^3-c^3) - a(b^3+b^2c-bc^2-c^3) \\ \hline abc(b-c) + bc(b^4-c^2) \\ \hline abc(b-c) + bc(b^2-c^2) \end{array} \right. \end{array}$$

The quotient is again

$$\begin{aligned} &= (b-c)\{a^2 - a(b+c) + bc\} \\ &= (b-c)\{a(a-b) - c(a-b)\} \\ &= (b-c)(a-c)(a-b) = -(b-c)(c-a)(a-b) \end{aligned}$$

$$\begin{aligned} 3 \quad \frac{x^3-y^3}{x^3+y^3} &= \frac{(1-y)(x^2+xy+y^2)}{(1+y)(x^2-xy+y^2)} = \frac{6\{(a+3)^2 + (a-3)^2 + (a^2-9)\}}{2a\{(a+3)^2 + (a-3)^2 - (a^2-9)\}} \\ &= \frac{3\{2(a^2+9) + (a^2-9)\}}{a\{2(a^2+9) - (a^2-9)\}} = \frac{3(3a^2+9)}{a(a^2+27)} = \frac{9(a^2+3)}{a(a^2+27)} \end{aligned}$$

$$\begin{aligned} 4 \quad &\frac{x^2}{y} - \frac{8x}{\sqrt{y}} + 24 - \frac{32\sqrt{y}}{1} + \frac{16y}{x^2} \left(\frac{1}{\sqrt{y}} - 4 + \frac{4\sqrt{y}}{x} \right) \\ &\frac{x^3}{y} \\ &\frac{2x}{\sqrt{y}} - 4 \left[\begin{array}{l} -\frac{8x}{\sqrt{y}} + 24 - \frac{32\sqrt{y}}{1} + \frac{16y}{x^2} \\ -\frac{8x}{\sqrt{y}} + 16 \end{array} \right] \\ &\frac{2x}{\sqrt{y}} - 8 + \frac{4\sqrt{y}}{x} \left[\begin{array}{l} 8 - \frac{32\sqrt{y}}{1} + \frac{16y}{x^2} \\ 8 - \frac{32\sqrt{y}}{1} + \frac{16y}{x^2} \end{array} \right] \end{aligned}$$

Thus the reqd root $= \frac{x}{\sqrt{y}} - 4 + \frac{4\sqrt{y}}{x}$

$$\begin{aligned}
 5 \quad & (a^2 + b^2 + c^2)(x^2 + y^2 + z^2) - (ax + by + cz)^2 \\
 &= a^2x^2 + a^2y^2 + a^2z^2 + b^2x^2 + b^2y^2 + b^2z^2 + c^2x^2 + c^2y^2 + c^2z^2 \\
 &\quad - 2(abxy + acxz + bcyz) \\
 &= (a^2y^2 - 2abxy + b^2x^2) + (b^2z^2 - 2bcxz + c^2x^2) \\
 &\quad + (c^2y^2 - 2acyz + a^2z^2) \\
 &= (ay - bx)^2 + (bz - cy)^2 + (cx - az)^2
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \frac{7 - 2\sqrt{5}}{4 - \sqrt{5}} = \frac{(7 - 2\sqrt{5})(4 + \sqrt{5})}{16 - 5} = \frac{28 - \sqrt{5} - 10}{11} = \frac{18 - \sqrt{5}}{11} \\
 & \text{and } \frac{15 + 6\sqrt{5}}{2 + \sqrt{5}} = \frac{3(5 + 2\sqrt{5})}{2 + \sqrt{5}} = 3\sqrt{5}
 \end{aligned}$$

Hence, their difference

$$= 3\sqrt{5} - \frac{18 - \sqrt{5}}{11} = \frac{33\sqrt{5} - 18 + \sqrt{5}}{11} = \frac{34\sqrt{5} - 18}{11}$$

$$\begin{aligned}
 7 \quad & 2^x \times 4^y = 32 \quad (i), \quad 3^x - 9^y = 3 \quad (ii) \\
 & \text{From (i), we have } 2^x \times 2^{2y} = 2^5, \quad x + 2y = 5 \quad (iii) \\
 & \text{From (ii), we have } 3^x \times 9^{-y} = 3 \quad \text{or, } 3^x \times 3^{-2y} = 3 \\
 & \text{or, } 3^{x-2y} = 3, \quad x - 2y = 1 \quad (iv) \\
 & \text{From (iii) and (iv), we have} \\
 & \quad 2x = 6 \quad \text{or, } x = 3 \\
 & \text{and } 4y = 4 \quad \text{or, } y = 1
 \end{aligned}$$

$$\begin{aligned}
 8 \quad & \frac{a}{b} = \frac{c}{d} = k \text{ (suppose),} \\
 & a = bk \text{ and } c = dk, \\
 & (a^2 + c^2)(b^2 + d^2) = (b^2k^2 + d^2k^2)(b^2 + d^2) = k^2(b^2 + d^2)^2 \\
 & \text{Also } (ab + cd)^2 = (b^2k + d^2k)^2 = k^2(b^2 + d^2)^2, \\
 & (a^2 + c^2)(b^2 + d^2) = (ab + cd)^2
 \end{aligned}$$

VI

1 The given expression

$$= \frac{2a(1-x)^2(1+x)^2}{y^2} \times \frac{y^2}{(1+x)^2(1-x)^2} \times \frac{z^2}{2ay^2(1-x)} = 1$$

$$2 \quad \left(a + b + \frac{b^2}{a} + \frac{a^2}{b}\right) \left(a - b + \frac{b^2}{a} - \frac{a^2}{b}\right)$$

$$\begin{aligned}
&= \left\{ \left(a + \frac{b^2}{a} \right) + \left(b + \frac{a^2}{b} \right) \right\} \left\{ \left(a + \frac{b^2}{a} \right) - \left(b + \frac{a^2}{b} \right) \right\} \\
&= \left(a + \frac{b^2}{a} \right)^2 - \left(b + \frac{a^2}{b} \right)^2 \\
&= a^2 + 2b^2 + \frac{b^4}{a^2} - \left(b^2 + 2a^2 + \frac{a^4}{b^2} \right) \\
&= b^2 - a^2 + \frac{b^4}{a^2} - \frac{a^4}{b^2}
\end{aligned}$$

3

$$\begin{aligned}
&\text{Quotient} = x^2 + (a-b)x - ab \\
&x^3 - (a+b)x^2 + abx^2 - 2bx^3 - (a^2 - b^2)x^2 + 2a^2bx - a^2b^2 \\
&\quad \underline{x^4 - (a+b)x^3 + abx^3} \\
&\quad (a-b)x^3 - (a^2 + ab - b^2)x^2 + 2a^2bx - a^2b^2 \\
&\quad \underline{(a-b)x^3 - (a^2 - b^2)x^2 + (a^2b - ab^2)x} \\
&\quad \quad - abx^2 + ab(a+b)x - a^2b^2 \\
&\quad \quad \underline{- abx^2 + ab(a+b)x - a^2b^2}
\end{aligned}$$

4

$$\begin{aligned}
&a^3 + b^3 + c^3 - bc - ca - ab \\
&= \frac{1}{2}(2a^3 + 2b^3 + 2c^3 - 2bc - 2ac - 2ab) \\
&= \frac{1}{2}\{(a^3 + b^3 - 2ab) + (b^3 + c^3 - 2bc) + (c^3 + a^3 - 2ac)\} \\
&= \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\} \\
&= \frac{1}{2}\{(y+z-x)^2 + (z+x-y)^2 + (x+y-z)^2\} \\
&= \frac{1}{2}\{(y-z)^2 + (z-y)^2 + (x-z)^2\} \\
&= \frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz) \\
&= x^2 + y^2 + z^2 - xy - yz - xz
\end{aligned}$$

5

$$5x^3 - 14x^2 + 16 = 5x^3 - 10x^2 - 4x^2 + 16 = 5x^2(x-2) - 4(x^2-4)$$

$$= (x-2)\{5x^2 - 4(x+2)\} = (x-2)(5x^2 - 4x - 8),$$

and

$$3x^3 - 2x^2 + 16x - 48 = 3x^3 - 6x^2 + 4x^2 - 8x + 24x - 48$$

$$= 3x^2(x-2) + 4x(x-2) + 24(x-2) = (x-2)(3x^2 + 4x + 24),$$

$$\frac{5x^2 - 14x^2 + 16}{3x^3 - 2x^2 + 16x - 48} = \frac{5x^2 - 4x - 8}{3x^2 + 4x + 24}$$

6

$$\text{We have } \frac{2}{x} + \frac{7}{y} - 29 = 0 \text{ and } \frac{5}{x} - \frac{6}{y} - 2 = 0,$$

by cross multiplication

$$\frac{1}{x(-14-174)} = \frac{1}{y(-145+4)} = \frac{1}{-12-35}$$

or

$$\frac{1}{188x} = \frac{1}{141y} = \frac{1}{47}, \quad x = \frac{1}{4} \text{ and } y = \frac{1}{4}$$

7 Multiplying the 2nd equation by 8, we have

$$56x - 32y + 8z - 64 = 0$$

and

$$2x + 3y - 8z + 35 = 0$$

1st equation

$$58x - 29y - 29 = 0$$

or,

$$2x - y - 1 = 0 \quad (4)$$

Again by multiplying the 2nd equation by 3, we have

$$21x - 12y + 3z - 24 = 0$$

From (3)

$$12x - 5y - 3z + 10 = 0$$

$$33x - 17y - 14 = 0$$

From (4)

$$34x - 17y - 17 = 0$$

$$x - 3 = 0, \quad x = 3,$$

$$\text{from (4), } 6 - y - 1 = 0 \quad y = 5,$$

$$\text{and from (2) } 21 - 20 + z - 80 = 0. \quad z = 7$$

8

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (suppose) ,}$$

$$a = bk, \quad c = dk \text{ and } e = fk,$$

$$\frac{a}{b} = \frac{bk}{b} = k$$

$$\text{Also, } \frac{\sqrt{m^2a^4 + n^2c^4 - p^2e^4}}{\sqrt{m^2b^4 + n^2d^4 - p^2f^4}} = \frac{\sqrt{m^2k^2b^4 + n^2k^2d^4 - p^2k^2f^4}}{\sqrt{m^2b^4 + n^2d^4 - p^2f^4}} = k,$$

$$a \quad b \quad \sqrt{m^2a^4 + n^2c^4 - p^2e^4} \quad \sqrt{m^2b^4 + n^2d^4 - p^2f^4}$$

VII

1

$$\text{Quotient} = 2x^3y^{-3} - 3x^4y$$

$$\begin{array}{r} -x^2y^{-6} + 7x^3y^{-1} + 8x^4y^3 \\ \hline -2x^6y^{-8} + 17x^0y^{-4} - 5x^7 - 24x^8y^4 \\ \hline -2x^6y^{-8} + 14x^0y^{-4} + 16x^7 \\ \hline 3x^0y^{-4} - 21x^7 - 24x^8y^4 \\ \hline 3x^0y^{-4} - 21x^7 - 24x^8y^4 \end{array}$$

2

$$e^{2x}a^3 + e^{2x} - a^3 - 1 = e^{2x}(a^3 + 1) - (a^3 + 1)$$

$$= (a^3 + 1)(e^{2x} - 1) = (a + 1)(a^2 - a + 1)(e^x - 1)(e^x + 1),$$

and

$$e^{2x}a^2 + 2e^xa^2 - e^{2x} - 2e^x + a^2 - 1$$

$$= e^{2x}a^2 + 2e^xa^2 + a^2 - e^{2x} - 2e^x - 1$$

$$= a^2(e^{2x} + 2e^x + 1) - (e^{2x} + 2e^x + 1)$$

$$= (a^2 - 1)(e^{2x} + 2e^x + 1) = (a + 1)(a - 1)(e^x + 1)^2,$$

$$\text{the required H C F} = (a + 1)(e^x + 1) = ae^x + e^x + a + 1$$

3 The left-hand expression

$$\begin{aligned}
 &= \frac{2ab + 2cd - a^2 - b^2 + c^2 + d^2}{2(ab + cd)} \\
 &= \frac{(c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab)}{2(ab + cd)} \\
 &= \frac{(c+d)^2 - (a-b)^2}{2(ab + cd)} = \frac{(c+d+a-b)(c+d-a+b)}{2(ab + cd)} \\
 &= \frac{(a+c+d-b)(b+c+d-a)}{2(ab + cd)}
 \end{aligned}$$

4 The numerator = $abx^2 + 1ya^2 + 1yb^2 + aby^2$

$$= a1(b1 + ay) + by(b1 + ay) = (bx + ay)(ax + by)$$

The denominator = $ab1^2 + 1ja^2 - 1yb^2 - aby^2$

$$= a1(b1 + ay) - by(b1 + ay) = (ax - by)(bx + ay)$$

$$\text{the required result} = \frac{ax + by}{ax - by}$$

$$5 \quad \frac{x-4}{x-5} - \frac{x-5}{x-6} = \frac{x-7}{x-8} - \frac{x-8}{x-9}$$

$$\text{or,} \quad 1 + \frac{1}{x-5} - 1 - \frac{1}{x-6} = 1 + \frac{1}{x-8} - 1 - \frac{1}{x-9}$$

$$\text{or,} \quad \frac{1}{x-5} - \frac{1}{x-6} = \frac{1}{x-8} - \frac{1}{x-9}$$

$$\text{or,} \quad \frac{x-6-1+5}{(x-5)(x-6)} = \frac{x-9-1+8}{(x-8)(x-9)}$$

$$\text{or,} \quad (x-5)(x-6) = (x-8)(x-9)$$

$$\text{or,} \quad x^2 - 11x + 30 = x^2 - 17x + 72$$

$$\text{or,} \quad 6x = 42, \quad x = 7$$

$$6 \quad \frac{x+a}{x^2+px+q} \left(\frac{x^2+px+q}{x^2+ax} \right) \left(\frac{x(p-a)+q}{x(p-a)+a(p-a)} \right) \left(\frac{a(a-p)+q}{a(a-p)+q} \right)$$

In order that x^2+px+q may be divisible by $x+a$

$$a(a-p)+q=0$$

Similarly $a(a-p')+q'=0$

$$a(p'-p)+q-q'=0 \quad \text{or,} \quad a = \frac{q-q'}{p-p'}$$

7 Let x be the number of guineas, then the number of half crowns $= x + 48$

$$\text{we have } \frac{21x}{20} + \frac{(x+48)}{8} = 100$$

$$\text{or, } 42x + 5x + 240 = 4000$$

$$\text{or, } 47x = 3760, \quad x = 80$$

Thus, the number of guinea $= 80$,

and the number of half crowns $= 128$

$$8 \quad \frac{a}{b} = \frac{c}{d} = k \text{ (suppose) }, \quad a = bk \text{ and } c = dk$$

$$\frac{4a^6 + 5b^6}{4c^6 + 5d^6} = \frac{4b^6k^6 + 5b^6}{4d^6k^6 + 5d^6} = \frac{b^6(4k^6 + 5)}{d^6(4k^6 + 5)} = \frac{b^6}{d^6}$$

$$\text{Also, } \frac{a^2b^4}{c^2d^4} = \frac{b^2k^2 \cdot b^4}{d^2k^2 \cdot d^4} = \frac{b^6}{d^6}$$

$$\frac{4a^6 + 5b^6}{4c^6 + 5d^6} = \frac{a^2b^4}{c^2d^4}$$

VIII

1 We have

$$\begin{aligned} & (ax + by + cz) + (cx - by + az) \\ &= x(a + c) + z(a + c) \\ &= (x + z)(a + c) \end{aligned}$$

Putting m for $ax + by + cz$ and n for $cx - by + az$ we have the given expression

$= m + n$, of which $m + n$ is a factor

Thus the given expression is divisible by $m + n$ i.e. $(x + z)(a + c)$

$$2 \quad (i) \quad (b + c)^2 - 6a(b + c) + 5a^2 = (b + c)^2 - a(b + c) - 5a(b + c) + 5a^2$$

$$= (b + c)(b + c - a) - 5a(b + c - a) = (b + c - a)(b + c - 5a)$$

$$(ii) \quad x^2 + 2xy + y^2 - 2ay = (x^2 + 2xy + y^2) - (y^2 + a^2 + 2ay)$$

$$= (x + y)^2 - (y + a)^2 = (x + y + y + a)(x + y - y - a)$$

$$= (x + 2y + a)(x - a)$$

$$3 \quad \frac{(a + b)\{(a + b)^2 - c^2\}}{4b^2c^2 - (a^2 - b^2 - c^2)^2} = \frac{(a + b)(a + b + c)(a + b - c)}{(2bc + a^2 - b^2 - c^2)(2bc - a^2 + b^2 + c^2)}$$

$$= \frac{(a + b)(a + b + c)(a + b - c)}{\{a^2 - (b - c)^2\}\{(b + c)^2 - a^2\}}$$

IX.

1 The given expression

$$\begin{aligned}
 &= \{1 - (2+3)^2\} - \{4 - (-3-1)^2\} - \{9 - (1-2)^2\} \\
 &= \{1 - (5)^2\} - \{4 - (-4)^2\} - \{9 - (-1)^2\} \\
 &= (1 - 25) - (4 - 16) - (9 - 1) = -24 + 12 - 8 = -20
 \end{aligned}$$

2 The given expression

$$\begin{aligned}
 &= \frac{x(x+1)^2 - (x-1)^2 - x(x^2+3)}{(x^2-1)^2} \\
 &= \frac{x(x^2+2x+1-x^2-3) - (x^2-2x+1)}{(x^2-1)^2} \\
 &= \frac{1(2x-2) - x^2+2x-1}{(x^2-1)^2} = \frac{2x^2-2x-x^2+2x-1}{(x^2-1)^2} \\
 &= \frac{x^2-1}{(x^2-1)^2} = \frac{1}{x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a^3 - b^3 + 3ab + 1 &= (a^3 + (-b)^3 + (1)^3 - 3a(-b)1) \\
 &= (a-b+1)(a^2+b^2+1+ab-a+b)
 \end{aligned}$$

$$4 \quad \frac{4x+3}{9} + \frac{7x-29}{5x-12} = \frac{8x+19}{18}$$

$$\text{or, } \frac{7x-29}{5x-12} = \frac{8x+19}{18} - \frac{4x+3}{9} = \frac{8x+19-8x-6}{18} = \frac{13}{18}$$

$$\text{or, } 126x - 522 = 65x - 156$$

$$\text{or, } 61x = 366, \quad x = 6$$

 5 Putting a for $y-z$, b for $z-x$ and c for $x-y$, the right hand side of the identity

$$= \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2 = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + 2\left(\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab}\right)$$

$$= \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{2(a+b+c)}{abc}$$

$$= \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

 [Since $a+b+c=0$]

$$= \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} + \frac{1}{(x-y)^2}$$

$$6 \quad \frac{x+y}{x-y} = \frac{5}{3} \quad (1)$$

$$\text{and} \quad x+5y=36 \quad (2)$$

From (1), by componendo, and dividendo, $\frac{x}{y} = \frac{8}{2} = 4$,

$$x = 4y$$

$$\text{from (2),} \quad 4y + 5y = 36$$

$$\text{or,} \quad 9y = 36$$

$$y = 4, \text{ and } x = 16$$

7 At 8 o'clock the hands are 40 minute-divisions apart, therefore they will be at right angles when the minute hand will have travelled $(40 - 15) = 25$ minute divisions more than the hour hand if x minutes past 8 o'clock be the required time, we get

$$x = 25 + \frac{x}{12}$$

$$\text{or,} \quad \frac{11x}{12} = 25 \quad x = \frac{300}{11} = 27 \frac{3}{11}$$

Thus the required time is $27\frac{3}{11}$ minutes past 8 o'clock

$$8 \quad \frac{a}{b} = \frac{b}{c} = k \text{ (suppose)} \quad a = bk \text{ and } b = ck,$$

$$a = ck^2,$$

$$(a+b+c)(a-b+c) = (ck^2 + ck + c)(ck^2 - ck + c)$$

$$= c^2(k^2 + k + 1)(k^2 - k + 1) = c^2(k^4 + k^2 + 1)$$

$$\text{Also,} \quad a^2 + b^2 + c^2 = c^2k^4 + c^2k^2 + c^2 = c^2(k^4 + k^2 + 1)$$

$$(a+b+c)(a-b+c) = a^2 + b^2 + c^2$$

X.

$$\begin{aligned} 1 \quad & 27a^3 - 8b^3 - 27c^3 - 54abc \\ &= (3a)^3 + (-2b)^3 + (-3c)^3 - 3 \cdot 3a \cdot -2b \cdot -3c \\ &= (3a - 2b - 3c)(9a^2 + 4b^2 + 9c^2 + 6ab + 9ac - 6bc), \\ &\text{the quotient} = 9a^2 + 4b^2 + 9c^2 + 6ab - 6bc + 9ac \end{aligned}$$

$$\begin{array}{r} 2 \quad x^5 + 11x + 12 \bigg) \frac{x^5}{x^5} + 11x^3 \quad - 54 \left(\begin{array}{l} + 11x + 12 \\ + 11x + 12 \end{array} \right) \\ \hline \quad \quad \quad 11 \mid 11x^3 - 11x - 66 \\ \quad \quad \quad - \quad \quad x^3 - x - 6 \end{array}$$

$$\begin{array}{r}
 x^3 - x - 6 \quad \left) \begin{array}{r} 1^6 \\ 1^5 - 1^4 - 61^2 \end{array} + 11x + 12 \left(\begin{array}{r} 1^2 + 1 \\ 1^4 + 61^2 + 111 + 12 \\ x^4 \quad \quad - 1 - 6 \\ 6 \mid 61^2 + 121 + 18 \\ \quad \quad \quad 2^2 + 21 + 3 \end{array} \right. \\
 x^3 + 21 + 3 \quad \left) \begin{array}{r} 1^3 \\ 1^3 + 21^2 + 31 \end{array} \left(\begin{array}{r} 1 - 2 \\ -2x^2 - 4x - 6 \\ -21^2 - 41 - 6 \end{array} \right.
 \end{array}$$

Thus the reqd H C F = $x^2 + 21x + 3$

$$\begin{aligned}
 3 \quad & (a^2 - b^2)(x^2 + y^2) + 2(a^2 + b^2)xy \\
 &= a^2(x^2 + y^2 + 2xy) - b^2(x^2 + y^2 - 2xy) = a^2(x + y)^2 - b^2(x - y)^2 \\
 &= \{a(1 + y) + b(1 - y)\}\{a(1 + y) - b(1 - y)\} \\
 &= \{(a + b)x + (a - b)y\}\{(a - b)x + (a + b)y\}
 \end{aligned}$$

4 The given expression

$$\begin{aligned}
 & \frac{a^4 + b^4 - 2a^2b^2}{a^2b^2} - \frac{a^2 - b^2}{ab} \\
 &= \frac{a^4 + b^4 + 2a^2b^2}{a^2b^2} - \frac{(a + b)^2 - 2ab}{ab} \\
 &= \frac{(a^2 + b^2)^2}{(a^2 + b^2)^2} - \frac{a^2 - b^2}{a^2 + b^2} = \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} \times \frac{a^2 + b^2}{a^2 - b^2} = \frac{a^2 - b^2}{a^2 + b^2}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & a^3(b + c) + b^3(c + a) + c^3(a + b) + abc(a + b + c) \\
 &= a^3(b + c) + a(b^3 + b^2c + bc^2 + c^3) + a^2bc + b^3c + bc^3 \\
 &= a^3(b + c) + a\{b^2(b + c) + c^2(b + c)\} + bc(a^2 + b^2 + c^2) \\
 &= a^3(b + c) + (b + c)(b^2 + c^2)a + bc(a^2 + b^2 + c^2) \\
 &= a(b + c)(a^2 + b^2 + c^2) + bc(a^2 + b^2 + c^2) \\
 &= (a^2 + b^2 + c^2)(ab + ac + bc)
 \end{aligned}$$

$$6 \quad \sqrt{9 + 2x} - \sqrt{21} = \frac{5}{\sqrt{9 + 2x}}$$

Multiplying both sides by $\sqrt{9 + 2x}$, we have

$$\begin{aligned}
 & 9 + 2x - \sqrt{18x + 4x^2} = 5 \\
 \text{or,} \quad & 4 + 2x = \sqrt{18x + 4x^2} \\
 \text{or,} \quad & (4 + 2x)^2 = 18x + 4x^2 \\
 \text{or,} \quad & 16 + 16x + 4x^2 = 18x + 4x^2 \\
 & 2x = 16, \quad x = 8
 \end{aligned}$$

7 Let one man do the work in τ days and one boy in y days

$$\frac{1}{\tau} + \frac{2}{y} = \frac{1}{12} \quad (1)$$

$$\text{and } \frac{3}{x} + \frac{1}{y} = \frac{1}{6} \quad (2)$$

Multiplying (2) by 2, we have

$$\frac{6}{x} + \frac{2}{y} = \frac{1}{3} \quad \text{and} \quad \frac{1}{\tau} + \frac{2}{y} = \frac{1}{12} \quad (1)$$

$$\frac{5}{x} = \frac{1}{4}, \quad x = 20$$

Thus one man can do the work in 20 days

$$8 \quad \frac{a}{b} = \frac{b}{c} = k \text{ (suppose),}$$

$$a = bk \text{ and } b = ck, \quad a = ck^2,$$

$$a^4 + a^2c^2 + c^4 = c^4k^8 + c^4k^4 + c^4 = c^4(k^8 + k^4 + 1)$$

$$\begin{aligned} \text{Also, } & b^2 \left(\frac{b^2}{c^2} - 1 + \frac{b^2}{a^2} \right) (a^2 + b^2 + c^2) \\ &= c^2k^2 \left(\frac{c^2k^2}{c^2} - 1 + \frac{c^2k^2}{c^2k^4} \right) (c^2k^4 + c^2k^2 + c^2) \\ &= (c^2k^4 - c^2k^2 + c^2)(c^2k^4 + c^2k^2 + c^2) \\ &= c^4(k^4 - k^2 + 1)(k^4 + k^2 + 1) = c^4(k^8 + k^4 + 1) \\ & \quad a^4 + a^2c^2 + c^4 = b^2 \left(\frac{b^2}{c^2} - 1 + \frac{b^2}{a^2} \right) (a^2 + b^2 + c^2) \end{aligned}$$

XI

$$\begin{aligned} 1 \quad & (x^2 + xy + y^2)^2 - 4xy(x^2 + y^2) = \{(x^2 + y^2) + xy\}^2 - 4xy(x^2 + y^2) \\ &= (x^2 + y^2)^2 - 2xy(x^2 + y^2) + x^2y^2 = \{(x^2 + y^2) - xy\}^2 \\ &= (x^2 - xy + y^2)^2 \end{aligned}$$

$$\begin{aligned} 2. \quad (1) \quad & a^3 - b^3 - c^3 + d^3 - 2(ad - bc) = a^3 - 2ad + d^3 - (b^3 - 2bc + c^3) \\ &= (a - d)^3 - (b - c)^3 = \{(a - d) + (b - c)\} \{(a - d) - (b - c)\} \\ &= (a + b - c - d)(a - b + c - d) \end{aligned}$$

$$\begin{aligned} (11) \quad & x^3 - y^3 - z^3 + 2yz + x + y - z = x^3 - (y^3 - 2yz + z^3) + (x + y - z) \\ &= x^3 - (y - z)^2 + (x + y - z) = (x + y - z)(x - y + z) + (x + y - z) \\ &= (x + y - z)(x - y + z + 1) \end{aligned}$$

$$\begin{array}{r}
 3 \quad \frac{9x^2}{a^2} - \frac{6x}{a} + 3 - \frac{2a}{3x} + \frac{a^2}{9x^2} \left(\frac{3x}{a} - 1 + \frac{a}{3x} \right. \\
 \left. \frac{9x^2}{a^2} \right. \\
 \left. \frac{6x}{a} - 1 \right) - \frac{6x}{a} + 3 - \frac{2a}{3x} + \frac{a^2}{9x^2} \\
 \left. - \frac{6x}{a} + 1 \right) \\
 \frac{6x}{a} - 2 + \frac{a}{3x} \left(2 - \frac{2a}{3x} + \frac{a^2}{9x^2} \right. \\
 \left. 2 - \frac{2a}{3x} + \frac{a^2}{9x^2} \right)
 \end{array}$$

Thus the reqd root = $\frac{3x}{a} - 1 + \frac{a}{3x}$

4 From (1), we have

$$2x + 4y + 6z = 12$$

$$\text{and } 2x + 4y + z = 7 \quad (2)$$

$$5z = 5, \quad z = 1$$

$$\text{From (1), } x + 2y + 3 = 6$$

$$\text{or, } x + 2y = 3 \quad (4)$$

$$\text{From (3), } 3x + 2y + 9 = 14$$

$$\text{or, } 3x + 2y = 5 \quad (5)$$

$$\text{From (4) and (5), } 2x = 2, \quad x = 1$$

$$\text{Hence also } z = 1$$

$$\text{Thus } x = y = z = 1$$

$$5 \quad x^4y - x^3y^2 - 15x^2y^3 + 38xy^4 - 14y^5$$

$$= y(x^4 - x^3y - 15x^2y^2 + 38xy^3 - 14y^4)$$

$$= y(x^4 - 7x^3y + 21x^2y^2 - 34xy^3 + 28y^4)$$

$$= x(x^4 - 7x^3y + 21x^2y^2 - 34xy^3 + 28y^4)$$

$$\begin{array}{r}
 x^4 - x^3y - 15x^2y^2 + 38xy^3 - 14y^4 \left(\begin{array}{l} x^4 - 7x^3y + 21x^2y^2 - 34xy^3 + 28y^4 \\ x^4 - x^3y - 15x^2y^2 + 38xy^3 - 14y^4 \end{array} \right) \\
 -6y \mid \begin{array}{l} -6x^3y + 36x^2y^2 - 72xy^3 + 42y^4 \\ x^4 - 6x^3y + 12xy^2 - 7y^3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 x^2 - 6r^2j + 12rj^2 - 7j^3 \Big) \frac{x^4 - 6r^2j + 12r^2j^2 - 71j^3}{x^4 - 6r^2j + 12r^2j^2 - 71j^3} \Big(r + 5j \\
 \frac{51j^2 - 271j^2 + 45rj^2 - 14j^4}{51j^2 - 271j^2 + 601j^3 - 35j^4} \\
 \frac{3j \Big) \frac{3r^2j^2 - 151j^3 + 21j^4}{r^2 - 51j + 7j^2}}{x^2 - 5xj + 7j^2} \Big) \frac{x^2 - 6x^2j + 12xj^2 - 7j^2}{x^2 - 5x^2j + 7rj^2} \Big(x - j \\
 \frac{-x^2j + 5rj^2 - 7j^3}{-x^2j + 51j^2 - 7j^3}
 \end{array}$$

Thus the reqd $H C F = x^2 - 51j + 7j^2$

6 Let him buy x oranges of the 1st sort, then he bought $(570 - x)$ oranges of the other sort

$$\text{We have } \frac{x}{16} + \frac{570 - x}{18} + 3 = \frac{570}{15}$$

$$\text{or, } x \left(\frac{1}{16} - \frac{1}{18} \right) = 570 \left(\frac{1}{15} - \frac{1}{18} \right) - 3$$

$$\text{or } x \left(\frac{9 - 8}{8 \times 18} \right) = 570 \left(\frac{18 - 15}{15 \times 18} \right) - 3$$

$$\text{or, } \frac{x}{8 \times 18} = \frac{19}{3} - 3$$

$$\text{or } \frac{x}{8 \times 18} = \frac{10}{3} \quad x = 480$$

Thus there were 480 oranges of the 1st sort and 90 oranges of the 2nd sort

7 The given expression

$$\begin{aligned}
 &= \frac{a^2}{(a-c)(a-b)} + \frac{b^2}{(b-a)(b-c)} + \frac{c^2}{(c-b)(c-a)} \\
 &= \frac{-a^2(b-c) - b^2(c-a) - c^2(a-b)}{(a-b)(b-c)(c-a)} \\
 &= \frac{-\{a^2(b-c) + b^2c - a\} + c^2(a-b)}{-\{a^2(b-c) + b^2c - a\} + c^2(a-b)} = 1
 \end{aligned}$$

$$8 \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = l \text{ (suppose)}$$

$$a = bk, c = dk \text{ and } e = fk,$$

$$(a^2 + c^2 + e^2)(b^2 + d^2 + f^2)$$

$$= (b^2k^2 + d^2k^2 + f^2k^2)(b^2 + d^2 + f^2)$$

$$= k^2(b^2 + d^2 + f^2)^2$$

$$\begin{aligned}
 \text{Also, } (ab+cd+ef)^2 & \\
 &= (b^2k + d^2l + f^2h)^2 \\
 &= k^2(b^2 + d^2 + f^2)^2 \\
 &= (a^2 + c^2 + e^2) b^2 + d^2 + f^2 = (ab+cd+ef)^2
 \end{aligned}$$

XII

$$\begin{aligned}
 1 \quad & x^2 + y^2 + z^2 - yz - zx - xy \\
 &= \frac{1}{2} \{ (x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2) \} \\
 &= \frac{1}{2} \{ (x-y)^2 + (y-z)^2 + (z-x)^2 \} \\
 &= \frac{1}{2} \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \} \\
 &= \frac{1}{2} (2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) \\
 &= a^2 + b^2 + c^2 - bc - ca - ab \\
 2 \quad & \text{The given expression} \\
 &= \frac{(y+z)(x^2-yz) + (z+x)(y^2-xz) + (x+y)(z^2-xy)}{(x^2-yz)(y^2-xz)(z^2-xy)} \\
 &= \frac{x^2(y+z-x-y) + y(z+x-z-x) + z^2(1+y-y-1)}{(x^2-yz)(y^2-xz)(z^2-xy)} = 0
 \end{aligned}$$

$$\begin{aligned}
 3 \quad (i) \quad & x^2 - 2ax + b^2 + 2ab \\
 &= x^2 - 2ax + a^2 - (a^2 + b^2 - 2ab) \\
 &= (x-a)^2 - (a-b)^2 \\
 &= (x-a+a-b)(x-a-a+b) \\
 &= (x-b)(x-2a+b)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & x^2 + (a+b+c)x + ab+ac \\
 &= x^2 + ax + (b+c)x + a(b+c) \\
 &= x(1+a) + (b+c)(1+a) \\
 &= (x+a)(1+b+c)
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 9x^4 + 80x^2 - 9 \\
 &= 9x^4 + 81x^2 - x^2 - 9 \\
 &= 9x^2(x^2+9) - (x^2+9) \\
 &= (9x^2-1)(x^2+9) \\
 &= (3x+1)(3x-1)(x^2+9)
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } & 6x^4 - 21x^3 + 9x^2 + 9x - 4 \\
 &= 6x^4 - 21x^3 + 9x^2 - 3x + 12x - 4 \\
 &= 2x^2(3x-1) + 3x(3x-1) + 4(3x-1) \\
 &= (2x^2+3x+4)(3x-1)
 \end{aligned}$$

Thus $3x-1$ is the reqd H C F

$$5 \quad \frac{6x+13}{15} - \frac{3x+5}{5x-25} - \frac{2x}{5} = 0$$

$$\text{or,} \quad \frac{6x+13}{3} - \frac{3x+5}{x-5} - 2x = 0$$

$$\text{or,} \quad \frac{6x+13-6x}{3} = \frac{3x+5}{x-5}$$

$$\text{or,} \quad 91+15=13x-65$$

$$\text{or,} \quad 4x=80, \quad x=20$$

6 Let A , B and C together do the work in 2 days

But A and B can do $\frac{1}{12}$ of the work in 1 day

A and C $\frac{1}{15}$

B and C $\frac{1}{20}$

A , B and C together can do

$(\frac{1}{12} + \frac{1}{15} + \frac{1}{20})$ of the work in 2 days,

$$\text{we have } \frac{2}{x} = \frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{5+4+3}{60} = \frac{12}{60} = \frac{1}{5},$$

$$x=10$$

Thus A , B and C can do the work in 10 days, all working together

$$\begin{aligned} 7 \quad & 4\sqrt{147} - 3\sqrt{75} - 6\sqrt{3} + 18\sqrt{\frac{1}{3}} \\ &= 4\sqrt{49 \cdot 3} - 3\sqrt{25 \cdot 3} - 2\sqrt{3} + \frac{18}{\sqrt{3}} \\ &= 4 \times 7\sqrt{3} - 3 \times 5\sqrt{3} - 2\sqrt{3} + \frac{6 \times 3}{\sqrt{3}} \\ &= 28\sqrt{3} - 15\sqrt{3} - 2\sqrt{3} + 2\sqrt{3} = 13\sqrt{3} \end{aligned}$$

$$8 \quad \frac{x}{y} = \frac{a}{b} = k \text{ (suppose),} \quad x = ky \text{ and } a = kb,$$

$$\begin{aligned} & \frac{x^2+a^2}{x+a} + \frac{y^2+b^2}{y+b} = \frac{k^2y^2+k^2b^2}{ky+kb} + \frac{y^2+b^2}{y+b} \\ &= \frac{k^2(y^2+b^2)}{k(y+b)} + \frac{y^2+b^2}{y+b} = k \frac{y^2+b^2}{y+b} + \frac{y^2+b^2}{y+b} \\ &= (k+1) \left(\frac{y^2+b^2}{y+b} \right) \end{aligned}$$

$$\begin{aligned}
 \text{Also, } & \frac{(x+y)^2 + (a+b)^2}{x+y+a+b} \\
 &= \frac{y^2(k+1)^2 + b^2(l+1)^2}{y(k+1) + b(l+1)} = \frac{(y^2 + b^2)(k+1)^2}{(k+1)(y+b)} \\
 &= (k+1) \left(\frac{y^2 + b^2}{y+b} \right) \\
 &= \frac{x^2 + a^2}{x+a} + \frac{y^2 + b^2}{y+b} = \frac{(x+y)^2 + (a+b)^2}{x+y+a+b}
 \end{aligned}$$

XIII

$$\begin{aligned}
 1 \quad & a(b-c)(s-a)^2 + b(c-a)(s-b)^2 + c(a-b)(s-c)^2 \\
 &= a(b-c)(s^2 - 2as + a^2) + b(c-a)(s^2 - 2sb + b^2) + c(a-b)(s^2 - 2sc + c^2) \\
 &= s^2\{a(b-c) + b(c-a) + c(a-b)\} - 2s\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\
 &\quad + a^3(b-c) + b^3(c-a) + c^3(a-b) \\
 &= 0 - (a+b+c)\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} + a^3(b-c) + b^3(c-a) \\
 &\quad + c^3(a-b) \\
 &= -\{a^3(b-c) + b^3(c-a) + c^3(a-b)\} \\
 &\quad - \{a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)\} \\
 &\quad + a^3(b-c) + b^3(c-a) + c^3(a-b) = 0
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & x^6 + x^3a^3 + a^6 = (x^2)^3 + (x^2a^{\frac{1}{2}})^2 + (a^{\frac{1}{2}})^4 \\
 &= \{(x^2)^2 + x^2a^{\frac{3}{2}} + (a^{\frac{3}{2}})^2\} \{(x^2)^2 - x^2a^{\frac{3}{2}} + (a^{\frac{3}{2}})^2\} \\
 &= (x^4 + x^2a^{\frac{3}{2}} + a^3)(x^4 - x^2a^{\frac{3}{2}} + a^3)
 \end{aligned}$$

$$\begin{aligned}
 \text{Again, } & x^3 + x^{\frac{3}{2}}a^{\frac{3}{2}} + a^3 \\
 &= (x^{\frac{3}{2}})^4 + (x^{\frac{3}{2}}a^{\frac{3}{4}})^2 + (a^{\frac{3}{4}})^4 \\
 &= \{(x^{\frac{3}{2}})^2 + x^{\frac{3}{2}}a^{\frac{3}{4}} + (a^{\frac{3}{4}})^2\} \{(x^{\frac{3}{2}})^2 - x^{\frac{3}{2}}a^{\frac{3}{4}} + (a^{\frac{3}{4}})^2\} \\
 &= (x^3 + x^{\frac{3}{2}}a^{\frac{3}{4}} + a^{\frac{3}{2}})(x^3 - x^{\frac{3}{2}}a^{\frac{3}{4}} + a^{\frac{3}{2}})
 \end{aligned}$$

$$\begin{aligned}
 \text{Lastly, } & x^{\frac{3}{2}} + x^{\frac{3}{4}}a^{\frac{3}{4}} + a^{\frac{3}{2}} \\
 &= (x^{\frac{3}{4}})^4 + (x^{\frac{3}{8}}a^{\frac{3}{8}})^2 + (a^{\frac{3}{8}})^4 \\
 &= \{(x^{\frac{3}{4}})^2 + x^{\frac{3}{4}}a^{\frac{3}{8}} + (a^{\frac{3}{8}})^2\} \{(x^{\frac{3}{4}})^2 - x^{\frac{3}{4}}a^{\frac{3}{8}} + (a^{\frac{3}{8}})^2\} \\
 &= (x^{\frac{3}{2}} + x^{\frac{3}{4}}a^{\frac{3}{8}} + a^{\frac{3}{2}})(x^{\frac{3}{2}} - x^{\frac{3}{4}}a^{\frac{3}{8}} + a^{\frac{3}{2}})
 \end{aligned}$$

Therefore, $x^6 + x^3a^3 + a^6$ is divisible by $x^{\frac{3}{2}} - x^{\frac{3}{4}}a^{\frac{3}{4}} + a^{\frac{3}{2}}$

3 The given expression

$$= \frac{(1^2 - yz)(y+z) + (y^2 - rz)(x+z) + (z^2 - xy)(x+y)}{(x+y)(y+z)(z+x)}$$

$$= \frac{1^2(y+z-x-y) + y^2(-z+x+z-x) + z^2(-y-x+x+y)}{(x+y)(y+z)(z+x)} = 0$$

4
$$\frac{x^2 - a^2}{x-a} + \frac{x^2 - b^2}{x-b} + \frac{x^2 - c^2}{x-c} = a+b+c-3x$$

or,
$$x+a+x+b+x+c-a-b-c-3x$$

or,
$$6x=0, \quad x=0$$

5. Let x be the reqd number of gallons His total cost
 $= 80 \times 15$ shillings, he must sell the mixture for

$$\frac{110}{100} \times 80 \times 15 = 11 \times 8 \times 15 \text{ shillings,}$$

we have $12(80+x) = 11 \times 8 \times 15$

or,
$$80+x=110, \quad x=30$$

Thus he must mix 30 gallons of water

6
$$\frac{x^{(a+b)^2} x^{b+c^2} x^{c+a^2}}{(x^a)^2 (x^b)^2 (x^c)^2}$$

$$= \frac{x^{a^2+ab+b^2+2b+c^2+ca}}{x^{2a+2b+2c}} = \frac{x^{a^2+ab+bc}}{x^{2a+2b+2c}} = 1$$

7
$$3\sqrt[3]{128} - 4\sqrt[3]{-686} + 2\sqrt[3]{54}$$

$$= 3\sqrt[3]{64 \times 2} - 4\sqrt[3]{-343 \times 2} + 2\sqrt[3]{27 \times 2}$$

$$= 3 \times 4\sqrt[3]{2} - 4 \times -7\sqrt[3]{2} + 2 \times 3\sqrt[3]{2}$$

$$= 12\sqrt[3]{2} + 28\sqrt[3]{2} + 6\sqrt[3]{2} = 46\sqrt[3]{2}$$

8
$$\frac{a}{b} = \frac{b}{c} = k \text{ (suppose),} \quad a = bk \text{ and } b = ck, \quad a = ck^2,$$

$$\frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{k^2 b^2 + k b k c + k^2 c^2}{b^2 + bc + c^2}$$

$$= \frac{k^2(b^2 + bc + c^2)}{b^2 + bc + c^2} = k^2$$

Also,
$$\frac{a}{c} = \frac{ck^2}{c} = k^2 \quad a^2 + ab + b^2 \quad b^2 + bc + c^2 \quad a \quad c.$$

XIV.

1 We have

$$a+b+c=2s \quad c-s=s-(a+b)$$

$$\begin{aligned} \text{Hence, } (a-s)^2 + (b-s)^2 + (c-s)^2 \\ &= (a-s)^2 + (b-s)^2 + \{s-(a+b)\}^2 \\ &= a^2 - 2as + s^2 + b^2 - 2bs + s^2 + s^2 - 2s(a+b) \\ &= 2(a^2 + b^2 + ab + s^2) + s^2 - 4s(a+b) \\ &= 4s(a+b) + s^2 - 4s(a+b) = s^2 \end{aligned}$$

$$2 \quad \frac{x+a}{x^2+ax} \cdot \frac{x^2+px+q}{x(a-p)+q} \cdot \frac{-x(a-p)-a(a-p)}{a(a-p)+q}$$

In order $x+a$ may be a factor of x^2+px+q , $a(a-p)+q=0$ (1)Similarly $a(a-l)+m=0$ (2)From (1) and (2), we have $a(l-p)-(m-q)=0$

$$a = \frac{m-q}{l-p}$$

$$3 \quad \frac{7+3\sqrt{5}}{7-3\sqrt{5}} + \frac{7-3\sqrt{5}}{7+3\sqrt{5}} = \frac{(7+3\sqrt{5})^2 + (7-3\sqrt{5})^2}{49-45} \\ = \frac{2(49+45)}{4} = \frac{94}{2} = 47$$

$$4 \quad \frac{a-b}{x-a} + \frac{a-b}{x-b} = \frac{a}{x-a} - \frac{b}{x-b}$$

$$\text{or, } \frac{(a-b)(x-b) + (a-b)(x-a)}{(x-a)(x-b)} = \frac{a(x-b) - b(x-a)}{(x-a)(x-b)}$$

$$\text{or, } -b(x-b) + a(x-a) = 0$$

$$\text{or, } (x-b)b = a(x-a)$$

$$\text{or, } xb - a^2 = ax - a^2$$

$$\text{or, } x(b-a) = b^2 - a^2$$

$$x = a+b$$

$$5 \quad \frac{y+z-x}{b+c} = \frac{z+x-y}{c+a} = \frac{x+y-z}{a+b} = 1$$

by addendo, we get

$$1 = \frac{x+y+z}{2(a+b+c)}$$

$$x + y + z = 2(a + b + c)$$

Also, $y + z - x = b + c$

$$2x = 2a + b + c$$

$$x = \frac{1}{2}(2a + b + c)$$

Again we have $z + x - y = c + a$

and, $x + y + z = 2(a + b + c)$

$$y = \frac{1}{2}(a + 2b + c)$$

Similarly $z = \frac{1}{2}(a + b + 2c)$

- 6 Suppose A worked for x days B worked for $(14 - x)$ days,
 A finished $\frac{x}{20}$ of the work and B did the remaining portion.

which is equal to $\frac{14 - x}{12}$

Therefore, we have $\frac{14 - x}{12} = 1 - \frac{x}{20}$

or, $70 - 5x = 60 - 3x$ or, $2x = 10$
 $x = 5$

Thus A worked for 5 days

7 $(x + a)(x + 2a)(x + 3a)(x + 4a)$
 $= \{(x + a)(x + 4a)\}\{(x + 2a)(x + 3a)\}$
 $= (x^2 + 5ax + 4a^2)(x^2 + 5ax + 6a^2)$
 $= \{(x^2 + 5ax + 5a^2) - a^2\}\{(x^2 + 5ax + 5a^2) + a^2\}$
 $= (x^2 + 5ax + 5a^2)^2 - a^4$

8 $a(y + z) = b(z + x) = c(x + y)$

$$\frac{a(y + z)}{abc} = \frac{b(z + x)}{abc} = \frac{c(x + y)}{abc}$$

or, $\frac{y + z}{bc} = \frac{z + x}{ac} = \frac{x + y}{ab} = k$ (suppose),

$$y + z = kbc \quad (1)$$

$$z + x = kac \quad (2)$$

$$x + y = kab \quad (3)$$

From (1) and (2), $x - y = ka(a - b)$,

$$\frac{x - y}{a(a - b)} = k$$

Similarly from (1) and (3), $\frac{z-r}{b(c-a)} = k$

and from (2) and (3) $\frac{y-r}{a(b-c)} = k$

Hence, $\frac{y-r}{a(b-c)} = \frac{z-r}{b(c-a)} = \frac{x-y}{c(a-b)}$

XV

$$\begin{array}{r}
 1 \quad x^4 + 2ax^3 + (a^2 + 8)x^2 + (4a + ab)x + 4b \quad \left(x^2 + ax + 4 \right) \\
 \underline{2x^2 + ar} \quad \left[2ax^3 + (a^2 + 8)x^2 + (4a + ab)x + 4b \right] \\
 \quad \quad \quad \underline{2ax^3 + a^2x^2} \\
 2x^2 + 2ar + 4 \quad \left[8x^2 + (4a + ab)x + 4b \right] \\
 \quad \quad \quad \underline{8x^2 + 8ax + 16} \\
 \quad \quad \quad \quad \quad \quad \underline{-(4a - ab)x + 4b - 16}
 \end{array}$$

In order the expression may be a perfect square, we must have

$$4 - ab = 0 \quad (1)$$

$$\text{also} \quad 4b - 16 = 0 \quad (2)$$

From both these conditions, we get $b = 4$

2 The left-hand expression

$$\begin{aligned}
 &= (b-c)\{1 + a(b+c) + a^2bc\} + (c-a)\{1 + b(c+a) + b^2ca\} \\
 &\quad + (a-b)\{1 + c(a+b) + c^2ab\} \\
 &= \{(b-c) + (c-a) + (a-b)\} + abc\{a(b-c) + b(c-a) + c(a-b)\} \\
 &\quad + \{a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)\} \\
 &= a(b^4 - c^3) + b(c^3 - a^3) + c(a^3 - b^3) \\
 &= -\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\
 &= -\{-(a-b)(b-c)(c-a)\} = (a-b)(b-c)(c-a)
 \end{aligned}$$

3 The given expression

$$\begin{aligned}
 &= \frac{2x^2 + 2}{x^4 + x^2 + 1} + \frac{x - \sqrt{x+1} + x + \sqrt{x+1}}{x^2 + x + 1} - \frac{1}{x^2 - x + 1} \\
 &= \frac{2x^2 + 2}{x^4 + x^2 + 1} + \frac{2x + 2}{x^2 + x + 1} - \frac{1}{x^2 - x + 1} \\
 &= \frac{2x^2 + 2 + 2x^3 + 2 - x^2 - x - 1}{x^4 + x^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2x^3 + x^2 - x + 3}{x^4 + x^2 + 1} = \frac{2x^3 + 3x^2 - 2x^2 - 3x + 2x + 3}{x^4 + x^2 + 1} \\
 &= \frac{x^2(2x + 3) - x(2x - 3) + (2x + 3)}{x^4 + x^2 + 1} \\
 &= \frac{(x^2 - x + 1)(2x + 3)}{(x^2 - x + 1)(x^2 + x + 1)} = \frac{2x + 3}{x^2 + x + 1}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 2x^3 + (2a - 3b)x^2 - (2b + 3ab)x + 3b^3 \\
 &= 2x^3 + 2ax^2 - 2bx - 3bx^2 - 3abx + 3b^3 \\
 &= 2x(x^2 + ax - b) - 3b(x^2 + ab - b) = (x^2 + ax - b)(2x - 3b) \\
 &\quad 2x^2 - (3b - 2c)x - 3bc = 2x^2 + 2cx - 3bx - 3bc \\
 &= 2x(x + c) - 3b(x + c) = (x + c)(2x - 3b) \\
 &\quad \text{the reqd H C F} = 2x - 3b
 \end{aligned}$$

$$5 \quad \text{We have } 1 = \frac{a^n + b^n}{2}$$

$$2m = na^n + nb^n$$

$$\begin{aligned}
 \text{the given exp} &= \frac{a^n}{2na^n - na^n - nb^n} + \frac{b^n}{2nb^n - na^n - nb^n} \\
 &= \frac{a^n}{na^n - nb^n} + \frac{b^n}{nb^n - na^n} = \frac{a^n - b^n}{na^n - nb^n} = \frac{1}{n}
 \end{aligned}$$

$$6 \quad \frac{201 + 36}{25} + \frac{51 + 20}{91 - 16} = \frac{41}{5} + \frac{86}{25}$$

$$\begin{aligned}
 \text{or,} \quad \frac{51 + 20}{91 - 16} &= \frac{41}{5} + \frac{86}{25} - \frac{201 + 36}{25} \\
 &= \frac{201 + 86 - 201 + 36}{25} = \frac{50}{25} = 2
 \end{aligned}$$

$$\text{or,} \quad 51 + 20 = 181 - 31$$

$$\text{or} \quad 131 = 52, \quad x = 4$$

7 Let x gallons be the contents of the vessel

there were $\frac{71}{10}$ gallons of spirit

of 9 gallons of the mixture, there were $\frac{79}{10}$ or $\frac{63}{10}$ gallons of spirit

After 9 gallons have been taken there remains in the vessel

$\left(\frac{7\epsilon}{10} - \frac{63}{10}\right)$ gallons of spirit

$$\text{we have, } \frac{7x-63}{10} = 58\frac{1}{10} \quad 100$$

$$\text{or, } \frac{7x-63}{10\tau} = \frac{175}{300}$$

$$\text{or, } \frac{7x-63}{1} = \frac{35}{6}$$

$$\text{or, } \frac{x-9}{x} = \frac{5}{6}$$

$$\text{or, } 6x-54=5x, \quad x=54$$

Thus the contents of the vessel = 54 gallons

$$8 \quad \frac{1-z}{y-z} = \frac{x^2}{y^2}, \quad \frac{x-y}{y-z} = \frac{x^2-y^2}{y^2}$$

$$\text{or, } \frac{1}{y-z} = \frac{x+y}{y^2}, \quad \frac{y}{y^2-yz} = \frac{x+y}{y^2} = \frac{1}{yz} \quad (\text{addendo}),$$

$$\frac{x+y}{y} = \frac{x}{z} \quad \text{or, } \frac{x+2y}{y} = \frac{x+z}{z} \quad (1)$$

$$\text{Again, } \frac{x+y}{z} = \frac{y}{z}, \quad \frac{2x+y}{z} = \frac{y+z}{z} \quad (2)$$

Hence, from (1) and (2), we have

$$\frac{x+z}{y+z} = \frac{\frac{x+2y}{y}}{\frac{y+z}{y}} = \frac{\frac{y}{x}}{\frac{y}{x}+2}$$

$$x+z \quad y+z = \frac{y}{x} + 2 \quad \frac{y}{x} + 2$$

XVI.

$$1 \quad \left. \begin{array}{l} x^6 + 2x^4 - 5x^3 - 7x + 3 \end{array} \right) \begin{array}{l} 3x^6 - 3x^4 - 18x^3 + x^2 + 2x + 3 \\ \hline 3x^6 - x^5 - 5x^4 - 18x^3 + 6x^2 + 9x \\ \hline 3x^6 + 6x^5 \quad \quad - 15x^4 - 21x^3 + 9x \\ \hline -x^2 - 7x^5 - 5x^4 - 3x^3 + 27x^2 \\ \hline 7x^3 + 5x^2 + 3x - 27 \end{array}$$

$$\begin{array}{r}
x^5 + 2x^4 - 5x^3 - 7x^2 + 3 \\
7 \overline{) 7x^5 + 14x^4 - 35x^3 - 49x^2 + 21} \left(x^2 \right. \\
\underline{7x^5 + 5x^4 + 3x^3 - 27x^2} \\
9x^4 - 3x^3 - 8x^2 - 49x + 21 \\
7 \overline{) 63x^4 - 21x^3 - 56x^2 - 343x + 147} \left(9x \right. \\
\underline{63x^4 + 45x^3 + 27x^2 - 243x} \\
-66x^2 - 83x^2 - 100x + 147 \\
7 \overline{) -462x^2 - 581x^2 - 700x + 1029} \left(-66 \right. \\
\underline{-462x^2 - 330x^2 - 198x + 1782} \\
-251 \mid -251x^2 - 502x - 753 \\
 x^2 + 2x - 3 \\
x^2 + 2x + 3 \overline{) 7x^3 + 5x^2 + 3x - 27} \left(7x - 9 \right. \\
\underline{7x^3 + 14x^2 + 21x} \\
-9x^2 - 18x - 27 \\
\underline{-9x^2 - 18x - 27}
\end{array}$$

Thus the reqd H C F $= x^2 + 2x + 3$

$$\begin{aligned}
2 \quad & \sqrt{2x-1} + \sqrt{3x-2} = \sqrt{4x-3} + \sqrt{5x-4} \\
\text{or, } & \sqrt{2x-1} - \sqrt{5x-4} = \sqrt{4x-3} - \sqrt{3x-2} \\
\text{or, } & 2x-1+5x-4-2\sqrt{10x^2-13x+4} \\
& = 4x-3+3x-2-2\sqrt{12x^2-17x+6} \\
\text{or, } & 10x^2-13x+4=12x^2-17x+6 \\
\text{or, } & 2x^2-4x+2=0 \\
\text{or, } & 2(x-1)^2=0, \quad x=1
\end{aligned}$$

$$3 \quad (a+b+c)x = (-a+b+c)y = (a-b+c)z = (a+b-c)w$$

$$\frac{1}{y} = \frac{-a+b+c}{a+b+c} \cdot \frac{1}{x}$$

$$\frac{1}{z} = \frac{a-b+c}{a+b+c} \cdot \frac{1}{x}$$

$$\frac{1}{w} = \frac{a+b-c}{a+b+c} \cdot \frac{1}{x}$$

$$\frac{1}{y} + \frac{1}{z} - \frac{1}{w} = \frac{a+b+c}{a+b+c} \cdot \frac{1}{x} = \frac{1}{x}$$

$$4 \quad \frac{5\sqrt{x+y}}{x} + \frac{5\sqrt{x+y}}{y} = 10\frac{2}{3} \quad (1)$$

$$\frac{3\sqrt{x-y}}{y} - \frac{3\sqrt{x-y}}{x} = \frac{4}{5} \quad (2)$$

$$\text{From (1), we have } \frac{5\sqrt{x+y}(x+y)}{xy} = \frac{20}{3} \quad (3)$$

$$\text{From (2), we have } \frac{3\sqrt{x-y}(x-y)}{xy} = \frac{4}{5} \quad (4)$$

$$\text{From (3) and (4), we have } \frac{5(x+y)^{\frac{3}{2}}}{3(x-y)^{\frac{3}{2}}} = \frac{10}{3} \quad \text{FI}$$

$$\text{or, } \left(\frac{x+y}{x-y}\right)^{\frac{3}{2}} = 8 = (4)^{\frac{3}{2}}$$

$$\text{or, } \frac{x+y}{x-y} = 4$$

$$\frac{x}{y} = \frac{5}{3} \quad x = \frac{5y}{3}$$

$$\text{from (4), we have } 3\left(\frac{5y}{3} - y\right)^{\frac{3}{2}} = \frac{4}{5} \frac{5y^{\frac{3}{2}}}{3}$$

$$\text{or, } 3\left(\frac{2y}{3}\right)^{\frac{3}{2}} = \frac{4y^{\frac{3}{2}}}{3} \quad \text{or, } \frac{9(2y)^{\frac{3}{2}}}{27} = \frac{16y^{\frac{3}{2}}}{9}$$

$$y = \frac{1}{2} = 1\frac{1}{2} \quad x = \frac{5}{2} \cdot \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$$

$$5 \quad ax(y^3 + b^3) + by(b^3 + a^3y) = b^3x^2y + ab^3x + axy^3 + a^3by^2 \\ = b^3x(1y + ab) + ay^2(xy + ab) = (1y + ab)(b^3x + ay^2)$$

$$6 \quad (\sqrt{a} + \sqrt{b} + \sqrt{c})(\sqrt{a} + \sqrt{b} - \sqrt{c}) \\ (\sqrt{a} - \sqrt{b} + \sqrt{c})(\sqrt{b} + \sqrt{c} - \sqrt{a}) \\ = \{(\sqrt{a} + \sqrt{b}) + \sqrt{c}\}\{(\sqrt{a} + \sqrt{b}) - \sqrt{c}\} \\ \{\sqrt{c} + (\sqrt{a} - \sqrt{b})\}\{\sqrt{c} - (\sqrt{a} - \sqrt{b})\} \\ = (a + 2\sqrt{ab} + b - c)(c - a + 2\sqrt{ab} - b) \\ = \{2\sqrt{ab} + (a + b - c)\}\{2\sqrt{ab} - (a + b - c)\} \\ = 4ab - (a + b - c)^2 \\ = 4ab - (a^2 + b^2 + c^2 + 2ab - 2ac - 2bc) \\ = -a^2 - b^2 - c^2 + 2ab + 2ac + 2bc$$

$$7 \quad \frac{a+b}{a-b} = \frac{c}{d} \qquad \frac{a}{b} = \frac{c+d}{c-d}, \qquad |$$

$$\text{we have, } \frac{a}{b} \cdot \frac{a+b}{a-b} = \frac{c+d}{c-d} \cdot \frac{c}{d}$$

$$\text{Thus, } \frac{a^2+ab}{ab-b^2} = \frac{c^2+cd}{cd-d^2}$$

8 Let there be x per cent wine in the 1st vessel and y per cent wine in the other vessel

from the 1st mixture, we get

$$x+y=(100-x)+(100-y)$$

$$\text{or, } 2x+2y=200$$

$$\text{or, } x+y=100 \qquad (1)$$

From the second mixture, we have

$$\frac{4x+y}{4(100-x)+(100-y)} = \frac{2}{3}$$

$$\text{or, } \frac{4x+y}{500-(4x+y)} = \frac{2}{3}$$

$$\text{or, } 3(4x+y)=1000-2(4x+y)$$

$$\text{or, } 5(4x+y)=1000$$

$$\text{or, } 4x+y=200 \qquad (2)$$

$$\text{Also, } x+y=100 \qquad (1)$$

$$3x=100, \qquad \frac{x}{100} = \frac{1}{3},$$

$$\text{from (1), } 3y=300-100=200, \qquad \frac{y}{100} = \frac{2}{3},$$

in the 1st vessel the wine is $\frac{1}{3}$ of the whole, in the second $\frac{2}{3}$

XVII

$$\begin{aligned} 1 \quad & x^6 + x^2 + 2x + 2 = x^2(x^2 + 1) + 2(x + 1) \\ & = (x + 1)\{x^2(x^2 + 1) + 2\} = (x + 1)(x^4 - x^2 + x^2 + 2) \\ & = (x + 1)\{(x^4 + x^2 + 1) - (x^2 - 1)\} \\ & = (x + 1)\{(x^2 + x + 1)(x^2 - x + 1) - (x - 1)(x^2 + x + 1)\} \\ & = (x + 1)(x^2 + x + 1)(x^2 - 2x + 2) \end{aligned}$$

$$\text{and } x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$$

Thus the reqd H C F. = $x^2 + x + 1$

2

$$\sqrt{y} - \sqrt[3]{y-x} = \sqrt{20-x} \quad (1)$$

$$\sqrt{y-x} + \sqrt{20-x} = 3 \quad (2)$$

$$\text{From (2), } \frac{\sqrt{y-x}}{\sqrt{20-x}} = \frac{3}{2}$$

$$\text{or, } \frac{y-x}{20-x} = \frac{9}{4}$$

$$\text{or, } 4y - 4x = 180 - 9x$$

$$\text{or, } 5x + 4y = 180 \quad (3)$$

$$\text{From (1), } \frac{\sqrt{y}}{\sqrt{20-x}} - \frac{\sqrt{y-x}}{\sqrt{20-x}} = 1$$

$$\text{or, } \frac{\sqrt[3]{y}}{\sqrt{20-x}} - \frac{3}{2} = 1$$

$$\text{or, } \frac{\sqrt{y}}{\sqrt{20-x}} = \frac{5}{2}, \quad \frac{y}{20-x} = \frac{25}{4},$$

$$4y = 500 - 25x$$

$$\text{or, } 25x + 4y = 500 \quad (4)$$

$$\text{also, } 5x + 4y = 180 \quad (3)$$

$$20x = 320 \quad x = 16$$

$$\text{from (4) } 4y = 500 - 400 = 100 \quad y = 25$$

3

$$\left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2 = \frac{x^2\{(x+1)^2 + (x-1)^2\}}{(x^2-1)^2}$$

$$= \frac{2x^2(x^2+1)}{(x^2-1)^2}$$

$$= \frac{2(n-1)\left(\frac{n-1}{n+1} + 1\right)}{\left(\frac{n-1}{n+1} - 1\right)^2}$$

$$= \frac{2(n-1)2n}{\left(\frac{-2}{n+1}\right)^2} = \frac{4n(n-1)}{4} = n(n-1)$$

4 The numerator

$$\begin{aligned}
 &= a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2) \\
 &= a^2(b^2 - c^2) + a^2bc(b - c) - b^2(c^2 - a^2) + b^2ac(c - a) \\
 &\quad + c^2(a^2 - b^2) + c^2ab(a - b) \\
 &= a^2(b - c)ab + ac - bc + b^2c - a^2(bc - ab + ac) \\
 &\quad + c^2(a - b)(ac + bc + ab) \\
 &= (ab + bc + ab)\{a^2(b - c) + b^2(c - a) + c^2(a - b)\} \\
 \text{the given fraction} &= ab + bc - ca
 \end{aligned}$$

$$\begin{aligned}
 5 \quad a + b - c &= 0 & a + b &= -c & (a + b)^2 &= c^2 \\
 a^2 + b^2 &= c^2 - 2ab & a^2 + b^2 - c^2 &= 2(c^2 - ab) \\
 (a^2 - b^2 + c^2)^2 &= 4(c^2 - ab)^2 = 4(c^2c^2 + a^2b^2 - 2c^2ab) \\
 &= 4\{c^2(a + b)^2 - 2c^2an - a^2b^2\} \\
 &= 4\{c^2(a^2 + b^2 + 2ab) - 2c^2ab - a^2b^2\} \\
 &= 4(a^2b^2 + a^2c^2 - b^2c^2)
 \end{aligned}$$

Putting a for $(j - z)$, b for $(z - x)$ and c for $(x - j)$,

$$\begin{aligned}
 \text{We get } a + b + c &= 0 & 4(a^2b^2 + b^2c^2 + c^2a^2) &= (a^2 - b^2 - c^2)^2 \\
 &4\{(j - z)^2(z - x)^2 + (z - x)^2(x - j)^2 + (x - j)^2(j - z)^2\} \\
 &= \{(j - z)^2 + (z - x)^2 + (x - j)^2\}^2 \\
 &= \{j^2 + z^2 - 2jz + z^2 + x^2 - 2xz - x^2 + j^2 - 2xj\}^2 \\
 &= 4\{z^2 + j^2 - z^2 - 2j - jz - 2xz\}^2 \\
 &(j - z)^2(z - x)^2 + (z - x)^2(x - j)^2 + (x - j)^2(j - z)^2 \\
 &= (z^2 + j^2 - z^2 - xj - jz - xz)^2
 \end{aligned}$$

6 Let x and $(1 - j)$ be the digits of the required number

$$\text{the number} = 10r - (1 - j) = 11r - j$$

$$\text{we get } 10(1 - j) + x = \frac{1}{5}(11r - j)$$

$$\text{or, } 881 - 400 = 33r - 15$$

$$\text{or, } 55x = 385 \quad x = 7,$$

the required number is 72

7 The numerator

$$\begin{aligned}
 &= 3x^3 - x^2 - 5x + 21 = 3x^3 - 7x^2 - 6x^2 - 14x - 9x + 21 \\
 &= x^2(3x + 7) - 2x(3x + 7) - 3(3x + 7) = (3x + 7)(x^2 - 2x + 3)
 \end{aligned}$$

The denominator

$$\begin{aligned}
 &= 6x^3 + 29x^2 + 26x - 21 = 6x^3 - 14x^2 - 15x^2 + 35x - 91 - 21 \\
 &= 2x^2(3x + 7) - 5x(3x + 7) - 3(3x + 7) \\
 &= (3x + 7)(2x^2 + 5x - 3)
 \end{aligned}$$

$$\text{Hence the expression} = \frac{x^2 - 2x + 3}{2x^2 + 5x - 3}$$

8

$$\begin{aligned}
 3(a^2 + b^2 + c^2) &= (a + b + c)^2 \\
 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\
 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca &= 0
 \end{aligned}$$

$$\text{or, } (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac) = 0$$

$$\text{or, } (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

Each of the terms being a perfect square, none of the terms is negative, therefore their sum cannot be zero unless each of them be zero

$$\text{We have } a - b = 0 \text{ or, } a = b \text{ and } b - c = 0 \text{ or, } b = c$$

$$a = b = c$$

XVIII

1 Putting a for $(x - y)$, b for $(y - z)$ and c for $(z - x)$,

we get $a + b + c = x - y + y - z + z - x = 0$

$$(a^2 + b^2 + c^2)^2 = 4(a^2b^2 + b^2c^2 + c^2a^2)$$

[See Ex 5 XVII Page 383]

$$a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2 = 4(a^2b^2 + b^2c^2 + c^2a^2)$$

$$a^4 + b^4 + c^4 = 2(a^2b^2 + b^2c^2 + c^2a^2)$$

$$2(a^4 + b^4 + c^4) = 4(a^2b^2 + b^2c^2 + c^2a^2) = (a^2 + b^2 + c^2)^2$$

$$2\{(x - y)^4 + (y - z)^4 + (z - x)^4\} = \{(x - y)^2 + (y - z)^2 + (z - x)^2\}^2$$

$$2 \frac{x + \sqrt{x^2 - a^2}}{x - \sqrt{x^2 - a^2}} = 4 \sqrt{2} \left\{ \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}} \right\}^{\frac{1}{2}}$$

$$\text{or, } \frac{2x + 2\sqrt{(x+a)(x-a)}}{2x - 2\sqrt{(x+a)(x-a)}} = 2^{\frac{5}{2}} \left\{ \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}} \right\}^{\frac{1}{2}}$$

$$\text{or, } \left\{ \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}} \right\}^2 = 2^{\frac{5}{2}} \left\{ \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}} \right\}^{\frac{1}{2}}$$

$$\text{or, } \left\{ \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}} \right\}^{\frac{5}{2}} = 2^{\frac{5}{2}}$$

$$\text{or, } \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}} = 2$$

$$\text{or, } \frac{\sqrt{x+a}}{\sqrt{x-a}} = \frac{3}{1}$$

$$\frac{x+a}{x-a} = 9 \quad \text{or, } \frac{x}{a} = \frac{10}{8} = \frac{5}{4} \quad x = \frac{5a}{4}$$

- 3 (i) $14x^3 - 37x + 5 = 14x^2 - 2x - 35x + 5$
 $= 2x(7x - 1) \quad 5(7x - 1) = (7x - 1)(2x - 5)$
- (ii) $(1+a)^2(1+c^2) - (1+c)^2(1+a^2)$
 $= (1+a^2+2a)(1+c^2) - (1+c^2+2c)(1+a^2)$
 $= (1+a^2)(1+c^2) + 2a(1+c^2) - (1+c^2)(1+a^2) - 2c(1+a^2)$
 $= 2(a+ac^2-c-a^2c) = 2\{(a-c) - ac(a-c)\}$
 $= 2(a-c)(1-ac)$
- (iii) $m^4 - n^4 + 2n(m^3 + n^3) - (m+n)^2 m - n$
 $= m^4 - n^4 - (m^3 - n^3)^2 + 2n(m^3 + n^3)$
 $= (m^2 + n^2)(m^2 + n^2 - m^3 + n^3) + 2n(m^3 + n^3)$
 $= 2n^2(m^2 + n^2) + 2n(m^3 + n^3)$
 $= 2n(m^2 + n)\{n(m-n) + m^2 - mn + n^2\}$
 $= 2n(m+n)(nm - n^2 + m^2 - mn + n^2)$
 $= 2m^2n(m+n)$

4 Out of every 4 lbs sold he cleared $\frac{1}{8}$ lb of the whole quantity sold he cleared $\frac{1}{8}$. Now if x be the required number of loaves he cleared $\frac{19x}{2 \times 12} + \frac{1}{16}$ shillings

$$\text{we have } \frac{19x}{2 \times 12 \times 16} = 100 - 5 = 95$$

$$x = 5 \times 2 \times 12 \times 16 = 1920$$

$$\begin{aligned} 5 \quad & \frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)} \\ &= \frac{-\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}}{(a-b)(b-c)(c-a)} \\ &= \frac{-\{(a-b)(b-c)(c-a)(a+b+c)\}}{(a-b)(b-c)(c-a)} \quad [\text{See page 125 Ex 9}] \\ &= a+b+c \end{aligned}$$

$$\begin{aligned} 6 \quad & \frac{a-b}{ay+bx} = \frac{b-c}{bz+cx} = \frac{c-a}{cy+ax} = \frac{a+b+c}{ar+by+cz} \\ & \text{each of the ratios} \\ &= \frac{(a-b) + (b-c) + (c-a) + (a+b+c)}{ay+bx+by+cz+cy+ax+az+by+cz} \\ &= \frac{a+b+c}{r(a+b+c) + y(a+b+c) + z(a+b+c)} = \frac{1}{r+y+z} \end{aligned}$$

$$7 \quad x(r+y+z)=24 \quad (1)$$

$$y(x+y+z)=48 \quad (2)$$

$$z(x+y+z)=72 \quad (3)$$

By adding the equations, we have $(x+y+z)^2=144$

$$x+y+z=12 \quad (4)$$

from (1) and (4), $x=2$,

from (2) and (4), $y=4$,

from (3) and (4), $z=6$

$$8 \quad a+c=\frac{b}{2}-d \quad (1)$$

$$a-c=\frac{d}{2}-bx \quad (2)$$

From (1), $d^2+(a+c)^2-b^2=0$

From (2), $bx^2+(a-c)x-d=0$

we have

$$\frac{x^2}{-d(a+c)+b(a-c)}=\frac{x}{-b^2+d^2}=\frac{1}{d(a-c)-b(a+c)}$$

$$\frac{x^2}{\{b(a-c)-d(a+c)\}\{d(a-c)-b(a+c)\}}=\frac{x^2}{(b^2-d^2)^2}$$

$$db(a-c)^2-d^2(a^2-c^2)-b^2(a^2-c^2)+bd(a+c)^2=(b^2-d^2)^2$$

$$\text{or, } bd\{(a-c)^2+(a+c)^2-(a^2-c^2)(b^2+d^2)\}=(b^2-d^2)^2$$

$$\text{or, } 2bd(a^2+c^2)-(a^2-c^2)(b^2+d^2)=(b^2-d^2)^2$$

$$\text{or, } c^2(b^2+d^2+2bd)-a^2(b^2+d^2-2bd)=(b^2-d^2)^2$$

$$\text{or, } c^2(b+d)^2-a^2(b-d)^2=(b^2-d^2)^2$$

$$\text{or, } \frac{c^2}{(b-d)^2}-\frac{a^2}{(b+d)^2}=1$$

XIX.

$$\begin{aligned} 1 \quad (r^2-2ar+3a^2)^{\frac{1}{2}}+(x^2-4ax+5a^2)^{\frac{1}{2}} \\ = (r^2-5ar+7a^2)^{\frac{1}{2}}+(r^2-7ar+9a^2)^{\frac{1}{2}} \\ (x^2-2ar+3a^2)^{\frac{1}{2}}-(r^2-7ar+9a^2)^{\frac{1}{2}} \\ = (r^2-5ax+7a^2)^{\frac{1}{2}}-(x^2-4ar+5a^2)^{\frac{1}{2}} \end{aligned}$$

$$\text{or, } x^2-2ar+3a^2+r^2-7ar+9a^2-2\{x^2-2ar+3a^2\}^{\frac{1}{2}} \\ \{x^2-7ar+9a^2\}^{\frac{1}{2}}$$

$$\begin{aligned}
 5 \quad x^2 + y^2 + z^2 - 3xyz &= (x+y+z)(x^2+y^2+z^2-xy-yz-zx) \\
 &= 2(a+b+c)\{x^2+y^2+z^2-xy-yz-zx\} \\
 &= (a+b+c)\{(x^2+y^2-2xy)+(x^2+z^2-2xz) \\
 &\quad + (y^2+z^2-2yz)\} \\
 &= (a+b+c)\{(x-y)^2+(x-z)^2+(y-z)^2\} \\
 &= (a+b+c)\{(b-a)^2+(a-c)^2+(c-b)^2\} \\
 &= 2(a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\
 &= 2(a^3+b^3+c^3-3abc)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \frac{1}{b^2(a-c)} + \frac{1}{a^2(b-c)} &= \frac{1}{ab(a-c)(b-c)} \\
 \text{or, } a^2(b-c) + b^2(a-c) &= a^2b^2 \\
 \text{or, } a^2(b-c) - a^2bc - a^2b^2 + a^2bc + ab^2c + ab^2 &- ab^2c - b^3c = 0 \\
 \text{or, } a^2(ab-ac-bc) - ab(ab-ac-bc) + b^2(ab-ac-bc) &= 0 \\
 \text{or, } (ab-ac-bc)(a^2-ab+b^2) &= 0 \\
 \text{Hence either } ab-ac-bc &= 0 \quad (1) \\
 \text{or, } a^2-ab+b^2 &= 0 \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 \text{From (1), } \frac{1}{a} + \frac{1}{b} &= \frac{1}{c} \\
 \text{and from (2), } a^2 + b^2 &= ab
 \end{aligned}$$

7 The given expression

$$\begin{aligned}
 &= \sqrt{\frac{(2\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{5+2\sqrt{3}^2}} = \sqrt{\frac{2(3-2)}{(\sqrt{3}+2)^2}} \\
 &= \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{3-2} = \sqrt{6}-2
 \end{aligned}$$

8 From the 1st and the 2nd equations, we have

$$\begin{aligned}
 \frac{1}{(c+a)(c+b)-(a+b)^2} &= \frac{y'}{(a+b)(a+c)-(b+c)^2} \\
 &= \frac{1}{(a+b)(b+c)-(c+a)^2} \\
 x &= \frac{(c+a)(c+b)-(a+b)^2}{(a+b)(b+c)-(c+a)^2} \\
 \text{and } y &= \frac{(a+b)(a+c)-(b+c)^2}{(a+b)(b+c)-(c+a)^2}
 \end{aligned}$$

from the 3rd equation, we have

$$\frac{(a+b)(b+c)(c+a) - (a+b)^3}{(a+b)(b+c) - (c+a)^2} + \frac{(a+b)(b+c)(c+a) - (b+c)^3}{(a+b)(b+c) - (c+a)^2} + (c+a) = 0$$

$$\text{we have } 3(a+b)(b+c)(c+a) - (a+b)^3 - (b+c)^3 - (c+a)^3 = 0$$

$$\text{or, } \{(a+b) + (b+c) + (c+a)\} \{(a+b)^2 + (b+c)^2 + (c+a)^2 - (a+b)(b+c) - (b+c)(c+a) - (c+a)(a+b)\} = 0$$

$$\text{or, } 2(a+b+c) \frac{1}{2} [\{(a+b) - (b+c)\}^2 + \{(b+c) - (c+a)\}^2 + \{(c+a) - (a+b)\}^2] = 0$$

$$\text{or, } (a+b+c) \{(a-c)^2 + (b-a)^2 + (c-b)^2\} = 0$$

$$\text{or, } (a+b+c) 2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

Thus $a^3 + b^3 + c^3 - 3abc = 0$ is the required Eliminant

XX

$$\begin{aligned} 1 \quad & a(b+c)^2 + b^2c + a)^2 + c(a+b)^2 - 4abc \\ & = a(b+c)^2 + b(c+a)^2 - 2abc + c(a+b)^2 - 2abc \\ & = a(b+c)^2 + b(c^2 + a^2) + c(a^2 + b^2) = a(b+c)^2 + a^2(b+c) + bc(b+c) \\ & = (b+c) \{a^2 + a(b+c) + bc\} = (b+c)(a+b)(a+c) \end{aligned}$$

$$\begin{aligned} 2 \quad & 1+a) \frac{a^2x^2 - b^2x^2 + ac^2x + 3a^3bc}{a^2x^2 + a^3x^2} \left(a^2x^2 - (a^3 + b^3)x + a(a^3 + b^3 + c^3) \right. \\ & \quad \left. - x^2(a^3 + b^3) + ac^2x + 3a^3bc \right. \\ & \quad \left. - x^2(a^3 + b^3) - a(a^3 + b^3)x \right. \\ & \quad \left. \frac{2a(a^3 + b^3 + c^3) + 3a^3bc}{2a(a^3 + b^3 + c^3) + a^2(a^3 + b^3 + c^3)} \right. \\ & \quad \left. - a^2(a^3 + b^3 + c^3 - 3abc) \right) \end{aligned}$$

$$\text{Hence, we have } a^2(a^3 + b^3 + c^3 - 3abc) = 0$$

$$\text{But since } a \text{ is not zero, } a^3 + b^3 + c^3 - 3abc = 0$$

$$\text{or, } a^3 + b^3 + c^3 = 3abc$$

3 The given expression

$$\begin{aligned} & = \frac{-\{b^2c^2(b^2 - c^2) + a^2c^2(c^2 - a^2) + a^2b^2(a^2 - b^2)\}}{abc(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)} \\ & = \frac{(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{abc(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)} = \frac{1}{abc} \end{aligned}$$

$$\begin{aligned}
 4 \quad & a^4(b-c) + b^4(c-a) + c^4(a-b) \\
 &= a^4(b-c) - a(b^4 - c^4) + bc(b^3 - c^3) \\
 &= (b-c)\{a^4 - a(b^3 + b^2c + bc^2 + c^3) + bc(b^2 + bc + c^2)\} \\
 &= (b-c)\{b^4c - a + b^2c(c-a) + bc^2(c-a) - a(c^3 - a^3)\} \\
 &= (b-c)(c-a)\{b^3 + b^2c + bc^2 - a(c^2 + ac + a^2)\} \\
 &= (b-c)(c-a)\{-c^2(a-b) - c(a^3 - b^3) - (a^3 - b^3)\} \\
 &= (b-c)(c-a)(a-b)\{-c^2 - c(a+b) - (a^2 + ab + b^2)\} \\
 &= (a-b)(b-c)(c-a)(-a^2 - b^2 - c^2 - ab - bc - ca) \\
 &\quad \text{the reqd quotient} = -(a^2 + b^2 + c^2 + ab + bc + ca)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & a+b+c=0, \quad a+b=-c \\
 & (a+b)^5 = (-c)^5 = -c^5 \\
 \text{or,} \quad & a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 = -c^5, \\
 & a^5 + b^5 + c^5 = -5ab(a^3 + 2a^2b + 2ab^2 + b^3) \\
 & \quad = -5ab\{(a^3 + b^3) + 2ab(a+b)\} \\
 & \quad = -5ab(a+b)(a^2 + b^2 - ab + 2ab) \\
 & \quad = -5ab(a+b)\{(a+b)^2 - ab\} \\
 & \quad = 5abc(c^2 - ab)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \frac{a}{x+a-c} + \frac{b}{x+b-c} = 2 \\
 \text{or,} \quad & 1 - \frac{a}{x+a-c} + 1 - \frac{b}{x+b-c} = 0 \\
 \text{or,} \quad & \frac{x-c}{x+a-c} + \frac{x-c}{x+b-c} = 0 \\
 \text{or,} \quad & (1-c) \left\{ \frac{1}{x+a-c} + \frac{1}{x+b-c} \right\} = 0
 \end{aligned}$$

either $1-c=0$, whence $x=c$,

$$\frac{1}{x+a-c} + \frac{1}{x+b-c} = 0$$

$$\frac{1}{x+a-c} = -\frac{1}{x+b-c}$$

$$-x-b+c = x+a-c$$

$$2x = 2c - b - a, \text{ whence } x = \frac{2c - b - a}{2}$$

$$= c - \frac{a+b}{2}$$

$$7 \quad ax + by + cz = a + b + c \quad (1)$$

$$\frac{ax}{b+c} + \frac{by}{a+c} = 1. \quad (2)$$

$$\frac{2x}{b+c} + \frac{2y}{a+c} = \frac{1}{a} + \frac{1}{b} \quad (3)$$

$$\text{From (2), } \frac{2ax}{b+c} + \frac{2by}{a+c} = 2$$

$$\text{From (3), } \frac{2bx}{b+c} + \frac{2by}{a+c} = \frac{b}{a} + 1$$

$$\frac{2x(a-b)}{b+c} = 1 - \frac{b}{a} = \frac{a-b}{a}, \quad x = \frac{b+c}{2a}$$

$$\text{from (2) } \frac{1}{2} + \frac{by}{a+c} = 1$$

$$\text{or, } \frac{by}{a+c} = \frac{1}{2}$$

$$\text{or, } 2by = a+c$$

$$y = \frac{a+c}{2b}$$

$$\text{Lastly, from (1), } \frac{b+c}{2} + \frac{a+c}{2} + cz = a+b+c$$

$$\text{or, } cz = a+b+c - \frac{a+b}{2} - c$$

$$= \frac{a+b}{2}, \quad z = \frac{a+b}{2c}$$

8 Let $x+y$ miles per hour be their respective rates of walking, and z miles be the required distance

Then we have,

$$\frac{z-1\frac{1}{2}}{x} = \frac{1\frac{1}{2}}{y} \quad (1)$$

$$\text{and } \frac{(z-1\frac{1}{2})+1}{y} = \frac{1\frac{1}{2}+(z-1)}{x} = 1 \quad (2)$$

From (2), we have

$$\frac{(z-1\frac{1}{2})+1+1\frac{1}{2}+(z-1)}{x+y} = 1$$

$$\text{or, } \frac{2z}{x+y} = 1$$

$$\text{or,} \quad \frac{z}{1+y} = \frac{1}{2} \quad (3)$$

$$\text{Also from (1),} \quad \frac{z}{1+y} = \frac{3}{2y}$$

$$\frac{3}{2y} = \frac{1}{2}, \quad y = 3$$

$$\text{from (2)} \quad \frac{z - \frac{1}{2}}{3} = 1$$

$$\text{or,} \quad z = 3\frac{1}{2},$$

$$\text{and from (3),} \quad x = 4$$

the reqd rates are 4 and 3 miles per hour respectively,
and the reqd distance = $3\frac{1}{2}$ miles

XXI

$$\begin{aligned} 1 \quad & \{(b+c)^2 + (c+a)^2 + (a+b)^2\} \{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\ &= 2(a^2 + b^2 + c^2 + ab + bc + ca) \{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\ &= 2\{a^2 + a(b+c) + \{b^2 + bc + c^2\}\} \{a^2(b-c) - a(b^2 - c^2) + bc(b-c)\} \\ &= 2\{a^3(b-c) + a^2(b^2 - c^2) + a^2(b^2 - c^2) - a^2(b^2 - c^2) - a^2(b^2 - c^2) - a^2(b^2 - c^2) \\ &\quad - a(b^2 - c^2) + a^2bc(b-c) + abcb^2 - c^2) + bc(b^2 - c^2)\} \\ &= 2\{a^3(b-c) + b^4(c-a) + c^4(a-b)\} \end{aligned}$$

2 The left-hand expression

$$\begin{aligned} &= \frac{(b-c)^2 + (c-a)^2 + (a-b)^2}{(a-b)(b-c)(c-a)} \\ &= \frac{b^2 - c^2 - 3bc(b-c) + c^2 - a^2 - 3ac(c-a) + a^2 - b^2 - 3ab(a-b)}{(a-b)(b-c)(c-a)} \\ &= \frac{-3\{bc(b-c) + ac(c-a) + ab(a-b)\}}{-\{bc(b-c) + ac(c-a) + ab(a-b)\}} = 3 \end{aligned}$$

$$3 \quad a+b+c=0$$

$$a+b=-c, \quad b+c=-a \text{ and } c+a=-b$$

$$a^2+ab+b^2=b^2+a(a+b)$$

$$=b^2+(-b-c)(-c)=b^2+bc+c^2$$

$$=c^2+b(b+c)$$

$$=c^2-(c+a)(-a)$$

$$=c^2+ac+a^2,$$

$$a^2+ab+b^2=b^2+bc+c^2=c^2+ac+a^2$$

Hence, by cross multiplication, we have

$$\frac{f^3}{-bc' + cb'} = \frac{f}{-ca' + ac} = \frac{1}{ab' - ba'}$$

we get $\frac{f^3}{cb' - bc'} \left(\frac{1}{ab' - ba} \right)^3 = \frac{f^3}{(ac' - ca')^3}$

$(ac' - ca)^3 = (ab' - ba')^3 (cb' - bc)$ is the required condition

8 $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 1$ (suppose) $a = bk, b = ck$ and $c = dk$

$$a = b k^2 = d k^3 \text{ and } b = d k^2$$

$$\frac{a}{d} = \frac{d k^3}{d} = k^3$$

$$\begin{aligned} \text{Also, } \sqrt{\frac{a^6 + b^2 c^2 + a^2 c^2}{b^4 c + d^4 + b^2 c d^2}} &= \sqrt{\frac{d^3 k^{16} + d^2 k^4 d^2 k^2 + d^2 k^9 d^2 k^2}{d^2 k^8 d k + d^4 + d^2 k^4 d k d^2}} \\ &= \sqrt{\frac{d^3 k^{16} + d^4 k^6 + d^6 k^{11}}{d^3 k^9 + d^4 + d^6 k^6}} = \sqrt{\frac{k^6 (d^3 k^9 + d^4 + d^6 k^6)}{d^3 k^9 + d^4 + d^6 k^6}} \\ &= \sqrt{k^6} = k^3 \end{aligned}$$

Therefore $a = d = \sqrt{a^2 + b^2 c^2 + a^2 c^2} = \sqrt{b^4 c + d^4 + b^2 c d^2}$

XXII

1 The left-hand expression

$$\begin{aligned} &= a(b-c)\{1 + a(b+c) + a^2 bc\} + b(c-a)\{1 + b(c+a) + b^2 ca\} \\ &\quad + c(a-b)\{1 + c(a+b) + c^2 ab\} \\ &= \{a(b-c) + b(c-a) + c(a-b)\} + \{a^2(b^2 - c^2) + b^2(c^2 - a^2) \\ &\quad + c^2(a^2 - b^2)\} + abc\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\ &= abc(a-b)(a-c)(b-c) \end{aligned}$$

2 $a + b + c = 0$ $a + b = -c$ $(a+b)^7 = (-c)^7$

or, $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7 = -c^7$

or, $a^7 + b^7 + c^7 = -7ab(a^5 + 3a^4b + 5a^3b^2 + 5a^2b^3 + 3ab^4 + b^5)$
 $= -7ab\{(a^5 + b^5) + 3ab(a^3 + b^3) + 5a^2b^2(a+b)\}$
 $= -7ab(a+b)\{a^4 - a^2b + a^2b^2 - ab^3 + b^4 + 3ab(a^2 - ab + b^2) + 5a^2b^2\}$

$$= 7abc(a^4 + 2a^3b + 3a^2b^2 + 2ab^3 + b^4)$$

$$= 7abc\{(a^4 + 2a^3b^2 + b^4) + 2ab(a^2 + b^2) + a^2b^2\}$$

$$\begin{aligned}
&= 7abc\{(a^2+b^2)^2+2ab(a^2+b^2)+a^2b^2\} \\
&= 7abc(a^2+b^2+ab)^2 \\
&= 7abc\{(a^2+b^2+2ab)-ab\}^2=7abc\{(a+b)^2-ab\} \\
&= 7abc(c^2-b)^2
\end{aligned}$$

$$\begin{aligned}
3 \quad & x+f \Big) \frac{ax^2+bx+c}{ax^2+afx} \Big(\frac{ax-(af-b)}{-x(af-b)+c} \\
& \qquad \qquad \qquad \frac{-x(af-b)+c}{-x(af-b)-f(af-b)} \\
& \qquad \qquad \qquad \frac{-x(af-b)-f(af-b)}{f(af-b)+c}
\end{aligned}$$

we must have $f(af-b)+c=0$

$$\text{or,} \qquad \qquad \qquad af^2-bf+c=0 \qquad (1)$$

Similarly by dividing $a'x^2+b'x+c'$

$$\text{we get} \qquad \qquad \qquad a'f'^2-b'f'+c'=0 \qquad (2)$$

From (1) and (2), by cross multiplication, we get

$$\frac{f^2}{-bc+b'c} = \frac{f}{ca'-ac'} = \frac{1}{-ab+ba'}$$

$$\text{or,} \quad \frac{f^2}{b'c-bc'} \cdot \frac{1}{a'b-ab} = \frac{f^2}{(ac'-ca')^2}$$

$$\text{or,} \quad (ac'-ca')^2 = (a'b-ab)(b'c-bc')$$

4 Let x yds be the length of the course Then A ran x yds while B ran $(x-1)-50$ yds

$$\frac{1}{20} = \frac{(x-1)-50}{19}$$

$$\text{or,} \quad 19x = 20x - 1020$$

$$x = 1020$$

Thus the length of the course is 1020 yds

$$\begin{aligned}
5 \quad & \frac{p+q+r}{p-q} - \frac{q(4p+3r)-r(p+r)}{p^2-q^2} \\
&= \frac{(p+q)(p+q+r)-q(4p+3r)+r(p+r)}{p^2-q^2} \\
&= \frac{(p+q)^2-4pq+r(p+q+p+r)-3qr}{p^2-q^2} \\
&= \frac{(p+q)^2-4pq+2r(p-q)+r^2}{p^2-q^2} \\
&= \frac{(p-q)^2+2r(p-q)+r^2}{p^2-q^2} = \frac{(p-q+r)^2}{p^2-q^2}
\end{aligned}$$

- 6 Putting x for $a^2 - b^2$, y for $b^2 - c^2$ and z for $c^2 - a^2$, we have
 the numerator $= x^2 + y^2 + z^2 = 3xyz$ [$x + y + z = 0$]
 $= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
 $= 3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$

Similarly putting x, y and z for $a - b, b - c$, and $c - a$ respectively we have

$$\begin{aligned} \text{the denominator} &= 3(a - b)(b - c)(c - a) \\ \text{the left-hand expression} &= (a + b)(b + c)(c + a) \end{aligned}$$

- 7 $x + y + z = 2a + 2b + 2c$ (1)
 $ax + by + cz = 2bc + 2ca + 2ab$ (2)
 $(b - c)x + (c - a)y + (a - b)z = 0$ (3)

From (1), $(x - 2b) + (y - 2c) + (z - 2a) = 0$

From (2), $a(x - 2b) + b(y - 2c) + c(z - 2a) = 0$

Hence, we have,

$$\frac{x - 2b}{c - b} = \frac{y - 2c}{a - c} = \frac{z - 2a}{b - a} = l \text{ (suppose)}$$

$$x = (c - b)l + 2b, y = l(a - c) + 2c \text{ and } z = l(b - a) + 2a$$

from (3), we get

$$-l(b - c)^2 + 2b(b - c) - l(c - a)^2 + 2c(c - a)$$

$$-l(a - b)^2 + 2a(a - b) = 0$$

$$\begin{aligned} \text{or, } l\{(a - b)^2 + (b - c)^2 + (c - a)^2\} &= 2(a^2 + b^2 + c^2 - ab - cb - ca) \\ &= 2\{(a - b)^2 + (b - c)^2 + (c - a)^2\} \end{aligned}$$

$$l = 1$$

$$x = c - b + 2b = b + c$$

$$y = a - c + 2c = a + c$$

$$\text{and } z = b - a + 2a = a + b$$

- 8 $ax + cy + bz = 0$ (1)
 $cx + by + az = 0$ (2)
 $bx + ay + cz = 0$ (3)

From (1) and (2), we get

$$\frac{x}{ac - b^2} = \frac{y}{bc - a^2} = \frac{z}{ab - c^2} = l \text{ (suppose)}$$

$$x = l(ac - b^2), y = l(bc - a^2) \text{ and } z = l(ab - c^2)$$

from (3), we get $l\{b(ac - b^2) + a(bc - a^2) + c(ab - c^2)\} = 0$

$$\text{or, } a^3 + b^3 + c^3 - 3abc = 0$$

XXIII

1 The left-hand expression

$$\begin{aligned}
 &= (b-c)\{1+a^2(b+c)+a^3bc\} + (c-a)\{1+b^2(c+a)+ab^2c\} \\
 &\quad + (a-b)\{1+c^2(a+b)+abc^2\} \\
 &= \{(a-b)+(b-c)+(c-a)\} + \{a^2(b^2-c^2)+b^2(c^2-a^2) \\
 &\quad + c^2(a^2-b^2)\} + abc\{a^2(b-c)+b^2(c-a)+c^2(a-b)\} \\
 &= 0+0+abc\{a^2(b-c)+b^2(c-a)+c^2(a-b)\} \\
 &= abc(a+b+c)(a-b)(a-c)(b-c)
 \end{aligned}$$

2 $21x^2-13x+2=21x^2-7x-6x+2=7x(3x-1)-2(3x-1)$
 $= (3x-1)(7x-2)$

$$28x^2-15x+2=28x^2-8x-7x+2$$

$$=4x(7x-2)-(7x-2)=(7x-2)(4x-1)$$

$$12x^2-7x+1=12x^2-4x-3x+1$$

$$=4x(3x-1)-(3x-1)=(3x-1)(4x-1)$$

$$\text{the reqd L C M} = (3x-1)(7x-2)(4x-1)$$

3 $(x+y)^7 - x^7 - y^7$

$$= 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6$$

$$= 7xy(x^5 + 3x^4y + 5x^3y^2 + 5x^2y^3 + 3xy^4 + y^5)$$

$$= 7xy\{(x^5+y^5) - 3xy(x^3+y^3) + 5x^2y^2(x+y)\}$$

$$= 7xy(x+y)\{x^4-x^2y+2x^2y^2-y^3+y^4+3xy(x^2-xy+y^2) + 5x^2y^2\}$$

$$= 7xy(x+y)\{(x^4+2x^2y^2+y^4) + 2xy(x^2+y^2) + x^2y^2\}$$

$$= 7xy(x+y)\{(x^2+y^2)^2 + 2xy(x^2+y^2) + x^2y^2\}$$

$$= 7xy(x+y)(x^2+y^2+xy)^2$$

$$(1+y)^7 - x^7 - y^7 \text{ is divisible by } (x^2+xy+y^2)^2$$

4 The left-hand expression

$$= 3x^4 - 2x^2(a^2+b^2+c^2) + (a^2b^2+b^2c^2+c^2a^2)$$

$$= 3x^4 - 4x^4 + a^2b^2 + b^2c^2 + c^2a^2 - (a^2b^2 + b^2c^2 + c^2a^2) - x^4$$

$$= (a^2b^2 + b^2c^2 + c^2a^2) - \frac{1}{4}(a^2+b^2+c^2)^2$$

$$= (a^2b^2 + b^2c^2 + c^2a^2) - \frac{1}{4}(a^4+b^4+c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2)$$

$$= \frac{1}{4}(2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4)$$

$$= \frac{1}{4}\{2b^2(a^2+c^2) - (a^2-c^2)^2 - b^4\}$$

$$= \frac{1}{4}\{b^2\{(a+c)^2 + (a-c)^2\} - (a^2-c^2)^2 - b^4\}$$

$$\begin{aligned}
 &= \frac{1}{4} \{ b^2(a+c)^2 - b^4 - (a^2 - c^2)^2 + (a-c)^2 b^2 \} \\
 &= \frac{1}{4} \{ b^2 \{ (a+c)^2 - b^2 \} - (a-c)^2 \{ (a+c)^2 - b^2 \} \} \\
 &= \frac{1}{4} \{ \{ (a+c)^2 - b^2 \} \{ b^2 - (a-c)^2 \} \} \\
 &= \frac{1}{4} (a+c+b)(a+c-b)(b+a-c)(b-a+c) \\
 &= \frac{1}{4} (2s)(a+b+c-2b)(a+b+c-2c)(a+b+c-2a) \\
 &= \frac{1}{4} (2s)(2s-2b)(2s-2c)(2s-2a) \\
 &= 4s(s-a)(s-b)(s-c)
 \end{aligned}$$

5 We have

$$\begin{aligned}
 (1 + x' + y')^2 &= (1 + x^2 + y^2)(1 + x'^2 + y'^2) \\
 \text{or, } 1 + x^2 x'^2 + y^2 y'^2 + 2xx' + 2yy' + 2x'y'y' \\
 &= 1 + x^2 x'^2 + y^2 y'^2 + x^2 + y^2 + x'^2 + y'^2 + x^2 y'^2 + x'^2 y^2 \\
 \text{or, } x^2 - 2xx' + x'^2 + y^2 - 2yy' + y'^2 + x^2 y'^2 + x'^2 y^2 - 2x'y'y' &= 0 \\
 \text{or, } (x - x')^2 + (y - y')^2 + (xy' - x'y)^2 &= 0
 \end{aligned}$$

In order that this condition be fulfilled it is necessary that each of these three terms be individually zero

$$\begin{aligned}
 x - x' &= 0 & \text{hence } x &= x' \\
 y - y' &= 0 & \text{hence } y &= y'
 \end{aligned}$$

6 The numerator

$$\begin{aligned}
 &= a^4 b - a^3 b^2 + a^2 b^3 - ab^4 + b^4 c - b^3 c^2 + b^2 c^3 - bc^4 + c^4 a - c^3 a^2 \\
 &\quad + c^2 a^3 - ca^4 \\
 &= a^4(b-c) - a^3(b^2-c^2) + a^2(b^3-c^3) - a(b^4-c^4) + b^3c(b-c) \\
 &\quad + bc^3(b-c) \\
 &= (b-c) \{ a^4 - a^3(b+c) + a^2(b^2+bc+c^2) \\
 &\quad - a(b^3+b^2c+bc^2+c^3) + b^3c+bc^3 \} \\
 &= (b-c) \{ b^3(c-a) - b^2a(c-a) + b(c^3-a^3) - ab^2(c-a) + a^2c(c-a) \\
 &\quad - a(c^3-a^3) \} \\
 &= (b-c)(c-a)(b^3-b^2a+bc^2+ab^2+a^3b-ab^2 \\
 &\quad + a^3c-ac^3-a^2c-a^3) \\
 &= (b-c)(c-a) \{ c^2(b-a) + b^2(b-a) + a^2(b-a) \} \\
 &= (b-c)(c-a)(b-a)(c^2+b^2+a^2)
 \end{aligned}$$

The denominator

$$\begin{aligned}
 &= a^3 b^2 - a^2 b^3 + b^3 c^2 - b^2 c^3 + a^2 c^3 - a^3 c^2 \\
 &= a^3(b^2-c^2) - a^2(b^3-c^3) + b^2c^2(b-c)
 \end{aligned}$$

$$\begin{aligned}
&= (b-c)\{a^2(b-c) - a^2(b^2+bc-c^2) + b^2c^2\} \\
&= (b-c)\{b^2(c^2-a^2) - a^2b(c-a) - a^2c(c-a)\} \\
&= (b-c)\{c-a\}\{b^2c-b^2a-a^2b-a^2c\} \\
&= (b-c)\{c-a\}\{ab(b-a) + c(b^2-a^2)\} \\
&= (b-c)\{c-a\}\{b-a\}ab + bc + ca)
\end{aligned}$$

the required result

$$= \frac{a^2 - b^2 + c^2}{ab - bc + ca}$$

$$\begin{aligned}
7 \quad a+b+c &= 0, & a+b &= -c & (a+b)^5 &= -c^5 \\
\text{or} \quad a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 &= -c^5 \\
\text{or,} \quad a^5 + b^5 - c^5 &= -5ab(a^3 + 2a^2b + 2ab^2 + b^3) \\
\frac{a^5 + b^5 - c^5}{5} &= -ab\{a^4 - b^4 + 2ab(a-b)\} \\
&= -ar(a+b)(a^3 - ab + b^3 - 2ab) \\
&= -ab(a+b)(a^2 + ab + b^2) \quad (A)
\end{aligned}$$

$$\begin{aligned}
\text{again} \quad a+b &= -c & (a-b)^5 &= -c^5 \\
\text{or} \quad a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5 &= -c^5 \\
\text{or,} \quad a^5 - b^5 - c^5 &= -5ab(a^3 - 2a^2b + 2ab^2 - b^3) \\
\frac{a^5 - b^5 - c^5}{5} &= -ab(a+b) \quad (B)
\end{aligned}$$

$$\begin{aligned}
\text{Lastly} \quad (a-b)^2 &= c^2 \\
\text{or} \quad a^2 - 2ab + b^2 &= c^2 \\
a^2 - b^2 + c^2 &= 2c^2 - 2ab \\
\frac{a^2 - b^2 - c^2}{2} &= c^2 - ab \\
&= (a+b)^2 - ab \\
&= a^2 - ab - b^2 \quad (C)
\end{aligned}$$

Thus from (A), (B) and (C), we have

$$\frac{a^5 - b^5 + c^5}{5} = \frac{a^2 - b^2 + c^2}{3} = \frac{a^2 - b^2 + c^2}{2}$$

$$8 \quad \text{From the 2nd equation we get } p^2x^2 + p^2y^2 - \frac{p^2q^2}{a^2 + b^2} = 0$$

$$\text{From the 3rd equation we get } x^2 + y^2 - 1 = 0$$

by cross multiplication, we get

$$\frac{x^2}{-p^2 - \frac{p^2q^2}{a^2 - b^2}} = \frac{y^2}{-\frac{p^2q^2}{a^2 - b^2} + q^2} = \frac{1}{q^2 - p^2}$$

$$x^2 = \frac{p^2(q^2 - a^2 - b^2)}{(a^2 + b^2)(q^2 - p^2)} \quad x = \frac{p \sqrt{q^2 - (a^2 + b^2)}}{\sqrt{(a^2 + b^2)(q^2 - p^2)}}$$

$$\text{and } y^2 = \frac{q^2(a^2 + b^2 - p^2)}{(a^2 + b^2)(q^2 - p^2)} \quad y = \frac{q \sqrt{(a^2 + b^2) - p^2}}{\sqrt{(a^2 + b^2)(q^2 - p^2)}}$$

from the 3rd equation, we get

$$ap \sqrt{q^2 - (a^2 + b^2)} + bq \sqrt{(a^2 + b^2) - p^2} = (a^2 + b^2) \sqrt{q^2 - p^2}$$

$$\begin{aligned} \text{or, } a^2 p^2 q^2 - a^2 p^2 (a^2 + b^2) + b^2 q^2 (a^2 + b^2) - b^2 p^2 q^2 \\ + 2apbq \sqrt{(a^2 + b^2)(p^2 + q^2) - p^2 q^2 - (a^2 + b^2)^2} \\ = (a^2 + b^2)^2 (q^2 - p^2) \end{aligned}$$

$$\begin{aligned} \text{or, } p^4 q^2 (a^2 - b^2) - (a^2 + b^2)(a^2 p^2 - b^2 q^2 + a^2 q^2 - a^2 p^2 + b^2 q^2 - b^2 p^2) \\ = -2apbq \sqrt{(a^2 + b^2)(p^2 + q^2) - p^2 q^2 - (a^2 + b^2)^2} \end{aligned}$$

$$\begin{aligned} \text{or, } p^4 q^4 (a^2 - b^2)^2 + (a^2 + b^2)^2 - (a^2 q^2 - b^2 p^2)^2 \\ - 2p^2 q^2 (a^2 - b^2)(a^2 q^2 - b^2 p^2) \\ = 4a^2 b^2 p^2 q^2 \{(a^2 + b^2)(p^2 + q^2) - p^2 q^2 - (a^2 + b^2)^2\} \\ \text{or, } (a^2 + b^2)^2 \{p^4 q^4 + (a^2 q^2 + b^2 p^2)^2\} \\ = 2p^2 q^2 (a^4 b^2 p^2 + 2a^4 b^2 q^2 + 2a^2 b^4 p^2 + 2a^2 b^4 q^2 \\ + a^6 q^2 - a^2 b^2 p^2 - a^2 b^4 q^2 + b^6 p^2) \\ = 2p^2 q^2 \{p^2 b^2 (a^4 + 2a^2 b^2 + b^4) + a^2 q^2 (a^4 + 2a^2 b^2 + b^4)\} \\ = 2p^2 q^2 (a^2 + b^2)^2 (p^2 b^2 + a^2 q^2) \end{aligned}$$

$$\text{or, } (a^2 q^2 + b^2 p^2)^2 + p^4 q^4 - 2p^2 q^2 (a^2 q^2 + b^2 p^2) = 0$$

$$\text{or, } (a^2 q^2 + b^2 p^2 - p^2 q^2)^2 = 0$$

$$\text{or, } a^2 q^2 + b^2 p^2 = p^2 q^2$$

$$\frac{a^2}{p^2} + \frac{b^2}{q^2} = 1 \text{ is the reqd Eliminant}$$

XXIV.

- 1 From the 1st equation, we get $(x-a) + (y-b) + (z-c) = 0$
 From the 2nd, we get $b(x-a) + c(y-b) + a(z-c) = 0$
 by cross multiplication, we get

$$\frac{x-a}{a-c} = \frac{y-b}{b-a} = \frac{z-c}{c-b} = k \text{ (suppose)}$$

From the 3rd equation, we get

$$c(x-a) + a(y-b) + b(z-c) = 0$$

$$/ \{c(a-c) + a(b-a) + b(z-c)\} = 0$$

or $k=0$ (Supposing the other factor to be not zero)

$$\text{we get } r-a=0, \quad \text{or, } x=a$$

$$y-b=0, \quad \text{or, } y=b$$

$$z-c=0, \quad \text{or, } z=c$$

2 Let r, y and z be the three parts

$$\text{we get } r+y+z=243$$

$$\text{Also, } \frac{r}{2} = \frac{y}{3} = \frac{z}{4} = k \text{ (suppose)}$$

$$x=2k, y=3k \text{ and } z=4k$$

$$\text{we get } k(2+3+4)=243$$

$$\text{or, } k=27$$

$$x=54, y=81 \text{ and } z=108$$

$$\begin{aligned} 3 \quad 4(a^2+b^2+c^2+d^2) &= (a+b+c+d)^2 \\ &= a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc \\ &\quad +2bd+2cd \end{aligned}$$

$$\text{or, } (a^2+b^2-2ab) + (b^2+c^2-2bc) + (c^2+d^2-2cd) \\ + (d^2+a^2-2ad) + (a^2+c^2-2ac) + (d^2+b^2-2bd) = 0$$

$$\text{or, } (a-b)^2 + (b-c)^2 + (c-d)^2 + (d-a)^2 + (a-c)^2 + (d-b)^2 = 0$$

In order that this expression may be equal to zero, each of these squares must be individually zero,

$$a=b=c=d$$

4 The given expression

$$\begin{aligned} &= s^2\{a(b-c) + b(c-a) + (a-b)\} - 2s\{a^2(b-c) + b^2(c-a) + c^2(a-b)\} \\ &\quad + \{a^3(b-c) + b^3(c-a) + c^3(a-b)\} \\ &= 0 - (a+b+c)(a-b)(a-c)(b-c) + (a+b+c)(a-b)(a-c)(b-c) \\ &= 0 \end{aligned}$$

5 We have

$$a-bx-cy=0 \quad (1)$$

$$az-b+cz=0 \quad (2)$$

$$\text{and } ay+bx-c=0 \quad (3)$$

from (1) and (2), we get

$$\frac{a}{-xz-y} = \frac{b}{-yz-x} = \frac{c}{-1+z^2} \quad (4)$$

From (2) and (3), we get

$$\frac{a}{1-z^2} = \frac{b}{zy+z} = \frac{c}{zx+y} \quad (5)$$

From (1) and (3), we get

$$\frac{a}{z+xy} = \frac{b}{-j^2+1} = \frac{c}{z+jx} \quad (6)$$

from (4) and (5), we get by multiplication,

$$\begin{aligned} \frac{a^2}{(1-z^2)(z+jx)} &= \frac{c^2}{(1-z^2)(zx+y)} \\ \frac{a^2}{1-z^2} &= \frac{c^2}{1-z^2} \end{aligned} \quad (7)$$

Similarly from (5) and (6), we get

$$\frac{a^2}{1-z^2} = \frac{b^2}{1-j^2} \quad (8)$$

from (7) and (8)

$$\frac{a^2}{1-z^2} = \frac{b^2}{1-j^2} = \frac{c^2}{1-z^2}$$

$$6 \quad a_1 + b_1 = 0 \quad (1)$$

$$z + j + zy = 0 \quad (2)$$

$$z^2 + j^2 - 1 = 0 \quad (3)$$

From (1) and (2), we get

$$\frac{r}{b} = \frac{j}{-a} = \frac{xy}{a-b}$$

$$\text{we get } z = \frac{a-b}{-a} \text{ and } y = \frac{a-b}{b}$$

from (3), we get

$$\frac{(a-b)^2}{a^2} + \frac{(a-b)^2}{b^2} - 1 = 0$$

$$\text{or, } b^2(a-b)^2 + a^2(a-b)^2 - a^2b^2 = 0$$

$$\text{or, } (a-b)^2(a^2+b^2) = a^2b^2$$

$$\text{or, } \frac{a^2+b^2}{a^2b^2} = \frac{1}{(a-b)^2}$$

$$\text{or, } \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{(a-b)^2}$$

Which is the reqd Eliminant

7 Let the common factor be $x-m$ Dividing the 1st expression by $x-m$, we get

$$\begin{array}{r} x-m \overline{) ax^2 - bx + c} \left(ar + (am - b) \right. \\ \underline{ax^2 - amx} \\ (am - b)x + c \\ \underline{(am - b)x - am^2 + bm} \\ am^2 - bm + c \end{array}$$

$$\text{We must have } am^2 - bm + c = 0 \quad (1)$$

Again dividing the 2nd expression by $x-m$, we get

$$\begin{array}{r} x-m \overline{) dx^3 - bx + c} \left(dx^2 + dm x + (dm^2 - b) \right. \\ \underline{dx^3 - dm^2 x^2} \\ dm x^2 - bx + c \\ \underline{dm x^2 - dm^2 x} \\ (dm^2 - b)x + c \\ \underline{(dm^2 - b)x - dm^3 + bm} \\ dm^3 - bm + c \end{array}$$

$$\text{We must have } dm^3 - bm + c = 0 \quad (2)$$

$$\text{From (1) and (2),} \quad am^2 = dm^3$$

$$\text{or,} \quad a = dm$$

$$\text{or,} \quad m = \frac{a}{d}$$

$$\text{from (2),} \quad d\left(\frac{a}{d}\right)^3 - b\frac{a}{d} + c = 0$$

$$\text{or,} \quad \frac{a^3}{d^2} - \frac{ab}{d} + c = 0$$

$$\text{or,} \quad a^3 - abd + cd^2 = 0$$

$$8 \quad (a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a) \quad (\text{Ex 6 Page 91})$$

$$(a+b)(b+c)(c+a) = 0$$

Hence at least one of these factors must be zero

Suppose $(a+b) = 0$, then $a+b+c = c$

Again $(a+b)$ is a factor of $a^{2n+1} + b^{2n+1}$

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$$a^{2n+1} + b^{2n+1} = 0$$

$$a^{2n+1} + b^{2n+1} + c^{2n+1} = c^{2n+1}$$

$$= (a+b+c)^{2n+1}$$

XXV.

1 We get

$$\begin{aligned}\frac{x^2-yz}{x+y+z} &= \frac{a^2+b^2c^2-2a^2bc-(b^2c^2-c^2a-ab^2+a^2bc)}{a^2+b^2+c^2-ab-bc-ca} \\ &= \frac{a(a^2+b^2+c^2-3abc)}{a^2+b^2+c^2-ab-bc-ca} \\ &= a(a+b+c)\end{aligned}$$

$$\frac{x^2-yz}{a} = (a+b+c)(x+y+z)$$

Similarly it can be proved that $\frac{y^2-zx}{b} = (a+b+c)(x+y+z)$

$$\text{and } \frac{z^2-xy}{c} = (a+b+c)(x+y+z)$$

$$\text{we get } \frac{x^2-yz}{a} = \frac{y^2-zx}{b} = \frac{z^2-xy}{c} = (a+b+c)(x+y+z)$$

$$\begin{aligned}2 \quad & 4(bc+ad)^2 - (b^2+c^2-a^2-d^2)^2 \\ &= \{2(bc+ad) + (b^2+c^2-a^2-d^2)\} \{2(bc+ad) - (b^2+c^2-a^2-d^2)\} \\ &= \{(b^2+c^2+2bc) - (a^2+d^2-2ad)\} \{(a^2+d^2+2ad) - (b^2+c^2-2bc)\} \\ &= \{(b+c)^2 - (a-d)^2\} \{(a+d)^2 - (b-c)^2\} \\ &= (b+c+a-d)(b+c-a+d)(a+d+b-c)(a+d-b+c) \\ &= (2s-2d)(2s-2a)(2s-2b)(2s-2c) \\ &= 16(s-a)(s-b)(s-c)(s-d)\end{aligned}$$

3 Putting x for $b+c-a$, y for $c+a-b$ and z for $a+b-c$,
we get $x+y+z=a+b+c$ and $x-y=2(b-a)$,
 $y-z=2(c-b)$ and $z-x=2(a-c)$

$$\begin{aligned}& (b+c-a)^2 + (c+a-b)^2 + (a+b-c)^2 \\ &= 3(b+c-a)(c+a-b)(a+b-c) = x^2+y^2+z^2-3xyz \\ &= \frac{1}{2}(x+y+z)\{(x-y)^2 + (y-z)^2 + (z-x)^2\} \\ &= \frac{1}{2}(a+b+c)\{4(b-a)^2 + 4(c-b)^2 + 4(a-c)^2\} \\ &= 4\left[\frac{1}{2}(a+b+c)\{(b-a)^2 + (c-b)^2 + (a-c)^2\}\right] \\ &= 4(a^2+b^2+c^2-3abc)\end{aligned}$$

$$\begin{aligned}
4 \quad & \left(\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c} \right) \left(\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} \right) \\
&= \left\{ \frac{bc(b-c) + ac(c-a) + ab(a-b)}{abc} \right\} \\
&\quad \left\{ \frac{a(c-a)(a-b) + b(b-c)(a-b) + c(b-c)(c-a)}{(a-b)(b-c)(c-a)} \right\} \\
&\quad \left\{ \frac{(a+c)-ac}{(a-b)(b-c)(c-a)} + \frac{c\{-c^2 + c(a+b) - ab\}}{(a-b)(b-c)(c-a)} \right\} \\
&= \left\{ \frac{-(a-b)(b-c)(c-a)}{abc} \right\} \left[\frac{a\{-a^2 + a(b+c) - bc\} + b\{-b^2 + b(c-a) - ca\}}{(a-b)(b-c)(c-a)} \right] \\
&= \frac{-1}{abc} \left[-\{a^3 + b^3 + c^3 - a^2(b+c) - b^2(c+a) - c^2(a+b) + 3abc\} \right] \\
&\quad + 9abc \\
&= \frac{(a^3 + b^3 + c^3 - 3abc) - \{a^2(b+c) + b^2(c+a) + c^2(a+b) + 3abc\}}{abc} \\
&\quad + 9abc \\
&= \frac{(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) - (a+b+c)(ab + bc + ca)}{abc} \\
&= \frac{0 - 0 + 9abc}{abc} = 9
\end{aligned}$$

$$\begin{aligned}
5 \quad & \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k \text{ (suppose)} \\
& x = ak, y = bk \text{ and } z = ck \\
& \frac{x^2 + a^2}{x+a} + \frac{y^2 + b^2}{y+b} + \frac{z^2 + c^2}{z+c} = \frac{a^2(k^2+1)}{a(k+1)} + \frac{b^2(k^2+1)}{b(k+1)} + \frac{c^2(k^2+1)}{c(k+1)} \\
&= \frac{k^2+1}{k+1}(a+b+c) \\
&\text{Also } \frac{(x+y+z)^2 + (a+b+c)^2}{x+y+z+a+b+c} \\
&= \frac{k^2(a+b+c)^2 + (a+b+c)^2}{k(a+b+c) + (a+b+c)} = \frac{(k^2+1)(a+b+c)^2}{(k+1)(a+b+c)} \\
&= \frac{k^2+1}{k+1}(a+b+c) \\
&\frac{x^2 + a^2}{x+a} + \frac{y^2 + b^2}{y+b} + \frac{z^2 + c^2}{z+c} = \frac{(x+y+z)^2 + (a+b+c)^2}{x+y+z+a+b+c}
\end{aligned}$$

$$6 \quad ax + by + cz = 0 \quad (1)$$

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0 \quad (2)$$

From (1), we get

$$(x^2 + y^2 + z^2 + rz + yz + rz)(ax + by + cz) = 0 \quad (3)$$

From (2), we get

$$ayz + brz + cz = 0$$

$$\text{or, } (1 + y + z)(ayz + brz + cz) = 0 \quad (4)$$

From (3) and (4), we get

$$\begin{aligned} ax^2 + by^2 + cz^2 + a(x^2y + x^2z + y^2 + rz^2 + 2xyz + y^2z + yz^2) \\ + b'x^2y + y^2z + rz^2 + 2xyz + y^2z + yz^2 \\ + c'x^2y + x^2z + y^2 + rz^2 + 2xyz + y^2z + yz^2 = 0 \end{aligned}$$

$$\text{or, } ax^2 + by^2 + cz^2 + (a + b + c)\{x^2(y + z) + x(y^2 + z^2 + 2yz) + yz(y + z)\} = 0$$

$$\text{or, } ax^2 + by^2 + cz^2 + (a + b + c)(y + z)\{x^2 + x(y + z) + yz\} = 0$$

$$\text{or, } ax^2 + by^2 + cz^2 + (a + b + c)(y + z)(x + y)(x + z) = 0$$

$$7 \quad (1) \quad ax + by + cz = 0 \quad (1)$$

$$hx + by + fz = 0 \quad (2)$$

$$gx + fy + cz = 0 \quad (3)$$

From (1) and (2), by cross multiplication,

$$\text{we get } \frac{x}{hf - gb} = \frac{y}{gh - af} = \frac{z}{ab - h^2} = k \text{ (suppose)}$$

$$x = k(hf - gb), y = k(gh - af) \text{ and } z = k(ab - h^2)$$

from (3), we get

$$k\{g(hf - gb) + f(gh - af) + c(ab - h^2)\} = 0$$

$$\text{or, } fgh - bg^2 + fgh - af^2 + abc - ch^2 = 0$$

$$\text{or, } abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \text{ is the reqd Eliminant}$$

$$(ii) \quad a(y + z) = r \quad (1)$$

$$b(z + x) = y \quad (2)$$

$$c(x + y) = z \quad (3)$$

$$\text{From (1), we get } r - ay - az = 0 \quad (4)$$

$$(2), \quad bx - y + bz = 0 \quad (5)$$

$$(3), \quad cx + cy - z = 0 \quad (6)$$

From (4), and (5), we get

$$\frac{x}{-ab - a} = \frac{y}{-ab - b} = \frac{z}{-1 + ab} = k \text{ (suppose)}$$

$$x = -k(ab + a), y = -k(ab + b) \text{ and } z = k(ab - 1)$$

$$\text{from (3), we get } k\{c(ab + a) + c(ab + b) + ab - 1\} = 0$$

$$\text{or, } ac + bc + ab + 2abc = 1 \text{ is the reqd Eliminant}$$

$$8 \quad al = bm = cn \quad (1)$$

$$l^2 + m^2 + n^2 = 1 \quad (2)$$

$$a^2 l^2 + b^2 m^2 + c^2 n^2 = a'^2 l + b'^2 m + c'^2 n \quad (3)$$

$$\text{From (1), we get } l^2 = \frac{c^2 n^2}{a^2} \quad (4)$$

$$\text{and } m^2 = \frac{c^2 n^2}{b^2} \quad (5)$$

$$\text{We get } l^2 + m^2 = c^2 n^2 \left(\frac{1}{a^2} + \frac{1}{b^2} \right) = \frac{c^2 n^2 (a^2 + b^2)}{a^2 b^2}$$

from (2), we get

$$\frac{c^2 n^2 (a^2 + b^2)}{a^2 b^2} + n^2 = 1 \quad (6)$$

$$\text{or, } n^2 = \frac{a^2 b^2}{a^2 b^2 + a^2 c^2 + b^2 c^2} \quad (7)$$

$$\text{from (4), } l^2 = \frac{c^2 b^2}{a^2 b^2 + a^2 c^2 + b^2 c^2} \quad (8)$$

Again from (1) and (3), we get

$$al^2 + bm^2 + cn^2 = \frac{a'^2}{a} + \frac{b'^2}{b} + \frac{c'^2}{c} \quad (9)$$

from (6), (7), (8), and (9), we get

$$\begin{aligned} & \frac{ab^3c^3}{a^2b^3 + a^3c^2 + b^2c^3} + \frac{bc^2a^3}{a^2b^3 + a^3c^2 + b^2c^3} + \frac{ca^2b^2}{a^2b^3 + a^3c^2 + b^2c^3} \\ &= \frac{a'^2}{a} + \frac{b'^2}{b} + \frac{c'^2}{c} \end{aligned}$$

$$\text{or, } abc(bc + ca + ab) = \left(\frac{a'^2}{a} + \frac{b'^2}{b} + \frac{c'^2}{c} \right) (a^2b^3 + b^2c^2 + c^2a^3)$$

$$\text{or, } \frac{abc(bc + ca + ab)}{a^2b^3c^2} = \left(\frac{a'^2}{a} + \frac{b'^2}{b} + \frac{c'^2}{c} \right) \left(\frac{a^2b^3 + b^2c^2 + c^2a^3}{a^2b^3c^2} \right)$$

$$\text{or, } \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \left(\frac{a'^2}{a} + \frac{b'^2}{b} + \frac{c'^2}{c} \right) \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right)$$

Exercise (99).

$$1 \quad 8x + \frac{7}{x} = \frac{65}{7}x \quad \text{or, } 56x^2 + 49 = 65x^2$$

$$\text{or, } 9x^2 = 49 \quad \text{or, } 3x = \pm 7 \quad x = \pm \frac{7}{3} = \pm 2\frac{1}{3}$$

$$2 \quad \frac{2x^2+10}{15} = 7 - \frac{50+x^2}{25} \quad \text{or,} \quad 10x^2+50=525-150-3x^2$$

$$\text{or,} \quad 13x^2=325 \quad \text{or,} \quad x^2=25 \quad x=\pm 5$$

$$3 \quad \frac{14x^2+16}{21} - \frac{2x^2+8}{8x^2-11} = \frac{2x^2}{3}$$

$$\text{or,} \quad 112x^4-128x^2-154x^2-176-42x^2-168=102x^4-154x^2$$

$$\text{or,} \quad 128x^2-42x^2=176+168 \quad \text{or,} \quad 86x^2=344$$

$$\text{or,} \quad x^2=4 \quad x=\pm 2$$

$$4 \quad \frac{x+7}{x(x-7)} - \frac{x-7}{x(x+7)} = \frac{7}{x^2-73}$$

$$\text{or,} \quad (x^2-73)\{(x+7)^2-(x-7)^2\}=7x(x^2-49)$$

$$\text{or,} \quad (x^2-73)47x=7x(x^2-49) \quad \text{or,} \quad (x^2-73)4=x^2-49$$

$$\text{or,} \quad 4x^2-292=x^2-49 \quad \text{or,} \quad 3x^2=292-49=243$$

$$\text{or,} \quad x^2=81 \quad x=\pm 9$$

$$5 \quad \frac{x^3-1}{(x-1)^2} - \frac{x^2+1}{(x+1)^2} = 6$$

$$\text{or,} \quad \frac{(x^2+1)+x}{x-1} - \frac{(x^2+1)-x}{x+1} = 6$$

$$\text{or,} \quad (x^2+1)(x+1)+x(x+1)-(x^2+1)(x-1)+x(x-1)$$

$$= 6(x^2-1)$$

$$\text{or,} \quad 2(x^2+1)+2x^2=6x^2-6 \quad \text{or,} \quad 2x^2=8$$

$$\text{or,} \quad x^2=4 \quad x=\pm 2$$

$$6 \quad \frac{1}{\sqrt{1-x}+1} + \frac{1}{\sqrt{1+x}-1} = \frac{1}{x}$$

$$\text{or,} \quad \frac{\sqrt{1-x}-1}{-x} + \frac{\sqrt{1+x}+1}{x} = \frac{1}{x}$$

$$\text{or,} \quad \sqrt{1+x}+1-\sqrt{1-x}+1=1$$

$$\text{or,} \quad \sqrt{1-x}-\sqrt{1+x}=1$$

$$\text{or,} \quad 1-x+1+x-2\sqrt{1-x^2}=1$$

$$\text{or,} \quad 2\sqrt{1-x^2}=1 \quad \text{or,} \quad 4(1-x^2)=1$$

$$\text{or,} \quad 4x^2=3 \quad \text{or,} \quad 2x=\pm\sqrt{3} \quad x=\pm\sqrt{\frac{3}{4}}$$

$$\begin{aligned}
 7 \quad & (1+r+x^2)^{\frac{1}{2}} = a - (1-r+x^2)^{\frac{1}{2}} \\
 \text{or,} \quad & (1+x+x^2)^{\frac{1}{2}} + (1-r+x^2)^{\frac{1}{2}} = a \\
 \text{or,} \quad & 1+x+x^2+1-r+r^2+2(1+r^2+x^4)^{\frac{1}{2}} = a^2 \\
 \text{or,} \quad & 2(1+r^2+x^4)^{\frac{1}{2}} = a^2 - 2(1+x^2) \\
 \text{or,} \quad & 4(1+x^2+x^4) = a^4 + 4(1+x^2)^2 - 4a^2(1+x^2) \\
 & \quad = a^4 + 4 + 8x^2 + 4x^4 - 4a^2 - 4a^2x^2 \\
 \text{or} \quad & a^4 + 4x^2 - 4a^2 - 4a^2x^2 = 0 \\
 \text{or,} \quad & 4x^2(a^2 - 1) = a^4 - 4a^2 = a^2(a^2 - 4) \\
 \text{or,} \quad & 2x\sqrt{a^2 - 1} = \pm a\sqrt{a^2 - 4} \\
 & x = \pm \frac{a\sqrt{a^2 - 4}}{2\sqrt{a^2 - 1}}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad & \frac{(z-a)(x-b)}{(z-ma)(x-mb)} = \frac{(z+a)(x+b)}{(z+ma)(x+mb)} \\
 \text{or,} \quad & \frac{z^2 - z(a+b) + ab}{z^2 - mz(a+b) + m^2ab} = \frac{z^2 + z(a+b) + ab}{z^2 + mz(a+b) + m^2ab} \\
 \text{or,} \quad & x^4 + x^3(a+b)(m-1) + x^2\{ab(m^2+1) - m(a+b)^2\} \\
 & \quad + m^2a^2b^2 - z mab(a+b)(m-1) \\
 & = z^4 - z^3(a+b)(m-1) + x^2\{ab(m^2+1) - m(a+b)^2\} \\
 & \quad + z mab(a+b)(m-1) \\
 \text{or,} \quad & 2z^3(a+b)(m+1) = 2x mab(a+b)(m-1) \\
 \text{or,} \quad & z^2 = mab \quad z = \pm \sqrt{mab}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad & \frac{ax+1+(a^2x^2-1)^{\frac{1}{2}}}{ax+1-(a^2x^2-1)^{\frac{1}{2}}} = \frac{b^2x}{2} \\
 \text{or,} \quad & \frac{ax+1}{(a^2x^2-1)^{\frac{1}{2}}} = \frac{b^2x+2}{b^2x-2} \\
 \text{or,} \quad & \frac{(ax+1)^2}{a^2x^2-1} = \frac{(b^2x+2)^2}{(b^2x-2)^2} \\
 \text{or,} \quad & \frac{(ax+1)^2 + (a^2x^2-1)}{(ax+1)^2 - (a^2x^2-1)} = \frac{(b^2x+2)^2 + (b^2x-2)^2}{(b^2x+2)^2 - (b^2x-2)^2} \\
 \text{or,} \quad & \frac{2ax(ax+1)}{2(ax+1)} = \frac{2(b^4x^2+4)}{8b^2x}
 \end{aligned}$$

$$\text{or, } ax = \frac{b^4x^3+4}{4b^2x} \quad \text{or, } 4ab^2x^3 = b^4x^3+4$$

$$\text{or, } b^2x^2(4a-b^2)=4 \quad \text{or, } bx\sqrt{4a-b^2}=\pm 2$$

$$x = \frac{2}{\pm b\sqrt{4a-b^2}}$$

$$\begin{aligned} 10 \quad & (a+x)^3 + (a-x)^3 + 9(a^2-x^2)^{\frac{1}{2}}(a^2-x^2)^{\frac{3}{2}} = 27(a^2-x^2) \\ \text{or, } & (a+x)^3 + (a-x)^3 + 9(a^2-x^2)^{\frac{1}{2}}(a^2-x^2)^{\frac{3}{2}} = 27(a^2-x^2) \\ \text{or, } & 2(a^3+x^3) + 9(a^2-x^2)^{\frac{1}{2}}(a^2-x^2)^{\frac{3}{2}} = 27(a^2-x^2) \\ \text{or, } & 2(a^3+x^3) = 18(a^2-x^2) \\ \text{or, } & a^3+x^3 = 9(a^2-x^2) \\ \text{or, } & 10x^2 = 8a^2 \\ \text{or, } & x^2 = \frac{4a^2}{5} \quad x = \pm \frac{2a}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} 11 \quad & \frac{5x^2+17}{x^2-11} + \frac{14x^2-117}{2x^2-9} = 12 \\ \text{or, } & 5 + \frac{72}{x^2-11} + 7 - \frac{54}{2x^2-9} = 12 \\ \text{or, } & \frac{72}{x^2-11} - \frac{54}{2x^2-9} = 0 \\ \text{or, } & \frac{4}{x^2-11} - \frac{3}{2x^2-9} = 0 \\ \text{or, } & 8x^2-36 = 3x^2-33 \\ \text{or, } & 5x^2 = 3 \quad x = \pm \sqrt{\frac{3}{5}} \end{aligned}$$

$$\begin{aligned} 12. \quad & \frac{x^3-1}{x^3-4} - \frac{x^2-5}{x^3-8} = \frac{x^3-2}{x^3-5} - \frac{x^2-6}{x^3-9} \\ \text{or, } & 1 + \frac{3}{x^3-4} - 1 - \frac{3}{x^3-8} = 1 + \frac{3}{x^3-5} - 1 - \frac{3}{x^3-9} \\ \text{or, } & \frac{x^3-8-x^2+4}{(x^3-4)(x^3-8)} = \frac{x^2-9-x^2+5}{(x^3-5)(x^3-9)} \\ \text{or, } & \frac{-4}{(x^3-4)(x^3-8)} = \frac{-4}{(x^3-5)(x^3-9)} \\ \text{or, } & x^4-14x^3+45 = x^4-12x^3+32 \\ \text{or, } & 2x^2 = 13 \quad x = \pm \sqrt{\frac{13}{2}} \end{aligned}$$

13 We get

$$\sqrt[3]{2(a+x)} = n \left\{ \frac{a+x}{a+(a^2-x^2)^{\frac{1}{2}}} \right\}^{\frac{1}{2}}$$

$$\text{or,} \quad \sqrt{2} = \frac{n}{\sqrt[3]{a+(a^2-x^2)^{\frac{1}{2}}}}$$

$$\text{or,} \quad 2\{a+(a^2-x^2)^{\frac{1}{2}}\} = n^2$$

$$\text{or,} \quad (a^2-x^2)^{\frac{1}{2}} = \frac{n^2}{2} - a$$

$$\text{or,} \quad a^2 - x^2 = \frac{n^4}{4} - an^2 + a^2$$

$$\text{or,} \quad x^2 = n^2 \left(a - \frac{n^2}{4} \right)$$

$$x = \pm n \left(a - \frac{n^2}{4} \right)^{\frac{1}{2}}$$

14

$$\frac{(1+2x)^{\frac{1}{2}} - 1}{(1-2x)^{\frac{1}{2}} + 1} + \frac{(1-2x)^{\frac{1}{2}} + 1}{(1+2x)^{\frac{1}{2}} - 1} = 2\sqrt{2}$$

$$\text{or,} \quad \frac{(1-4x^2)^{\frac{1}{2}} - (1+2x)^{\frac{1}{2}} - (1-2x)^{\frac{1}{2}} - 1}{1-2x-1}$$

$$+ \frac{(1-4x^2)^{\frac{1}{2}} + (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} - 1}{1+2x-1} = 2\sqrt{2}$$

$$\text{or,} \quad -(1-4x^2)^{\frac{1}{2}} + (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} - 1 + (1-4x^2)^{\frac{1}{2}} \\ + (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} + 1 = 4\sqrt{2}x$$

$$\text{or,} \quad (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} = 2\sqrt{2}x$$

$$\text{or,} \quad 1+2x+1-2x+2(1-4x^2)^{\frac{1}{2}} = 8x^2$$

$$\text{or,} \quad (1-4x^2)^{\frac{1}{2}} = 4x^2 - 1 \quad \text{or,} \quad 1-4x^2 = 16x^4 - 8x^2 + 1$$

$$\text{or,} \quad 16x^4 = 4x^2 \quad \text{or,} \quad 4x^2 = 1$$

$$\text{or} \quad 2x = \pm 1 \quad x = \pm \frac{1}{2}$$

Exercise (100)

1 & 2—worked out in the book.

$$\begin{aligned}
 3 \quad & 87 - 95x + 30x^2 - 16x^3 \\
 \text{or,} \quad & 16x^3 - 125x^2 + 87x - 87 \\
 \text{or,} \quad & x^3 - 6x^2 = -\frac{87}{16} \\
 \text{or,} \quad & x^3 - 6x + 16 = -\frac{87}{16} + 16 \\
 \text{or,} \quad & (x-4)^3 = \left(\frac{1}{4}\right)^3 \\
 & x-4 = \pm \frac{1}{4} \quad x = 4 \pm \frac{1}{4} = 7\frac{1}{4} \text{ or } 3\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 17x^3 - 85x^2 + 216x - 65x^3 - 8x^3 \\
 \text{or,} \quad & 25x^3 - 150x^2 - 216 \\
 \text{or,} \quad & x^3 - 6x = -\frac{216}{25} \\
 \text{or,} \quad & x^3 - 6x + 9 = -\frac{216}{25} + 9 \\
 \text{or,} \quad & (x-3)^3 = -\frac{27}{5} \quad x-3 = \pm \sqrt[3]{\frac{27}{5}} \\
 & x = 3 \pm \sqrt[3]{\frac{27}{5}} = 3\sqrt[3]{\frac{27}{5}} \text{ or } 2\sqrt[3]{\frac{27}{5}}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \frac{x^2+8}{11} = 5x - x^2 - 5 \\
 \text{or,} \quad & x^2 + 8 - 55x - 11x^2 - 55 \\
 \text{or,} \quad & 12x^2 - 55x = -63 \\
 \text{or,} \quad & x^2 - \frac{55}{12}x = -\frac{63}{12} \\
 \text{or,} \quad & x^2 - \frac{55}{12}x + \left(\frac{55}{24}\right)^2 = -\frac{63}{12} + \left(\frac{55}{24}\right)^2 \\
 \text{or,} \quad & \left(x - \frac{55}{24}\right)^2 = \frac{1}{144} \\
 & x - \frac{55}{24} = \pm \frac{1}{12} \\
 & x = \frac{55}{24} \pm \frac{1}{12} = 2\frac{1}{2} \text{ or } 2\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & 4\left(x^3 - 3\frac{3}{5}x\right) = 10\left(x^2 - 4\frac{2}{5}x - 6\right) + 3\left(\frac{x}{5} - \frac{5}{3}\right) \\
 \text{or,} \quad & 4x^3 - 14\frac{3}{5}x = 10x^2 - 44x - 60 + \frac{1}{5}x - 5 \\
 \text{or,} \quad & 6x^3 - 29x = 65 \\
 \text{or,} \quad & x^3 - \frac{29}{6}x = \frac{65}{6} \\
 \text{or,} \quad & x^3 - \frac{29}{6}x + \left(\frac{29}{18}\right)^2 = \frac{65}{6} + \left(\frac{29}{18}\right)^2 \\
 \text{or,} \quad & \left(x - \frac{29}{18}\right)^2 = \frac{1491}{162} \\
 & x - \frac{29}{18} = \pm \sqrt{\frac{1491}{162}} \quad x = \frac{29}{18} \pm \sqrt{\frac{1491}{162}} = 6\frac{1}{2} \text{ or } -1\frac{1}{2}
 \end{aligned}$$

$$7 \quad 4(5x^2 - 35x) = 5(x^2 - 7x + 12) + \frac{8(x-9)}{9}$$

$$\text{or,} \quad 20x^2 - 151x = 5x^2 - 35x + 60 + \frac{8}{9}x - 8$$

$$\text{or,} \quad 15x^2 + 19x = 52$$

$$\text{or} \quad x^2 + \frac{19x}{15} = \frac{52}{15}$$

$$\text{or,} \quad x^2 + \frac{19}{15}x + (\frac{19}{30})^2 = \frac{52}{15} + (\frac{19}{30})^2$$

$$\text{or,} \quad (x + \frac{19}{30})^2 = \frac{3481}{900}$$

$$x + \frac{19}{30} = \pm \sqrt{\frac{3481}{900}}$$

$$x = -\frac{19}{30} \pm \frac{59}{30} = 1^{\frac{2}{3}} \text{ or } -2^{\frac{2}{3}}$$

$$8 \quad 21 + 02 = 245x - x^2$$

$$\text{or,} \quad x^2 - 45x = -02$$

$$\text{or,} \quad x^2 - 45x + (225)^2 = -02 + (225)^2$$

$$\text{or,} \quad (x - 225)^2 = 030625$$

$$x - 225 = \pm 175$$

$$x = 225 \pm 175 = 4 \text{ or } 05$$

$$9 \quad 4(x^2 + 23x - 24) = 29x^2 - 8x + 1$$

$$\text{or,} \quad 4x^2 + 92x - 96 = 29x^2 - 8x + 1$$

$$\text{or,} \quad 25x^2 - 100x = -97$$

$$\text{or,} \quad x^2 - 4x = -\frac{97}{25}$$

$$\text{or,} \quad x^2 - 4x + 4 = -\frac{97}{25} + 4$$

$$\text{or,} \quad (x - 2)^2 = \frac{27}{25}$$

$$x - 2 = \pm \sqrt{\frac{27}{25}} \quad x = 2 \pm \frac{3}{5}\sqrt{3}$$

10 Worked out in the text

$$11 \quad (2x - 5)(3x - 7) - (x - 1)(4x - 5) = x^2 - 3(x + 14)$$

$$\text{or,} \quad 6x^2 - 29x + 35 - 4x^2 + 9x - 5 = x^2 - 3x - 42$$

$$\text{or,} \quad x^2 - 17x = -72$$

$$\text{or,} \quad x^2 - 17x + (\frac{17}{2})^2 = -72 + (\frac{17}{2})^2$$

$$\text{or,} \quad (x - \frac{17}{2})^2 = \frac{1}{4}$$

$$x - \frac{17}{2} = \pm \frac{1}{2}$$

$$x = \frac{17}{2} \pm \frac{1}{2} = 9 \text{ or } 8$$

$$\begin{aligned}
 12 \quad & (31-11)(x-2) + (21-3)(1+4) + 13x = 10(21-1)^2 + 12 \\
 \text{or,} \quad & 31^2 - 171 + 22 + 21^2 + 51 - 12 + 131 = 401^2 - 401 + 10 + 12 \\
 \text{or,} \quad & 35x^2 - 411 = -12 \\
 \text{or,} \quad & x^2 - \frac{41x}{35} = -\frac{12}{35} \\
 \text{or,} \quad & x^2 - \frac{41}{7}x + \left(\frac{41}{7}\right)^2 = -\frac{12}{7} + \left(\frac{41}{7}\right)^2 \\
 \text{or,} \quad & \left(x - \frac{41}{7}\right)^2 = \frac{1600}{49} \\
 & x - \frac{41}{7} = \pm \frac{40}{7} \\
 & x = \frac{41}{7} \pm \frac{40}{7} = \frac{1}{7} \text{ or } \frac{81}{7}
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & (1-\frac{1}{2})(1-\frac{1}{2}) + (1-\frac{1}{4})(1-\frac{1}{4}) = (1-\frac{1}{4})(1-\frac{1}{4}) \\
 \text{or,} \quad & (1-\frac{1}{2})(1-\frac{1}{2}) = (1-\frac{1}{4})(1-\frac{1}{2}) - (1-\frac{1}{4}) \\
 \text{or,} \quad & (1-\frac{1}{2})(1-\frac{1}{2}) = (1-\frac{1}{4})(1-\frac{1}{2}) - (1-\frac{1}{4}) \\
 \text{or,} \quad & x^2 - \frac{3}{2}x + \frac{1}{4} = 1 - \frac{1}{2}x - \frac{1}{4} \\
 \text{or,} \quad & x^2 - \frac{5}{4}x = -\frac{1}{4} = -1 \\
 \text{or,} \quad & x^2 - \frac{5}{4}x + \left(\frac{5}{8}\right)^2 = -\frac{1}{4} + \left(\frac{5}{8}\right)^2 \\
 \text{or,} \quad & \left(x - \frac{5}{8}\right)^2 = \frac{9}{64} \\
 & x - \frac{5}{8} = \pm \frac{3}{8} \\
 & x = \frac{5}{8} \pm \frac{3}{8} = \frac{1}{2} \text{ or } \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad & \frac{1}{15} + \frac{40}{3(10-1)} = \frac{3(10+1)}{95} \\
 \text{or,} \quad & \frac{40}{3(10-1)} = \frac{3(10+1)}{95} - \frac{1}{15} = \frac{90+91-19}{15 \times 19} = \frac{90-10}{3 \times 95} \\
 \text{or,} \quad & \frac{4}{10-1} = \frac{9-1}{95} \\
 \text{or,} \quad & 380 = 90 - 191 + 1^2 \\
 \text{or,} \quad & x^2 - 191 = 290 \\
 \text{or,} \quad & x^2 - 191 + \left(\frac{19}{2}\right)^2 = 290 + \left(\frac{19}{2}\right)^2 \\
 \text{or,} \quad & \left(x - \frac{19}{2}\right)^2 = \frac{1764}{4} \\
 & x - \frac{19}{2} = \pm \frac{42}{2} \\
 & x = \frac{19}{2} \pm \frac{42}{2} = 29 \text{ or } -10
 \end{aligned}$$

$$15 \quad \frac{21}{15} + \frac{31-50}{3(10+1)} = \frac{12x+70}{190}$$

$$\text{or, } \frac{3x-50}{3(10+x)} = \frac{12x+70}{190} - \frac{2x}{15} = \frac{36x+210-76x}{190 \times 3} = \frac{210-40x}{190 \times 3}$$

$$= \frac{21-4x}{19 \times 3}$$

$$\frac{3x-50}{10+x} = \frac{21-4x}{19}$$

$$\text{or, } 57x-950=210+21x-40x-4x^2$$

$$\text{or, } 4x^2+76x=1160 \quad \text{or, } x^2+19x=290$$

$$\text{or, } x^2+19x+(19)^2=290+(19)^2$$

$$\text{or, } (x+19)^2=15^2+1$$

$$x+19=\pm 19^{\frac{1}{2}} \quad x=-19^{\frac{1}{2}} \pm 19^{\frac{1}{2}}=10 \text{ or } -29$$

16 Worked out in the book

$$17 \quad \frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6}$$

$$\text{or, } \frac{x}{x+1} - 1 + \frac{x+1}{x} - 1 = \frac{1}{6}$$

$$\text{or, } -\frac{1}{x+1} + \frac{1}{x} = \frac{1}{6}$$

$$\text{or, } \frac{-x+x+1}{x(x+1)} = \frac{1}{6}$$

$$\text{or, } x^2+x=6$$

$$\text{or, } x^2+x+(1)^2=6+1$$

$$\text{or, } (x+\frac{1}{2})^2=\frac{35}{4} \quad x+\frac{1}{2}=\pm\frac{\sqrt{35}}{2}$$

$$x=-\frac{1}{2} \pm \frac{\sqrt{35}}{2}=2 \text{ or } -3$$

18 Worked out in the book

$$19 \quad \frac{x-2}{x+2} + \frac{x+2}{x-2} = \frac{2(x+3)}{x-3}$$

$$\text{or, } 1 - \frac{4}{x+2} + 1 + \frac{4}{x-2} = 2 + \frac{12}{x-3}$$

$$\text{or, } -\frac{1}{x+2} + \frac{1}{x-2} = \frac{3}{x-3}$$

$$\text{or, } \frac{-x+2+x+2}{x^2-4} = \frac{3}{x-3}$$

$$\text{or, } 4x-12=3x^2-12$$

$$3x^2=4x \quad x=0 \text{ or } \frac{4}{3}$$

20 We have

$$\left(\frac{x+2}{x-2}-1\right)-\left(\frac{x-2}{x+2}-1\right)=\frac{5}{6}$$

$$\text{or,} \quad \frac{1}{x-2} + \frac{4}{x+2} = \frac{5}{6}$$

$$\text{or,} \quad \frac{4(x+2+x-2)}{x^2-4} = \frac{5}{6}$$

$$\text{or,} \quad 48x = 5x^2 - 20$$

$$\text{or,} \quad 5x^2 - 48x = 20$$

$$\text{or,} \quad x^2 - \frac{48}{5}x = 4$$

$$\text{or,} \quad x^2 - \frac{48}{5}x + \left(\frac{24}{5}\right)^2 = 4 + \left(\frac{24}{5}\right)^2$$

$$\text{or,} \quad \left(x - \frac{24}{5}\right)^2 = \frac{676}{25}$$

$$x - \frac{24}{5} = \pm \frac{26}{5} \quad x = \frac{24}{5} \pm \frac{26}{5} = 10 \text{ or } -\frac{2}{5}$$

$$21 \quad \frac{x-6}{x-12} - \frac{x-12}{x-6} = \frac{5}{6}$$

$$\text{or,} \quad 1 + \frac{6}{x-12} - 1 + \frac{6}{x-6} = \frac{5}{6}$$

$$\text{or,} \quad \frac{6(x-6+x-12)}{(x-12)(x-6)} = \frac{5}{6}$$

$$\text{or,} \quad 72x - 648 = 5x^2 - 90x + 360$$

$$\text{or,} \quad 5x^2 - 162x = -1008$$

$$\text{or,} \quad x^2 - \frac{162}{5}x = -\frac{1008}{5}$$

$$\text{or,} \quad x^2 - \frac{162}{5}x + \left(\frac{81}{5}\right)^2 = -\frac{1008}{5} + \left(\frac{81}{5}\right)^2$$

$$\text{or,} \quad \left(x - \frac{81}{5}\right)^2 = \frac{1600}{25} \quad x - \frac{81}{5} = \pm \frac{40}{5}$$

$$x = \frac{81}{5} \pm \frac{40}{5} = 24 \text{ or } 8\frac{1}{5}$$

$$22 \quad \frac{2x-9}{2x-7} - \frac{2x-7}{2x-9} = \frac{7}{12}$$

$$\text{or,} \quad 1 - \frac{2}{2x-7} - 1 + \frac{2}{2x-9} = \frac{7}{12}$$

$$\text{or,} \quad -24(2x-9+2x-7) = 7(2x-7)(2x-9)$$

$$\text{or,} \quad -96x + 384 = 28x^2 - 224x + 441$$

$$\text{or,} \quad 28x^2 - 128x = -57$$

$$\text{or,} \quad x^2 - \frac{32}{7}x = -\frac{57}{28}$$

$$\text{or,} \quad x^2 - \frac{32}{7}x + \left(\frac{16}{7}\right)^2 = -\frac{57}{28} + \left(\frac{16}{7}\right)^2$$

$$\text{or,} \quad \left(x - \frac{16}{7}\right)^2 = \frac{925}{49}$$

$$x - \frac{16}{7} = \pm \frac{30.5}{7} \quad x = \frac{16}{7} \pm \frac{30.5}{7} = 4\frac{1}{4} \text{ or } \frac{1}{2}$$

23

$$\frac{x+6}{x+7} - \frac{x+1}{x+2} = \frac{1}{3x+1}$$

$$\text{or, } 1 - \frac{1}{x+7} - 1 + \frac{1}{x+2} = \frac{1}{3x+1}$$

$$\text{or, } \frac{-x-2+x+7}{(x+7)(x+2)} = \frac{1}{3x+1}$$

$$\text{or, } 15x+5 = x^2+9x+14$$

$$\text{or, } x^2-6x+9=0$$

$$(x-3)^2=0 \quad x=3$$

24 We have

$$\left(\frac{2x}{x-4}-2\right) + \left(\frac{2x-5}{x-3}-2\right) = 4\frac{1}{3}$$

$$\text{or, } \frac{8}{x-4} + \frac{1}{x-3} = \frac{13}{3}$$

$$\text{or, } 24x-72+3x-12=13x^2-91x+156$$

$$\text{or, } 13x^2-118x=-240$$

$$\text{or, } x^2-\frac{118}{13}x=-\frac{240}{13}$$

$$\text{or, } x^2-\frac{118}{13}x+\left(\frac{59}{13}\right)^2=-\frac{240}{13}+\left(\frac{59}{13}\right)^2$$

$$\text{or, } \left(x-\frac{59}{13}\right)^2=\frac{161}{169}$$

$$x-\frac{59}{13}=\pm\frac{11}{13} \quad x=\frac{59}{13}\pm\frac{11}{13}=6 \text{ or } 3\frac{1}{13}$$

25

$$\frac{x}{x+5} - \frac{11x}{11x-8} + \frac{7}{6-4x} = 0$$

$$\text{or, } \frac{11x^2-8x-11x^2-55x}{(x+5)(11x-8)} + \frac{7}{6-4x} = 0$$

$$\text{or, } \frac{-63x}{(x+5)(11x-8)} + \frac{7}{6-4x} = 0$$

$$\text{or, } -9x(6-4x) + (x+5)(11x-8) = 0$$

$$\text{or, } -54x+36x^2+11x^2-8x+55x-40=0$$

$$\text{or, } 47x^2-7x=40$$

$$\text{or, } x^2-\frac{7}{47}x=\frac{40}{47}$$

$$\text{or, } x^2-\frac{7}{47}x+\left(\frac{7}{94}\right)^2=\frac{40}{47}+\left(\frac{7}{94}\right)^2$$

$$\text{or, } \left(x-\frac{7}{94}\right)^2=\frac{7569}{(94)^2}$$

$$x-\frac{7}{94}=\pm\frac{87}{94}$$

$$x=\frac{7}{94}\pm\frac{87}{94}=1 \text{ or } -\frac{40}{47}$$

26 We get

$$\frac{x+2a}{x+a} = -\frac{2x+5a}{x+2a}$$

$$\text{or, } x^2 + 4ax + 4a^2 = -(2x^2 + 7ax + 5a^2)$$

$$\text{or, } 3x^2 + 11ax = -9a^2$$

$$\text{or, } x^2 + \frac{11ax}{3} = -3a^2$$

$$\text{or } x^2 + \frac{11ax}{3} + \left(\frac{11a}{6}\right)^2 = -3a^2 + \left(\frac{11a}{6}\right)^2$$

$$\text{or } \left(x + \frac{11a}{6}\right)^2 = \frac{13a^2}{36}$$

$$x + \frac{11a}{6} = \pm \frac{\sqrt{13}a}{6}$$

$$x = -\frac{11a}{6} \pm \frac{\sqrt{13}a}{6}$$

$$= \frac{-11 \pm \sqrt{13}}{6} a$$

Exercise (101)

Note — The roots of an equation of the form $ax^2 + bx + c = 0$

are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the following exercises are to be

solved by applying this formula

$$1 \quad 3x^2 - 17x + 24 = 0$$

$$x = \frac{17 \pm \sqrt{289 - 288}}{6} = \frac{17 \pm 1}{6}$$

$$= 3 \text{ or } 2\frac{2}{3}$$

$$2 \quad x^2 + 9x + 20 = 0$$

$$x = \frac{-9 \pm \sqrt{81 - 80}}{2} = \frac{-9 \pm 1}{2} = -4 \text{ or } -5$$

3

$$6x^2 = 20 - 7x$$

$$6x^2 + 7x - 20 = 0$$

$$x = \frac{-7 \pm \sqrt{49 + 480}}{12} = \frac{-7 \pm 23}{12}$$

$$= \frac{4}{3} \text{ or } -\frac{5}{2}$$

4

$$-9x^2 + 25 = 6x - 10$$

or,

$$9x^2 + 6x - 35 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 1260}}{18} = \frac{-6 \pm \sqrt{1296}}{18}$$

$$= \frac{-6 \pm 36}{18} = \frac{5}{3} \text{ or } -\frac{7}{3}$$

5

$$8x^2 = 14x + 15$$

or,

$$8x^2 - 14x - 15 = 0$$

$$x = \frac{14 \pm \sqrt{196 + 480}}{16} = 14 \pm \sqrt{676}$$

$$= \frac{14 \pm 26}{16} = \frac{10}{19} \text{ or } -\frac{12}{16} = \frac{5}{2} \text{ or } -\frac{3}{4}$$

6

$$-3x^2 + 20x = 25$$

or,

$$3x^2 - 20x + 25 = 0$$

$$x = \frac{20 \pm \sqrt{400 - 300}}{6} = \frac{20 \pm 10}{6} = \frac{30}{6} \text{ or } \frac{10}{6}$$

$$= 5 \text{ or } \frac{5}{3}$$

7

$$5 + x - 4x^2 = 0$$

or,

$$4x^2 - x - 5 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 80}}{8} = \frac{1 \pm 9}{8} = \frac{10}{8} \text{ or } -\frac{8}{8}$$

$$= \frac{5}{4} \text{ or } -1$$

Exercise (102)

1

$$2x^2 + 9x = 18$$

$$4(2x)^2 + 4 \times (2x) \times 9 + 9^2 = 144 + 81 = 225$$

or,

$$(4x + 9)^2 = (15)^2$$

$$4x + 9 = \pm 15$$

$$41 = \pm 15 - 9 = 6 \text{ or } -24$$

$$r = \frac{3}{2} \text{ or } -6$$

2

or,

or,

$$15t^2 - 28 = 1$$

$$15t^2 - r = 28$$

$$4(15r)^2 - 4 \times (15t) + 1 = 4 \times 15 \times 28 + 1 = 1681$$

$$(30r - 1)^2 = (41)^2$$

$$30r - 1 = \pm 41$$

$$30r = \pm 41 + 1 = 42 \text{ or } -40$$

$$r = \frac{7}{5} \text{ or } -\frac{4}{3}$$

3

or,

or,

$$16r^2 + 100t = 3t^2 + 1 + 40$$

$$13r^2 + 99t = 40$$

$$4(13r)^2 + 4 \times (13t) \times 99 + (99)^2$$

$$= 4 \times 13 \times 40 + 9801$$

$$= 2080 + 9801 = 11881$$

$$(26r + 99)^2 = (109)^2$$

$$26r + 99 = \pm 109$$

$$26r = -99 \pm 109 = -208 \text{ or } 10$$

$$r = -8 \text{ or } \frac{5}{13}$$

4

or,

or,

$$r^2 + 50t = 102 - 15t - r^2$$

$$2r^2 + 65t = 102$$

$$4(2r)^2 + 4 \times (2t) \times 65 + (65)^2 = 8 \times 102 + 4225$$

$$= 816 + 4225 = 5041$$

$$(4t + 65)^2 = (71)^2$$

$$4t + 65 = \pm 71$$

$$4t = -65 \pm 71 = 6 \text{ or } -136$$

$$t = \frac{3}{2} \text{ or } -34$$

5

$$17x^2 + 19t = 1848$$

$$4(17r)^2 + 4 \times (17t) \times 19 + (19)^2 = 4 \times 17(1848) + (19)^2$$

$$= 125664 + 361 = 126025$$

$$(34t + 19)^2 = (355)^2$$

$$34t + 19 = \pm 355$$

$$34t = \pm 355 - 19$$

$$= 336 \text{ or } -374$$

$$t = 9\frac{16}{17} \text{ or } -11$$

6

$$\begin{aligned}
 2cx^2 - acx &= 3(2x - a) \\
 \text{or, } 2cx^2 - 1(ac + 6) &= -3a \\
 4(2cx)^2 - 4(2cx)(ac + 6) + (ac + 6)^2 & \\
 &= (ac + 6)^2 - 24ac \\
 (4cx - ac - 6)^2 &= (ac - 6)^2 \\
 4cx - (ac + 6) &= \pm (ac - 6) \\
 4cx &= 2ac \text{ or } 12 \\
 x &= \frac{a}{2} \text{ or } \frac{3}{c}
 \end{aligned}$$

7

$$\begin{aligned}
 x^2 + ax &= ab(3x + a) - 2x^2 \\
 \text{or, } 3x^2 + 1(a - 3ab) &= a^2b \\
 4(3x)^2 + 4(3x)(a - 3ab) + (a - 3ab)^2 & \\
 &= (a - 3ab)^2 + 4(3a^2b) \\
 (6x + a - 3ab)^2 &= (a + 3ab)^2 \\
 6x + a - 3ab &= \pm (a + 3ab) \\
 6x &= 6ab \text{ or } -2a \\
 x &= ab \text{ or } -\frac{a}{3}
 \end{aligned}$$

Exercise (103)

1

$$\begin{aligned}
 3x^2 - 12x + 1 &= 6x - 23 \\
 \text{or, } 3x^2 - 18x + 24 &= 0 \\
 \text{or, } x^2 - 6x + 8 &= 0 \\
 (x - 2)(x - 4) &= 0 \\
 \text{Hence either } x - 2 &= 0, & x = 2, \\
 \text{or, } x - 4 &= 0, & x = 4
 \end{aligned}$$

2

$$\begin{aligned}
 4x^2 - 4x &= 80 \\
 \text{or, } x^2 - x - 20 &= 0 \\
 (x + 4)(x - 5) &= 0 \\
 \text{Hence either } x + 4 &= 0 & x = -4, \\
 \text{or, } x - 5 &= 0 & x = 5
 \end{aligned}$$

- 3 $1 + 2 - \frac{6}{x+2} = 1$
 or, $(1+2)^2 - (1+2) - 6 = 0$
 or, $\{(x+2) - 3\}\{(x+2) + 2\} = 0$
 $(x-1)(x+4) = 0$
 Hence either $x-1=0$, $x=1$,
 or, $x+4=0$, $x=-4$
- 4 $x^2 + 9x - 52 = 0$
 $(x+13)(x-4) = 0$
 Hence either $x+13=0$, $x=-13$,
 or, $x-4=0$, $x=4$
- 5 $x^2 - 7x - 4 = 0$
 or, $3x^2 - 5x - 12 = 0$
 $(x-3)(3x+4) = 0$
 Hence either $x-3=0$, $x=3$,
 or $3x+4=0$, $x=-\frac{4}{3}$
- 6 $6x^2 + 5x - 4 = 0$
 $(2x-1)(3x+4) = 0$
 Hence either $2x-1=0$, $x=\frac{1}{2}$,
 or, $3x+4=0$, $x=-\frac{4}{3}$
- 7 $3(x-2)^2 = 18 + (8x+1)$
 or, $3x^2 - 12x + 12 = 19 + 8x$
 or, $3x^2 - 20x - 7 = 0$
 $(x-7)(3x+1) = 0$
 Hence either $x-7=0$, $x=7$,
 or, $3x+1=0$, $x=-\frac{1}{3}$
- 8 $x - \frac{x^3-8}{x^2+5} = 2$
 or, $x^4 + 5x - x^3 + 8 = 2x^3 + 10$
 or, $2x^2 - 5x + 2 = 0$
 $(x-2)(2x-1) = 0$
 Hence either $x-2=0$, $x=2$,
 or, $2x-1=0$, $x=\frac{1}{2}$

$$9 \quad \frac{21x^2 - 16}{3x^2 - 4} - 7x = 5$$

$$\text{or,} \quad 21x^3 - 16 - 21x^3 + 28x = 15x^2 - 20$$

$$\text{or,} \quad 15x^2 - 28x - 4 = 0$$

$$(1-2)(15x+2)=0$$

$$\text{Hence either} \quad 1-2=0,$$

$$1=2,$$

$$\text{or,} \quad 15x+2=0,$$

$$1=-\frac{2}{15}.$$

$$10 \quad x^2 - (a+b)x + ab = 0$$

$$(x-a)(x-b)=0$$

$$\text{Hence either} \quad x-a=0,$$

$$x=a,$$

$$\text{or,} \quad x-b=0,$$

$$x=b$$

$$11 \quad (a-b)x^2 - (a+b)x + 2b = 0$$

$$\text{or,} \quad (a-b)x^2 - (a-b)x - 2bx + 2b = 0$$

$$(x-1)\{(a-b)x - 2b\} = 0$$

$$\text{Hence either} \quad x-1=0,$$

$$x=1,$$

$$\text{or,} \quad (a-b)x - 2b = 0,$$

$$x = \frac{2b}{a-b}$$

$$12 \quad \frac{x^2}{\frac{1}{a^2} + \frac{1}{b^2}} - (a^2 - b^2)x = \frac{1}{(ab^2)^{-\frac{1}{2}} + (a^2b)^{-\frac{1}{2}}}$$

$$= \frac{1}{\frac{1}{a^{\frac{1}{2}}b} + \frac{1}{ab^{\frac{1}{2}}}} = \frac{ab}{a^{\frac{1}{2}} + b^{\frac{1}{2}}}$$

$$\text{or,} \quad x^2 - (a-b)x - ab = 0$$

$$(1-a)(x+b)=0$$

$$\text{Hence either} \quad 1-a=0,$$

$$1=a,$$

$$\text{or,} \quad x+b=0,$$

$$x=-b$$

$$13 \quad \frac{2x(a-1)}{3a-2x} = \frac{a}{4}$$

$$\text{or,} \quad 8ax - 8x^2 = 3a^2 - 2ax$$

$$\text{or,} \quad 8x^2 - 10ax + 3a^2 = 0$$

$$\text{or,} \quad 8x^2 - 6ax - 4ax + 3a^2 = 0$$

$$(4x-3a)(2x-a)=0$$

Hence either $4x - 3a = 0$ $x = \frac{3a}{4},$

or, $2x - a = 0$ $x = \frac{a}{2}$

14

$$\frac{10}{x^{\frac{1}{2}}} + \frac{1}{2} = \frac{6}{x^{\frac{1}{2}}}$$

or $32 + x^2 = 12x$

or, $x^2 - 12x + 32 = 0$

or, $x^2 - 8x - 4x + 32 = 0$

$$(x-8)(x-4) = 0$$

Hence either $x - 8 = 0$ $x = 8$

or, $x - 4 = 0$ $x = 4$

15

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{1-c}$$

or, $\frac{a}{x-a} - \frac{c}{1-c} + \frac{b}{x-b} - \frac{c}{1-c} = 0$

or $\frac{x(a-c)}{(1-a)(1-c)} + \frac{x(b-c)}{(x-b)(x-c)} = 0$

or, $\frac{x}{x-c} \left\{ \frac{a-c}{1-a} + \frac{b-c}{1-b} \right\} = 0$

Hence either $\frac{x}{x-c} = 0$, $x = 0$,

or, $\frac{a-c}{1-a} + \frac{b-c}{1-b} = 0$

Whence $x(a-c+b-c) = b(a-c) + a(b-c)$

$$x = \frac{2ab - ac - bc}{a + b - 2c}$$

16

$$\frac{a - \sqrt{2ax - x^2}}{a + \sqrt{2ax - x^2}} = \frac{x}{a - x}$$

$$\frac{a}{-\sqrt{2ax - x^2}} = \frac{a}{2x - a}$$

$$\text{or,} \quad \frac{1}{2ax - x^2} = \frac{1}{(2x - a)^2}$$

$$\text{or,} \quad 4x^2 - 4ax + a^2 = 2ax - x^2$$

$$\text{or,} \quad 5x^2 - 6ax + a^2 = 0$$

$$\text{or,} \quad 5x^2 - 5ax - ax + a^2 = 0$$

$$(x - a)(5x - a) = 0$$

$$\text{Hence either} \quad x - a = 0$$

$$x = a,$$

$$\text{or,} \quad 5x - a = 0$$

$$x = \frac{a}{5}$$



